DESCRIPTION OF AND SELECTED RESULTS FROM THE MAA NATIONAL STUDY OF CALCULUS[1]

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In fall 2010, the Mathematical Association of America undertook the first large-scale study of postsecondary Calculus I instruction in the United States, employing multiple instruments. This report describes the motivation behind the study, its methodology, and several preliminary results.

Keywords: Calculus, Survey

INTRODUCTION

In the fall term of 2010, the Mathematical Association of America (MAA) conducted a national survey of Calculus I instruction across a stratified random sample of two- and four-year colleges and universities under the title *Characteristics of Successful Programs of College Calculus* (CSPCC).[2] The survey was restricted to what is known in the United States as "mainstream" calculus, the calculus course that is designed to prepare students for the study of engineering or the mathematical or physical sciences. Within mathematics, it serves as the initial introduction to calculus, preceding a first course of analysis.

In the United States, Calculus I is viewed as a university-level course. Each fall, approximately 300,000 college or university students (Kirkman, 2012), most of them in their first post-secondary year, take this course. This number has been essentially constant over the past quarter century, as has the number of students graduating with degrees in engineering, the physical sciences, or the mathematical sciences (Lutzer *et al*, 2007; NCES, 2011).

Calculus I is famously perceived to be a filter, discouraging all but the very strongest students from pursuing a career in science or engineering. However, we have had very little data about what happens in Calculus I and the effect of this course on student attitudes toward mathematics or on career intentions. While the Conference Board of the

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Mathematical Sciences (CBMS), the umbrella organization of all professional mathematics societies in the United States, conducts a survey of undergraduate mathematics instruction every five years (see Lutzer *et al*, 2007 for latest published report), the collected data are limited to enrolment numbers, class sizes, and whether the instructor is tenured/tenure track, other full time, part time, or a graduate student. Information as basic as passing rates, anticipated majors, or mathematical preparation of the students has not been available on a national level. CSPCC was conceived to address and correct this lack of knowledge.

In particular, CSPCC has five goals:

1. To improve our understanding of the demographics of students who enrol in calculus,
2. To measure the impact of the various characteristics of calculus classes that are believed to influence student success,
3. To conduct explanatory case study analysis of exemplary programs in order to identify why and how these programs succeed,
4. To develop a theoretical framework that articulates the factors under which students are likely to succeed in calculus, and
5. To use the results of these studies and the influence of the MAA to leverage improvements in calculus instruction across the United States.

To address the first two goals, data were collected through a succession of surveys conducted online during the summer and fall of 2010, supplemented with the collection of final examinations and grades and a follow-up survey conducted in fall 2011. The focus of this report will be on these data. Additional phases of CSPCC will address the other three goals.

The next section describes the survey design, sampling methodology, variables that were studied, and primary method of analysis. The following sections report some of the demographic and attitudinal data that were collected and describe some of the analyses of these data that are now in progress.

SURVEY DESIGN AND SAMPLING METHODOLOGY

Five major online surveys were constructed: one for the calculus coordinator, two for the calculus instructors of which one was administered immediately before the start of the course and the other immediately after it ended, and two for the students in the course (one at the start of the term and one at the end). In addition, instructors reported on the distribution of final grades and were asked to submit a copy of the final exam. One year after the surveys were administered, a short follow up survey was sent to those students who had volunteered their email addresses. No incentives were given for completing the surveys. Links to the surveys can be found at www.maa.org/cspcc.

For the purposes of surveying post-secondary mathematics programs in the United States, CBMS separates colleges and universities into four types, characterized by the highest mathematics degree that is offered: associate's degree (hereafter referred to as two-year colleges), bachelor's degree (referred to as undergraduate colleges), master's degree (referred to as masters universities), and doctoral degree (referred to as research universities). Because enrollments vary so greatly even within each type of institution, CBMS provides further stratification according to the number of full-time equivalent (FTE) undergraduate students. We sampled most heavily at the institutions with the largest enrollments. No for-profit colleges or universities were included in the study. In all, we selected 521 colleges and universities; 213 participated in the study. See Table 1.

Table 1. Sampling and response rates for colleges and universities

Institution type	**Number of institutions**	**Sample size (sample rate)**	**Participants[a] (response rate)**	**Number of substrata by FTE**	**Range of response rates[b]**
Two-year colleges	1121	207 (18%)	54 (26%)	8	17%–42%
Undergraduate colleges	1015	134 (13%)	60 (45%)	5	29%–52%
Masters universities	181	60 (33%)	26 (43%)	4	33%–54%
Research universities	197	120 (61%)	73 (61%)	6	46%–88%

Notes: a. Number of colleges or universities that provided data and percentage of the sample that provided data. b. Range of percentages, by substrata, of the sampled institutions that provided data.

For the purpose of analyzing the effect of the Calculus I program, we had six dependent variables, four of which were measured at both the start and the end of the term. See Table 2. For the first three variables, students were presented with the following statements and asked to indicate their level of agreement:

- I am confident in my mathematical abilities.
- I enjoy doing mathematics.
- This course has increased my interest in taking more mathematics.

Table 2. Dependent Variables

Variable	**How measured**	**Start of term**	**End of term**
Confidence	6-option Likert scale, strongly disagree to strongly agree	X	X
Enjoyment	6-option Likert scale, strongly disagree to strongly agree	X	X
Increased interest	6-option Likert scale, strongly disagree to strongly agree		X
Desire to continue studying mathematics	4-option Likert scale, If I had a choice: "would not continue to study math" to "would continue …"	X	X
Intention to continue Calculus	Options: yes, no, not sure	X	X
Final grade of C or higher	Instructor reported grades		X

The control variables were of two types: student characteristics (*e.g.* gender, parental education, race/ethnicity) and academic background (*e.g.* secondary school experience in mathematics including which courses were taken and what grades were received, score on the Advanced Placement (AP) Calculus exam, scores in SAT and/or ACT college admission exams, year at university, prior mathematics courses at university, and career intention).

The independent variables were collected at the student, instructor/classroom, and institutional levels. See Table 3.

Table 3. Independent variables by level

Level	**Variables**	**Instrument**
Student	Beliefs and attitudes about learning mathematics, study habits, level of intellectual engagement with the course, experience with technology (graphing calculators and/or computer software)	Student surveys
Instructor	Experience and background; beliefs, attitudes, and interests	Instructor surveys
Classroom	Class size; instructional practices; assessment practices; out of class interactions with students; use of technology including use of web resources; textbook; additional instructional resources	Instructor surveys
Classroom	Student perceptions of instructional practices, use of technology, assessment practices, intellectual community outside of class	Student surveys
Institutional	Placement procedures, technological support, institutional support for students, institutional support for instructors	Coordinator survey

From the 213 colleges and universities that participated in the study, there were 663 instructors and over 14,000 students who responded to at least one of the surveys. We have both start and end of term surveys that are linked to each other for 7,260 of the students. There is complete data (all five surveys completed and linked with each other) for 3103 students enrolled with 309 instructors at 125 colleges or universities.

Philip Sadler and Gerhard Sonnert of the Harvard-Smithsonian Center for Astrophysics are undertaking the statistical modeling of the data. The primary tool is a multiple linear regression with hierarchical linear modeling to account for the fact that the data are nested at three levels: student, instructor/classroom, and institution. Sadler and Sonnert are using propensity weighting (Rosenbaum & Rubin, 1983), recognizing, for example, that students who studied calculus in high school probably differ from those who did not on a host of characteristics. Propensity matching or propensity weighting is a statistical procedure that evens out the differences between those who studied calculus in high school and those who did not, providing a clearer picture of the difference attributable to studying calculus in high school. They also have applied methods of variable reduction, such as factor analysis or multidimensional scaling, to create robust composite variables.

While we do not yet have the results from the Sadler and Sonnert analysis, we expect that they will appear soon. In the meantime, we can report on summative data.

RESULTS OF THE INITIAL STUDENT SURVEY

From the fall 2010 CBMS survey (Kirkman, 2012), we know that there were just over 298,000 students who studied Calculus I in college or university. These students are distributed among the four types of institutions as follows: 110,000 students at the research universities, which are dominated by the large state universities; 41,000 at masters universities, primarily public universities that draw their students from within their state and often focus on teacher preparation; 82,000 at undergraduate colleges, a mix of public and private institutions, some with strong national reputations, others with a more regional draw; and 65,000 at two-year colleges, almost entirely public institutions that serve a local community.

In this section, we present several types of summative data: basic demographic data, information on prior course taking, expectations, distribution of scores on college admissions examinations, and intended careers. All of these are organized by type of institution. Table 4 presents a selection of the data that have been collected on demographics, prior course taking, and expectations.

Table 4. Data on demographics, prior course taking, and expectations of entering Calculus I students. Percentages are relative to all Calculus I at that institution type.

Student characteristic		**Research** N = 8077 (73)[a]	**Masters** N = 602 (26)	**Undergrad** N = 1938 (60)	**2year** N = 849 (54)	**Total** N = 11,466 (213)
Female		46%	47%	47%	34%	44%
Age	Mean	18.3	20.5	18.8	22.0	19.7
	SD	2.39	5.29	2.93	7.41	3.48
Race/ethnicity[b]	White	81%	82%	86%	73%	81%
	Black	5%	8%	8%	10%	7%
	Asian	17%	10%	10%	14%	13%
	Hispanic	9%	7%	11%	18%	11%
College year[c]	Freshman	83%	50%	72%	25%	63%
	Sophomore	10%	27%	16%	40%	20%
	Junior	4%	13%	7%	14%	9%
	Senior	0%	5%	3%	4%	2%
	Graduate	0%	2%	0%	3%	1%
Enrolled full time		98%	91%	98%	76%	92%
Took calculus in high school		70%	43%	53%	24%	52%
Earned 3 or higher, AP Calculus exam		26%	9%	14%	5%	16%
Took precalculus in college		13%	31%	17%	60%	27%
Expect to take Calculus II (start of term answer)	Yes	70%	67%	69%	71%	69%
	Don't know	19%	18%	19%	15%	18%

Calculus II is required for intended major	Yes	59%	60%	56%	60%	58%
	Don't know	20%	15%	22%	15%	19%
Believe that to succeed in college calculus, must have studied calculus in high school		49%	36%	40%	37%	42%

Notes: a. First number is the number of students who answered the survey (8077), second is the number of institutions that they represent (73). b. Categories of race and ethnicity are not mutually exclusive. c. Reported percentages do not include category "other".

The differences between institution types in terms of academic preparation of calculus students in also reflected in the scores on the college admission examinations: the SAT Math and the ACT Math. For the purpose of combining these scores, we record percentiles rather than the actual SAT or ACT scores. See Figure 1.

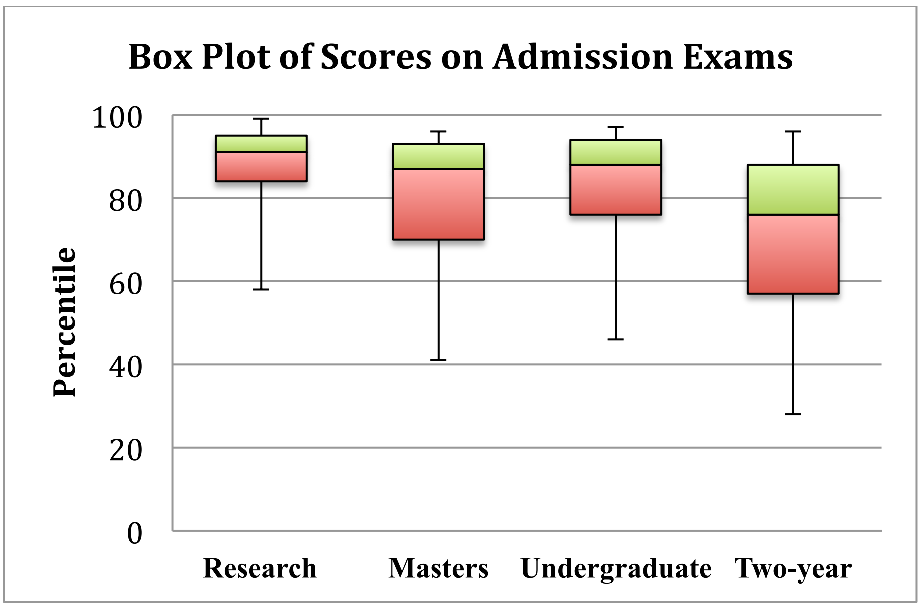

Figure 1: Box plots of scores of Calculus I students on college admission examinations in mathematics.[3] These represent 7,138 students at research universities, 457 students at masters universities, 1,653 student at undergraduate colleges, and 427 students at two-year colleges.

Not all students take a college admission examination. In the survey, exam scores were reported by 88% of the students at research universities, 76% of those at masters universities, 85% of those at undergraduate colleges, and only 50% of the students at two-year colleges.

It is interesting to note that the top half of the students in Calculus I at all of the four-year programs are very comparable in terms of their scores on admission examinations. The differences occur in the spread of scores and are most pronounced at the first quartile. Students in Calculus I at research universities have by far the narrowest range of scores for their college admission examinations.

[3] Whiskers extend from the scores of the students at the 5th percentile to the score of the students at the 95th percentile. In all cases, the actual minimum is 0 and the actual maximum is 100.

Most of the students who take Calculus I in college are heading into a career in Science, Technology, Engineering or Mathematics (STEM). See Figure 2.

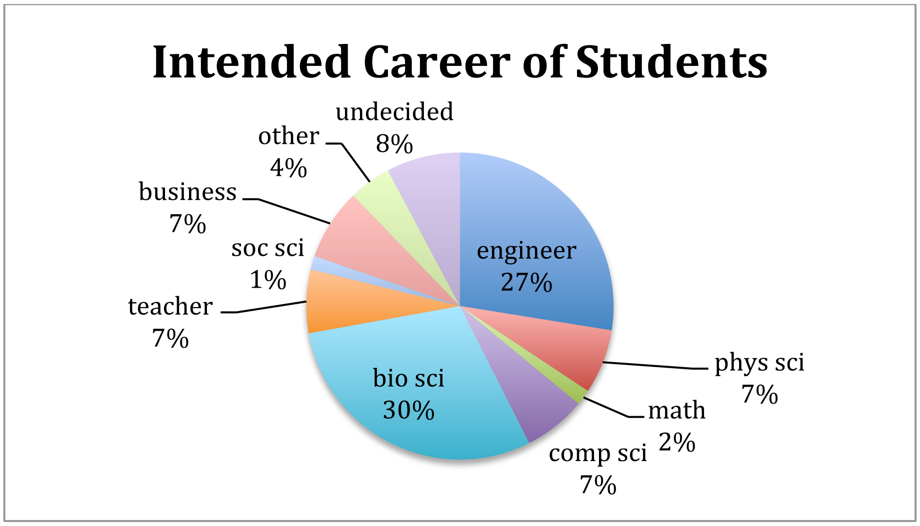

Figure 2. Intended career of students at start of Calculus I

The distribution of intended careers varies by type of institution. At research universities, 76% of Calculus I students are or intend to be STEM majors, with 34% expecting to major in engineering and 31% in the biological sciences. At the other extreme, only 65% of Calculus I students at undergraduate colleges expect to enter a STEM field, 20% heading to engineering and 30% to the biological sciences.

The common Calculus I curriculum in use today generally follows the model adopted by George Thomas at MIT in the early 1950s (Thomas, 1951). It is a curriculum that was designed for students in engineering or the physical sciences, yet these students constitute barely a third of those who take Calculus I today. Many of the students who are heading into careers in engineering or the physical or mathematical sciences begin their college study of mathematics with Calculus II or higher. It is not at all clear that the traditional Calculus I curriculum is appropriate for students going into the biological sciences, many of whom will take only a single course in calculus.

STUDENT GRADES

While we did collect individual grades for students where we could, issues of student confidentiality made this difficult. We only have instructor reported grades matched to student surveys for 1224 students. However, at the end of the term, we asked the instructors to report the number of students who received each of the grades A, B, C, D, or F or withdrew from the course after the second week (designated as W). The grades DFW are grouped because they all indicate that the student is not prepared to continue to any course that has Calculus I as a prerequisite. We do have good data about the distribution of grades by type of institution. See Figure 3. Note the grade distributions at research universities and undergraduate colleges are comparable, as are the grade distributions at masters universities and two-year colleges.

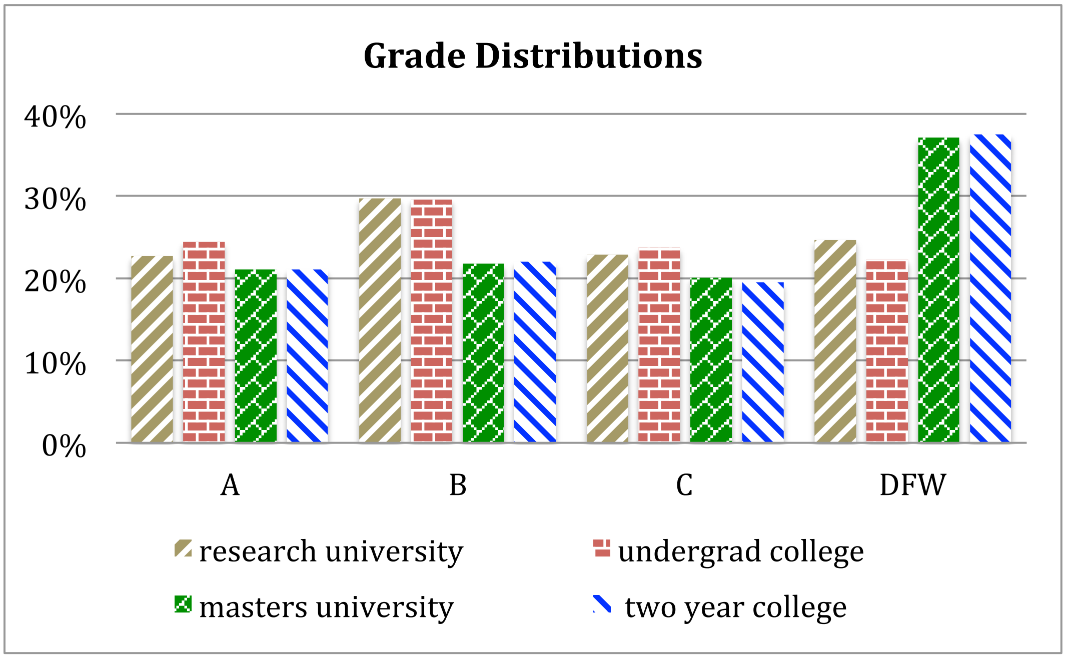

Figure 3. Grade distributions by type of institution.

The number of instructors reporting grades and the number of students for whom these grades were reported are presented in Table 5.

Table 5. Number of instructors reporting grades and number of students for whom grades were reported.

	Research	**Masters**	**Undergraduate**	**2-year**	**Total**
# of instructors	360	73	118	112	663
# of students	17,170	2,220	3,290	2,184	24,864

SHIFTS IN STUDENT CONFIDENCE, ENJOYMENT, AND DESIRE TO CONTINUE

As previously described, one of the purposes of these surveys was to determine what happens to student confidence in mathematical abilities, enjoyment of mathematics, and desire to continue studying mathematics. Identical questions addressing these issues were asked at the start and end of the course. Because of the need to maintain student anonymity, we matched surveys by asking students for their five-digit zip code (postal code) at the time they graduated from high school and for their birth date.

Not surprisingly, the response rates were much lower at the end of the term than they had been at the beginning. There also was a very noticeable selection bias: The students who were inclined to answer the survey at the end of the term were those students who had been doing well in the course. Of the students for whom it was possible to match the start of term and end of term surveys, roughly 40% were earning A, 40% B, and 20% C.[4] Although these represented the top half of the class, there was a very dramatic decrease in confidence, a significant decrease in enjoyment, and some decrease in desire to continue in mathematics. See Tables 6–10. The effect size is the fraction of a standard deviation that the mean shifted.

Table 6. Change in dependent variables from start to end of term. All institutions. N = 3436 students, 168 institutions.

Variable	Pre-mean (SD)	Post-mean (SD)	Effect size of change
Confidence[a]	4.89 (1.01)	4.42 (1.18)	– 0.47
Enjoyment[a]	4.63 (1.27)	4.28 (1.37)	– 0.28
Desire for more mathematics[b]	2.93 (1.02)	2.84 (1.08)	– 0.09

Notes: a. Measured on 1–6 Likert scale. B. Measured on 1–4 Likert scale.

Table 7. Change in dependent variables from start to end of term. Research universities. N = 2019 students, 66 universities.

Variable	Pre-mean (SD)	Post-mean (SD)	Effect size of change
Confidence[a]	4.93 (1.01)	4.40 (1.19)	– 0.52
Enjoyment[a]	4.69 (1.24)	4.28 (1.35)	– 0.33
Desire for more mathematics[b]	2.97 (1.00)	2.83 (1.07)	– 0.14

Notes: a. Measured on 1–6 Likert scale. B. Measured on 1–4 Likert scale.

Table 8. Change in dependent variables from start to end of term. Masters universities. N = 193 students, 21 universities.

Variable	Pre-mean (SD)	Post-mean (SD)	Effect size of change
Confidence[a]	4.85 (1.09)	4.51 (1.15)	– 0.31
Enjoyment[a]	4.72 (1.35)	4.51 (1.33)	– 0.15
Desire for more mathematics[b]	3.00 (1.03)	3.10 (0.97)	0.10

Notes: a. Measured on 1–6 Likert scale. B. Measured on 1–4 Likert scale.

Table 9. Change in dependent variables from start to end of term. Undergraduate colleges. N = 952 students, 41 colleges.

Variable	Pre-mean (SD)	Post-mean (SD)	Effect size of change
Confidence[a]	4.80 (1.00)	4.44 (1.17)	– 0.36
Enjoyment[a]	4.43 (1.33)	4.15 (1.43)	– 0.21
Desire for more mathematics[b]	2.76 (1.07)	2.72 (1.07)	– 0.04

Notes: a. Measured on 1–6 Likert scale. B. Measured on 1–4 Likert scale.

[4] These are self-reported anticipated grades.

Table 10. Change in dependent variables from start to end of term. Two-year colleges. N = 272 students, 40 colleges.

Variable	Pre-mean (SD)	Post-mean (SD)	Effect size of change
Confidence[a]	4.93 (1.02)	4.50 (1.13)	– 0.41
Enjoyment[a]	4.87 (1.15)	4.73 (1.18)	– 0.12
Desire for more mathematics[b]	3.21 (0.95)	3.20 (0.97)	– 0.01

Notes: a. Measured on 1–6 Likert scale. B. Measured on 1–4 Likert scale.

As previously stated, these are the changes in attitudes of the best students, mostly those who finished in the top half of the class

It is intriguing that students at two-year colleges are the ones who start and end Calculus I with the greatest confidence, enjoyment, and desire for more mathematics. Students at masters universities show the least decrease in these variables and even some increase in the desire for more mathematics, though the relatively small N does raise questions about the robustness of these data. It is the students at the research universities who show the greatest losses, even though they are the best prepared.

We expect that the model now being developed by Sadler and Sonnert will provide some insight into what is happening and suggest possible causal relationships.

OTHER ANALYSES OF THE SURVEY DATA

Members of the CSPCC team have undertaken two analyses of selected pieces of the data that have been collected. Chris Rasmussen, Jess Ellis and Kristin Duncan have analyzed survey data from 7,260 students for whom the start of term and end of term student surveys could be matched to better understand who switches out of a STEM major and their reasons for doing so (Rasmussen, 2012; Bressoud *et* al, 2012). As mentioned previously, the students for whom we have matched surveys are predominantly A, B, or—to some extent—C students, generally in the top half of the class. Rasmussen *et al* found that even in this group of successful students, of those who entered Calculus I with the intention of majoring in a STEM discipline that required a second semester of calculus, one out of eight changed their mind about continuing on to the next course of calculus. The percentage of such "switchers" was twice as high for women as for men and was considerably higher at research universities with enrollments over 20,000 (17.5%) than at other types of post-secondary institutions.

As part of their study, Rasmussen *et al* analyzed the responses to the surveys that were sent out in fall 2011 to those who had volunteered their email addresses. This was one year after the initial study. This survey was returned by 1230 students. Among the representative comments from those who had decided not to continue on a STEM trajectory, students reported that the teaching of Calculus I was ineffective and uninspiring, the course was "over stuffed" with content and delivered at too fast a pace, assessments were poorly aligned with what was taught, and the instructor lacked connection to students and the course.

Michael Tallman and Marilyn Carlson have analyzed the final exams that were submitted by instructors immediately after the completion of the course (Tallman *et al.*, preprint). There were 253 submitted exams of which 150 were randomly selected for analysis. Among their findings was that most exam questions are straightforward, requiring no more than recall and application of a procedure, and that this was true to a much greater extent than instructors had realized when asked to describe the nature of their examinations. Most instructors had estimated that such questions constituted 40–70% of their examinations. Analysis of the examinations found that such questions accounted for 71–88%.[5]

CONCLUSION

These are still early days in the analysis of the massive data set collected in the CSPCC surveys. The greatest challenge will be to identify those factors at the student, instructor/classroom, and institutional level that are correlated with student success in calculus and desire to continue on a trajectory toward a career in a STEM discipline.

As evidenced in Sandra Laursen's study of Inquiry Based Learning (IBL) (Laursen *et al*, 2011), the effectiveness of certain classroom strategies may be dependent on student characteristics. In particular, she found that employing IBL in mathematics classes made the greatest positive difference for women and students from groups that are traditionally underrepresented in STEM disciplines. There is every reason to expect that our study also will uncover strategies that have their greatest impact on particular subpopulations. The distinct demographics of calculus students in two-year colleges and masters universities suggest that these institutions may present a different set of challenges from those of the research universities or the undergraduate colleges.

It is particularly discouraging that research universities, which teach Calculus I to more students than any other category of post-secondary institution and which teach this class to the best prepared of our students, also seem to be doing the worst job in maintaining student confidence in their mathematical abilities, enjoyment of mathematics, and interest in continuing with the mathematics that is needed to pursue their intended careers. These universities deserve particular attention. Fortunately, this is also the group for which we had the best response rates and have collected the most complete data.

Finally, for the very first time we know the national failure rate in Calculus I: 28% D, F, or W. This amounts to almost 85,000 students each fall. We view this to be disturbingly high. Something is wrong either in the procedures for deciding who is admitted to this course or in its instruction, or both.

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[5] [40%–70%] and [71%–88%] are the interquartile ranges of the instructor estimates and the results of the exam analyses, respectively.

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