The Impact of Instructor Pedagogy on College Calculus Students' Attitude Toward Mathematics

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Abstract

College calculus teaches students important mathematical concepts and skills. The course also has a substantial impact on students' attitude toward mathematics, affecting their career aspirations and desire to take more mathematics. This national study of 3,103 students at 123 colleges and universities tracks changes in students' attitudes toward mathematics during a "mainstream" calculus course, while controlling for student backgrounds. The attitude measure combines students' self-ratings of their mathematics confidence, interest in, and enjoyment of, mathematics. Three major kinds of instructor pedagogy, identified through the factor analysis of 61 student-reported variables, are investigated for impact on student attitude: (1) Instructors who employ generally accepted "good teaching" practices (e.g., clarity in presentation and answering questions, useful homework, fair exams, help outside of class), are found to have the most positive impact, particularly with students who began with a weaker initial attitude. (2) Use of educational "technology" (e.g., graphing calculators, for demonstrations, in homework), on average, is found to have no impact on attitudes, except when used by graduate student instructors, which negatively affects students' mathematics attitudes. (3) "Progressive pedagogy" (e.g., group work, word problems, "flipped" reading, student explanations of thinking) has a small negative impact on student attitudes, while being a relatively more constructive influence only on students who already enjoyed a positive attitude toward mathematics and in classrooms with a large number of students. This study provides support for efforts to improve calculus teaching through the training of faculty and graduate students to use traditional "good teaching"

practices through professional development workshops and courses. As currently implemented, technology and progressive pedagogical practices, while no doubt effective in certain classrooms, do not appear to have a reliable, positive impact on student attitudes toward mathematics.

I. Introduction

Each year, more than 300,000 2- and 4-year college and university students enroll in introductory college calculus (Kirkman, 2012). Calculus I is required for most STEM (Science, Technology, Engineering, Mathematics) majors, as well as those in science-related fields or programs (e.g., pre-medicine). For many students, introductory calculus is but the first in a sequence of college mathematics courses that they are required to take. Their prior experience in high school often motivates them to continue their studies in mathematics, as most have done well in earlier coursework and standardized tests (SAT, ACT). However, continuing with mathematics is thought to be influenced by their earlier performance, but also their attitude towards mathematics, i.e., whether their confidence and interest is maintained (or even increased) in college courses, and whether they find the learning of mathematics that they study enjoyable.

Those who teach college calculus are aware that, for many students, introductory calculus is a daunting course, one in which the pace is often quick and the coverage wide. Many professors go to great pains to teach well, using new techniques and innovations that are reported at professional meetings, or incorporating the latest in technological tools and supports. Others rely more upon traditional methods, striving to improve the clarity and organization of their lectures, while insuring that additional support and resources are available for those students who

desire it. While much has been written about the need for improvements in the teaching of calculus, there have been precious few multi-institution studies in which the impact of the innovations promoted are measured. The goal of this study is to measure changes in student attitudes from the start to the end of Calculus I courses in a national sample of colleges and students and to relate any differences to the specific teaching approaches used in the course, while controlling for variables beyond the purview of the calculus professor.

II. Literature review

What is the goal of mathematics education? Certainly, gaining mathematical knowledge needed for careers or citizenship are most frequently touted (Moses, 1995). Yet, the affective components of learning may be equally important in that they provide desire and motivation to continue to pursue study (Atkin & Helms, 1993; Neale, 1989; Wilkins & Ma, 2003). The attitudes of mathematics learners have long been studied (Zan, 2008). Walberg (1984) found that three groups of variables predicted students' attitudes toward schoolwork: instructional (e.g., type, amount, quality), environmental (e.g., classroom, peers), and personal (e.g., ability, development, motivation). Walberg's findings provide a theoretical framework for the study of changes in attitude experienced by students enrolled in college calculus. The instruction received by calculus students impacts their attitudes through the decisions that their instructor makes about teaching approach (e.g., lecture, discussion, use of technology) and time spent on different aspects of the course (e.g., class time, homework, project work). However, factors beyond the instructor's control may well have a large influence on changes in student attitudes and confound any study unless a methodology is used in which such factors are controlled. Environmental factors might include the number of students in the class, availability of technology for teaching, and even the background of the instructor assigned to teach the course (graduate student or

professor, years teaching). Personal factors may include the students' facility with the prerequisite mathematics with which they enter the calculus classroom, their attitudes at the start of class (i.e., also product of earlier experiences learning mathematics), and the students' intention to major in a particular field.

We know something about the attitudes that mathematics students hold. Those ready to take pre-calculus or calculus in high school view mathematics more positively than do their peers who avoid these advanced courses (Wilkins & Ma, 2003). Teacher skills in presenting lessons with clarity impact students by reducing apprehension and increasing positive affect (Cheseboro, 2003). College students studying to be elementary teachers exhibit a tremendously wide range of affect concerning mathematics, from viewing mathematics as fun to "feeling physically sick before a math class (Harkness, D'Ambrosio & Morrone, 2007, p. 243)." Struggling to understand and keep up was a theme for many studying to be teachers. Yet, struggle followed by an end result of success was positive when students were "allowed the freedom to share ideas, ask questions, and make mistakes (p. 250)."

The metaphor of a "leaky pipeline" is often used when describing the steady decrease in the number of students who wish to pursue STEM careers (Sadler, Sonnert, Hazari & Tai, 2012). Retention is a particular issue with under-represented minorities (Riegle-Crumb, Moore, & Ramos-Wada, 2011). This loss impacts the entire nation, because the American educational system does not produce enough young people to fill all available STEM jobs and because the lack of diversity may adversely impact the solution to pressing scientific and engineering challenges (Sadler et. al, 2011). Critical to maintaining the desire for a STEM career is that

individuals grow to exhibit high levels of engagement and interest in science or mathematics (Bandura, 1986; Fouad & Smith, 1996; Fouad, Smith, & Zao, 2002; Lent, Brown, & Hackett, 1994, 1996; Tai, Liu, Maltese & Fan, 2006) leading to positive self-perceptions and identity as a science or math "person" (Hazari, Sonnert, Sadler, & Shanahan, 2010). Because a STEM major requires both enrolling in and passing many mathematics courses (often beginning with calculus), the degree to which this gateway course positively (or negatively) affects engagement, interest, confidence, and enjoyment has an outsized impact on students' subsequent career trajectories into (or out of) STEM.

III. Research questions

The goal of this research is twofold. First, we examine a host of pedagogical variables reported on by calculus students and reduce them to a small number of underlying characteristics using factor analysis. Second, we investigate to what extent these characteristics influence students' attitudes about mathematics when present in a college calculus course, using hierarchical linear modeling.

IV. Data and Methods

Data

The data used in this article are drawn from the Characteristics of Successful Programs in College Calculus (CSPCC) project, a major national study of "mainstream"[1] college calculus conducted by the Mathematical Association of America (MAA) with support from the National

[1] "Mainstream Calculus" are courses that are the prerequisites for more advanced mathematics courses and are required for most STEM. "Non-Mainstream Calculus" are other calculus courses, often taught for business or social science majors.

Science Foundation (Bressoud, Carlson, Mesa & Rasmussen, 2013). This study included two student surveys (one at the beginning and the other at the end of the calculus course), two instructor surveys (again one at the beginning and the other at the end of the calculus course), and one survey of course coordinators at participating institutions. The distribution of introductory calculus students among U.S. institutions of higher learning is best thought of as being stratified by the highest degree offered by each. CBSS (Conference Board of the Mathematical Sciences) classifications were used to categorize schools (Blair, Kirkman, Maxwell, 2013): associate's degree (also referred to as two-year colleges), bachelor's degree (referred to as undergraduate colleges), master's degree (referred to as masters universities), and doctoral degree (referred to as research universities).

Half of American students taking mainstream calculus are enrolled in universities offering graduate degrees in mathematics (Table 1). These make up only 15% of all calculus teaching institutions in the U.S. Almost 600 students are enrolled in calculus, on average, in each Ph.D. granting university and more than 200 in each M.S. granting institution. This can necessitate large lecture-style courses, or alternatively many smaller sections at each school. At the other end of the scale, only 22% of students taking calculus do so at 2-year colleges, nationwide. However, these courses are taught at more than 1,100 junior and community colleges. Classes there may be much smaller or less frequently taught.

[insert table 1]

Stratified random sampling was carried out to provide a national sample of colleges and universities for the CSPCC project. Institutions were recruited by the Mathematics Association of American (MAA). Details of the recruitment process and yield are described by Bressoud,

Carlson, Mesa, and Rasmussen (2013). The complexity of getting students to take both a pre- and post-survey, instructors to take a pre-survey and post-survey, and the institution department chair, or calculus coordinator to contribute to a survey led to many incomplete observations. Overall, 13,965 students engaged in some way attending 213 institutions (Table 2). However, the final sample with all five surveys completed, which was used in this article, included 3,103 students in 308 classrooms at 123 institutions.

[insert table 2]

Ph.D. granting universities ended up being over-represented in the sample. They constitute half of our sample of institutions, as compared with 8% of all institutions offering mainstream calculus. Likewise, students taking calculus at Ph.D. granting institutions make up 60% of the CSPCC sample, whereas 37% of those taking calculus in the U.S. are taking them at these institutions (Figure 1). This disparity suggested to use statistical methods for analysis that control for differences by institution.

[insert figure 1]

A second way to check on the representativeness of the sample used in this study is to compare it to another national study of introductory college calculus. The "Factors Influencing College Success in Mathematics" (FICSMath) project conducted a study using similar stratification criteria to build a national sample with three categories of size based on number of students (i.e., <5,400; between 5,401 and 14,800; >14,800) and whether they were 2- and 4-year institutions (Barnett, Sonnert, & Sadler, 2012). FICSMath more closely represented the national statistics of student enrollment in calculus. Student compliance was excellent because only a single pre-survey was required in the study and was filled out in calculus class rather than

outside of class. The FICSMath sample collected data from nearly all students within the first two weeks of the semester. The FICSMath and CSPCC samples are compared in Table 3. While MAA-CSPCC had a greater number of institutions and instructors involved in pre-surveying than FICSMath, the average number of students/instructor was substantially lower (21 versus 30). This is probably the result of the difference in administration, on-line for CSPCC versus in-class for FICSMath. Also, the larger number of 2-year college students in the FICSMath sample is reflected in the statistics of student characteristics for each study. Fewer FICSMath thanCSPCC students had taken calculus (40% vs. 54%) or precalculus (78% vs. 88%) in high school, as well as precalculus in college (12% vs. 32%) or were taking college calculus for a second time (16% vs 11%). Only 48% of FICSMath were in their first college year versus 73% of CSPCC students. FICSMath subjects also had lower SAT and ACT scores.

When using longitudinal data collection one must consider any differences in the group who fill out the initial survey compared to those who complete all surveys. In this data, a substantial decrease in participation is evident from pre- to post-survey in the number of: institutions (-17%), instructors (-27%), and students (-66%). The reduced number of students taking a post-survey halved the average number of students per instructor. Luckily, there was little difference between the background of these two groups in HS coursework, prior college calculus, gender, race/ethnicity, year in college, or standardized test scores. This is reassuring in that those students who did not return a post-survey appear little different from those who did, with the exception that those who enrolled previously in college precalculus were far less represented (4% vs. 12%). It is unknown whether they disproportionately dropped out of the courses or for some other reason did not fill out a post-survey.

[insert table 3]

Variables

Dependent variables

The dataset contained four variables that were measured in both the pre- and post-survey and concerned the students' attitude toward mathematics. Three of them were about the students' personal view of mathematics, dealing with confidence, enjoyment, and persistence. The fourth asked for the students' intention to take Calculus II, the next mathematics course in the calculus sequence. Because each question was asked on the pre-survey and again on the post-survey, we could measure any change in attitude that took place over the term. Students rated these four attitude statements:

- "I am confident in my mathematics abilities" (6-point Likert scale from "strongly disagree" to "strongly agree"
- "I enjoy doing mathematics." (6-point Likert scale from "strongly disagree" to "strongly agree"
- "If I had a choice, I would never take another mathematics course," to "If I had a choice, I would continue to take mathematics." (on a 4-point scale)
- "Do you intend to take Calculus II?" (yes, no, don't know yet)

If these four variables represent different aspects of the students' underlying attitude towards mathematics, it was reasonable to assume that they might be combined into composite variables that represent broader and more robust indicators of that attitude. Factor analysis of the post-survey variables, employing Varimax rotation after normalizing each variable, resulted in a two-factor model (Table 5) in which confidence, enjoyment, and choice are grouped together in

factor D1 and CalcII was placed in a separate factor, D2. This result surprised us in that the desire to take Calculus II appeared unrelated to the other student attitudes.

[insert table 4]

This result also prompted us to examine in greater detail the institution exhibiting the strongest gains on that latter variable over the course of the semester. To our surprise, we found that it is an institution at which *every* student is required to take Calculus II. We surmised that these measured “gains” had little to do with the pedagogy used in Calculus I, but were caused by initially oblivious students wising up to requirements. In general, because course taking intentions may be strongly influenced by study requirements and/or the students' awareness thereof, this variable appears less suited as indicators of the quality of students' educational experience in Calculus I. Factor D2 is better thought of as being related to students' awareness of institutional or major requirements and appears only indirectly related to their attitude toward mathematics. Because of this problem with the variable that became obvious during our analysis, we do not report on it. We also did not use the grade earned in college calculus because we were interested in the effects of instructors' pedagogical characteristics, and grades are to a large extent controlled by the instructors themselves. Putting it hyperbolically, the "worst" calculus instructor in America awards grades of “A” in his or her class, just as does the "best" instructor—and the average grades achieved in their respective classes might not even differ by much.

All three original post-survey variables loading on D1 were standardized, with a mean of zero and a standard deviation of 1, and added; then the summed variable was standardized again. The pre-survey counterpart of mathematics attitude was constructed analogously. Finally, the

mean of post-survey composite was adjusted downward to compensate for the average decline in attitude, and its standard deviation was adjusted to be in units of the pre-survey attitude composite. The resulting composite forms the main dependent variable (mathematics attitude) for subsequent analyses.

Control variables

In correlational studies such as this, it is important to control for variables that can be reasonably expected to affect the outcome variable. These variables include students' background characteristics, such as their gender (0=female; 1=male) and race/ethnicity, represented by the dummy variables for Hispanic, Black, Asian, and Other, with White serving as the baseline. (All students identifying as Hispanic were put into the Hispanic category and counted in no other category, so that the other categories are, more precisely, Non-Hispanic Black, Non-Hispanic Asian, etc., though we drop the "Non-Hispanic" part for convenience.) Students' socioeconomic status (SES) was modeled by the average of their parents' educational levels (on a 5-point scale: 1=Did not finish high school; 2=High school; 3=Some college; 4=Four years of college; 5=Graduate school). If educational information was missing for one parent, the information for the other parent was used as the SES indicator. Several variables described the students' prior mathematics experience and preparation in high school. These included the students' grades in their most advanced mathematics class during high school (4.33=A+, 4=A, 3.67=A-, etc.) and dummies indicating whether the student had attended a non-AP calculus, AP Calculus AB, or AP Calculus BC class, respectively.

The students' SAT mathematics score (200-800) was also used in our analysis. If students reported no SAT mathematics score, but an ACT mathematics score, the latter was mapped onto

the SAT mathematics scale, following the College Board (1999) concordance. Furthermore, we included potentially relevant college-related characteristics of students, such as the students' year of college, represented by the dummies sophomore, junior, senior, graduate student, and special student, with freshman as the baseline, and whether the students had previously taken a college precalculus class or a college calculus class, also represented by two dummies. The students' career plans were modeled by three dummies—engineering (including computing), science and mathematics, and medicine (including health)—with other career intentions serving as the baseline. The mentioned beginning-of-semester attitudinal counterpart to the end-of semester dependent variable, of course, also appeared as a control in our models.

In addition, two control variables at the classroom/instructor level were included: the number of students in the college calculus class and the academic status of the instructor. The latter was modeled by a dummy variable distinguishing between graduate teaching assistants (coded as 1) and instructors of all other statuses (coded as 0).

Variables of interest

The student post-survey asked a multitude of questions about their calculus instructor's pedagogical practices, behaviors, and characteristics. We had available 61 variables in this area. The means of the student ratings of their individual instructor on each of these variables were used as indicators of the instructor's characteristics. To reduce this large number of variables to a smaller number of meaningful and robust composites, exploratory factor analysis was used. A series of solutions with a different number of factors was run. Guiding considerations were parsimony and interpretability of the factors. We decided upon a three-factor solution (which explained 49.3% of the variance). The three factors were VARIMAX rotated, and variables with

loadings below .4 were excluded from the composites to be formed. Thus, for the first composite, 22 variables were retained; for the second, 17; and for the third, 14, whereas 8 variables were dropped from inclusion in any of these three factors.

As indicated by the factor analysis, the appropriate variables were then combined into three composites of instructor characteristics. To make the different formats of the individual variables commensurate, all variables were standardized (with a mean of zero and a standard deviation of 1), before adding them together. (Variables that loaded negatively on a factor were inverted before addition.) The resulting composites were again standardized to ease comparison and interpretation.

Table 6 shows means and standard deviations of the variables used in our analysis.

[insert table 6]

Method

The fact that the CSPCC sample is not representative in terms of the type of institution and number of students taking introductory calculus nationally requires heightened attention to the scope of possible statements and a carefully controlled method of analysis. Whereas general characterizations of the American calculus students are problematic because of over- and under-representations, statements about the relationship between variables are still possible if controlled for over- and under-represented student characteristics. We are primarily interested in explaining the variance in students' attitude at the end of their calculus course through differences at the course/instructor-level. This suggests the employment of a statistical method that helps to account for differences at the institutional level, while controlling for subjects' background. Hierarchical linear modeling (HLM) is an appropriate method here, because it

allows us to analyze data that is structured at several levels (in our case, students within classrooms within institutions).

Issues of analysis

Several issues arise in analysis of the CSPCC dataset that are discussed below.

Missing Values

Even though the individual variables included in our multivariate models typically have quite low percentages of missing values (the average percentage being 2.3% and the highest percentage being 13.3%), the percentages accumulate and lead to rather high losses in the number of subjects (28.5%), when using the common method of listwise deletion of observations with any missing data. Instead, the method of multiple imputation of missing values was used (Rubin 1976, 1987, 1996). This approach creates multiple datasets, each of which is identical to the others as far as non-missing values are concerned. In place of a missing value, however, the method inserts a value that it imputes. This replacement value is different for each of the datasets, with the distribution of the replacement values representing the uncertainty about the missing value. Thus, the method creates several datasets—each of which is complete, but differs from the others in the values that have replaced the originally missing values. Each of these datasets is then subjected to the desired statistical procedures, which of course produce somewhat different parameter estimates in each case. In the last step, the different parameter estimates are combined to produce final estimates with appropriate standard errors (see Allison, 2002). The multiple imputation was implemented through PROC MI and PROC MIANALYZE in the SAS 9.3 statistical software package.

Goodness of Fit

Establishing how well our models fit the data is rather tricky. Whereas, for ordinary least square multiple regression, the R^2-statistic serves as the widely accepted measure for the goodness-of-fit of the estimated model (intuitively interpretable as the proportion of the variance in the dependent variable that is explained by the model). Measuring model quality becomes more complicated for hierarchical models that partition the overall variance. For our hierarchical linear models, we chose a pseudo-R^2 that consists of the squared correlation between the observed and the predicted values of the dependent variable (Singer & Willett, 2003, p. 102).

V. Results

Simple statistics

Before building multivariate models to investigate what effects different calculus classroom pedagogical practices had on students' mathematics attitudes, we note the changes in students' mathematics attitudes (i.e., mathematics confidence, enjoyment, and persistence) that occurred between the beginning and end of their calculus courses. All changes were in the negative direction. The students' mathematics confidence dropped, on average, 0.47 points on the 6-point rating scale, corresponding to an effect size of -0.47 (in units of the standard deviation of the pre-course rating). The students' self-reported enjoyment of mathematics dropped 0.35 points (or -0.27 in the effect size metric). The students' desire to persist in studying mathematics fell 0.09 points of the 4-point rating scale (effect size: -0.09). Finally, the mathematics attitude change composite was -0.31 (in units of the standard deviation of the pre-course composite).

[insert Table 5]

Multivariate analysis

As mentioned, the nested nature of this dataset (students within classes within institutions) suggested a hierarchical modeling approach. Because the dependent variable is continuous, hierarchical linear models are appropriate. At the beginning of the analysis, unrestricted means models were run to determine how the outcome variance was partitioned into the three levels (students, classrooms, institutions). As one might expect, the lion's share of the variance of the dependent variable (students' mathematics attitude) was associated with the student level: Prior experience with, preparation in, and, most of all, prior attitudes towards mathematics powerfully shape student outcomes. Nonetheless, there was significant variance at the institutional and class (or instructor) levels, and this article focuses on the effects of pedagogical characteristics at the latter level.

Our factor analysis (described in the Data and Methods section) reduced the multitude of measured classroom and instructor characteristics to three factors. Table 7 shows all the component variables of the resulting three composite indicators of instructor characteristics, together with the rotated factor loadings of these variables. Inspection of the variables loading on the first factor suggests that they represent what can be considered traditionally accepted good teaching practices (e.g., providing explanations that the students understood, and listening carefully to students' questions and comments). We call the composite derived from this factor "good teaching." The second factor appears to aggregate variables related to the use of instructional technology (e.g., using graphing calculators, or computers). We call it "technology." Finally, the constituents of the third factor appear to be associated with pedagogical reform and novel approaches that aim at increasing the interactivity of the classroom experience and its

relevance (e.g., emphasis on group work, students' explaining their thinking, and having class discussions). We call this composite "progressive pedagogy."

[insert table 7]

Table 8 summarizes the results of our regression models for the mathematics attitude composite. All models resulted in only two levels (students and classrooms) because no variance remained associated with the third level (institutions). While this may appear at odds to our desire that the model include an institutional level to control for the deviation of the sample from national representativeness, the inclusion of a classroom level appears to have the effect of controlling by institution. It absorbs the variance at the institutional level, because these two levels are related. In many colleges, there is but a single instructor teaching calculus, so the addition of an institutional level in model can explain little additional variance. Also, multiple classrooms within a single institution may be very similar to each other (e.g., number of students, level of instructor, access to technology) so that the institutional level would add little after taking the classroom/instructor level into account. In this way, the classroom level does control for differences from the national population of calculus takers. (We also ran a parallel set of models in which we included the institutional, but not the classroom/instructor level, which yielded very similar results.)

We present both a main effects and an interaction model in Table 8. Each of these models of mathematics attitude explains more than 50% of the variance.

[insert table 8]

Surveying briefly the statistically significant main effects of the various control variables on the outcomes, male students had significantly higher ratings on the mathematics attitude

composite. Considering the students' racial/ethnic backgrounds, only the "Other" racial group reported lower scores than did Whites on the attitude composite. As expected, the effects of strong prior mathematics experiences and preparation, such as taking more rigorous calculus courses in high school and achieving a good grade in the most advanced high school mathematics class, were powerful and pervasive: For instance, a higher grade in high school mathematics predicted a more positive mathematics attitude.

It is noteworthy that having taken either a *college* precalculus or *college* calculus course previously had no significant effect. Sophomores and graduate students reported a more positive mathematics attitude than did freshmen. The students' career plans had the expected effects, with students bound for a science or mathematics career scoring higher on mathematics attitude than did students with a career interest outside the STEM fields.

Among the two classroom/instructor-level variables, class size was not significant, whereas having a graduate teaching assistant proved deleterious to students' mathematics attitude.

The students' initial attitude is a powerful predictor of their attitude at the end of the semester. This suggests that this attitude is fairly viscous, which one might expect if one realizes that mathematics education is a protracted and incremental process that, at that point, has been going on for more than a decade.

We now turn to the variables of interest. In the main effects models, "good teaching" had a positive effect. The use of "technology" was not significant. "Progressive pedagogy" had a negative effect. In terms of relative effect sizes, the positive effect of "good teaching" far

outweighed the negative effect of "progressive pedagogy," the former being nearly three times as large.

We also estimated a series of models with various sets of interactions: among the instructor characteristics themselves, between the instructor characteristics and the students' prior attitudes, between the instructor characteristics and the students' SAT/ACT mathematics score, and between the instructor characteristics and the classroom/instructor level controls (class size and instructor status). All interactions thus identified as significant were then entered into a single model, and the ones that dropped below significance level were deleted. Model 2 includes the remaining four significant interactions.

Two of them are between the students' prior attitudes and instructor characteristics. Good teaching improves the mathematics attitudes of students with initially negative attitudes more than it does the attitudes of students who already came to the calculus class with a more positive attitude. By contrast, progressive teaching is more beneficial (in terms of influencing students' mathematics attitudes) for students with initially more positive attitudes than for students with initially more negative attitudes. In addition, a graduate teaching assistant who uses a lot of technology is a particularly bad combination. Finally, progressive teaching works better in larger than in smaller classrooms.

VI. Discussion

The students' mathematics attitudes took decreased substantially (close to a third of a standard deviation), on average, during their college calculus course (Table 5). To some degree, this trend might be owed to a general "reality check" that occurs when students enter college. The weaker high school students are no longer present, and the levels of challenge and expected

performance are raised. Some calculus professors may accept the decline in students' mathematics attitudes as a fact of life, others may bemoan it—but, as we found, professors do exert at least some degree of influence on the trend. It would be interesting to compare the calculus results with those in other disciplines, such as physics and other STEM fields, to determine to which extent the drop in students' mathematics attitudes parallels trends in other fields, and to which extent it might be unique to mathematics.

Going beneath the surface level of declining averages, our multivariate analyses explored what difference students' experiences in the college calculus classroom made in shaping their mathematics attitudes, after controlling for many of their background characteristics and prior experiences. In college calculus, the amount of technology use does not influence students' attitudes about mathematics one way or the other. In this respect, the "math wars" controversies concerning the use of technology turn out to be not really relevant. As to "good teaching," it may not come as a surprise that these practices improve students' attitudes about mathematics. The underlying characteristics are what most people would consider traits and behaviors of good teachers—and what students typically appreciate in their teachers. What may require more probing is why "progressive pedagogy" is negatively related to students' attitudes. Is it that students tend to dislike professors' attempts at engaging them in more active modes of participation? Do they resent having to talk or listening to other students talk in class? Do students not share their instructors' ideas about what is good for them (the students)? Is progressive pedagogy often implemented poorly or in ways that put students off?

In this respect, it might be useful to go beyond the "one size fits all" approach. Progressive pedagogy appears to be better for some students than for others. Here is where the interaction effects between student and classroom characteristics (which hierarchical models

allow to estimate) become crucial The finding that "progressive pedagogy" had a more positive effect on students' mathematics attitude if they had scored high on it at the outset than if they had scored low might suggest that "progressive pedagogy" is best targeted toward students who do not have a problematic prehistory with mathematics. These students may be more interested in mathematics and willing to put in the additional effort that progressive pedagogy often requires of students. Those with less interest and confidence may be somewhat overwhelmed with their instructor requiring them to do more and a different kind work than they expect.

The students initially scoring lower in attitude appear to benefit more from conventional "good teaching," as the interaction of initial mathematics attitude level and "good teaching" indicated. These students may have experienced less than stellar instruction previously, resulting in less positive attitudes. As a result, they may be particularly appreciative of good teaching at the college level.

Teaching characteristics were also found to interact with certain structural features of the calculus class. Progressive pedagogy appears to work better—or, put more somberly, to do less damage—in larger classes. This may be because smaller classes already experience a higher level of student engagement. Larger classes are often lecture-based and progressive pedagogy may increase the connection of students with each other and with the instructor.

Finally, while the instructor being a graduate teaching assistant generally depresses the students' mathematics attitude, this negative effect is even heightened if that teaching assistant heavily relies on technology. For those so new to teaching, a focus on technology (with which graduate students may be more familiar than teaching) may redirect attention away from their students. Novice teacher may better use their time perfecting the traditional skills of a good

teacher than overly focusing on new technologies. Because ours is a correlational study, this does not necessarily mean that one could improve students' mathematics attitudes simply by prohibiting teaching assistants from using technology, but it may be a worthwhile strategy to emphasize good teaching in the training of graduate teaching assistants, while de-emphasizing the use of technology.

We readily acknowledge that the outcomes discussed in this article are only half of the story. The other half is, of course, the amount of mathematics knowledge and skills learnt in the class. We already discussed that the grade awarded in a class is a problematic indicator of the pedagogical quality of the class, and nationally standardized test results are not available for college calculus courses. Nonetheless, the students' attitudes toward mathematics are also crucial because they may influence future career choices. In addition, if more students emerge from their college mathematics education with a sense of confidence and enjoyment, and fewer with a sense of dread, this will help make the general societal outlook on mathematics more favorable—itself a necessary condition of success for a society grounded in high tech and science.

Acknowledgements to be added

References

Allison, P. D. (2002). Missing Data. Thousand Oaks, CA: Sage Publications

Atkin, J. M., & Helms, J. (1993). Getting Serious About Priorities in Science Education1. *Studies in Science Education*, *21*, 1-20.

Bandura, A. (1986). Social foundations of thought and action: A social cognitive theory. Englewood Cliffs, NJ: Prentice Hall.

Barnett, M. D., Sonnert, G. & Sadler, P. M. (2012). More Like Us: The Effect of Immigrant Generation on College Success in Mathematics. *International Migration Review.* 46(4), 891-918.

Blair, R. M., Kirkman, E. E., Maxwell, J. W., & American Mathematical Society. (2013). *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States: Fall 2010 CBMS Survey*.

Bressoud, D. M., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The calculus student: insights from the Mathematical Association of America national study. *International Journal of Mathematical Education in Science and Technology*, *44*(5), 685-698.

Chesebro, J. L. (2003). Effects of teacher clarity and nonverbal immediacy on student learning, receiver apprehension, and affect. *Communication Education*, *52*(2), 135-147.

College Board Office of Research and Development (1999). Concordance Between SAT I and ACT Scores for Individual Students. Report RN-07 (June 1999). New York: The College Board.

Fouad, N.A., & Smith, P.L. (1996). A test of a social cognitive model for middle school students: Math and science. *Journal of Counseling Psychology*, 43(3), 338–346.

Fouad, N., Smith, P.L., & Zao, K. (2002). Across academic domains: Extensions of the social–cognitive career model. *Journal of Counseling Psychology*, 49(2), 164–171.

Goodykoontz, E. N. (2008). *Factors that Affect College Students' Attitude toward Mathematics* (Doctoral dissertation, West Virginia University).

Harkness, S. S., D'ambrosio, B., & Morrone, A. S. (2007). Preservice elementary teachers' voices describe how their teacher motivated them to do mathematics. *Educational Studies in Mathematics*, *65*(2), 235-254.

Hazari, Z., Sonnert, G., Sadler, P. and Shanahan, M-C. (2010). Connecting High School Physics Experiences, Outcome Expectations, Physics Identity, and Physics Career Choice: A Gender Study. *Journal of Research in Science Teaching*, 47(8), 978-1003.

Lent, R., Brown, S., & Hackett, G. (1996). Career development from a social cognitive perspective. In: D. Brown, L. Brooks, et al. (Eds.), Career choice and development (3rd ed.). San Francisco: Jossey-Bass, pp. 373–422.

Lutzer, D.J., Rodi, S.B., Kirkman, E.E., and Maxwell, J.W. 2007. *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*. Conference Board of the Mathematical Sciences. Washington, DC.

Moses, R. (1995, May). Algebra, the new civil right. In *The algebra initiative colloquium* (Vol. 2, pp. 53-67).

Neale, D. C. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher*, 631-640.

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2003). Framing students' mathematics-related beliefs. In *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Springer Netherlands.

Riegle-Crumb, C., Moore, C., & Ramos-Wada, A. (2011). Who wants to have a career in science or math? Exploring adolescents' future aspirations by gender and race/ethnicity. Science Education, 95(3), 458 – 476.

Royster, D. C., KIM HARRIS, M., & Schoeps, N. (1999). Dispositions of college mathematics students. *International Journal of Mathematical Education in Science and Technology*, *30*(3), 317-333.

Rubin, D. B. (1976). Inference and missing data. Biometrica, 63, 581-592.

Rubin, D. B. (1987). Multiple Imputation for Nonresponse in Surveys. New York: John Wiley & Sons.

Rubin, D. B. (1996). Multiple imputation after 18+ years. Journal of the American Statistical Association, 91, 473-489.

Sadler, P.M., Sonnert, G., Hazari, Z., & Tai, R.H. (2012) Stability and Volatility of STEM Career Interest in High School: A Gender Study, *Science Education.* 96(3), 411-427.

Singer, J. D., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. New York: Oxford University Press.

Tai, R., Liu, C., Maltese, A. & Fan, X. (2006) Planning Early for Careers in Science, *Science*, 312(5777), 1143-1144.

Walberg, H. J. (1984). Improving the productivity of America's schools. *Educational leadership*, *41*(8), 19-27.

Wilkins, J. L., & Ma, X. (2003). Modeling change in student attitude toward and beliefs about mathematics. *The Journal of Educational Research*, *97*(1), 52-63.

Zan, R., & Di Martino, P. (2008). Attitude Toward Mathematics. *Beliefs and Mathematics: Festschrift in Honor of Guenter Toerner's 60th Birthday*, *3*, 197.

Highest Math Degree Offered	Associate's	Bachelor's	Master's	Doctoral	Total
Mainstream Calculus Students	65,000	82,000	42,000	111,000	300,000
% of Students in U.S. Taking Calculus	21.70%	27.30%	14.00%	37.00%	100.00%
# of institutions	1121	1015	181	197	2514
% of institutions in U.S. Teaching Calculus	44.60%	40.40%	7.20%	7.80%	100.00%
students/institution	58	81	232	563	119

Table 1. Mainstream Calculus I Enrollment in 2010 (Blair, Kirkman, Maxwell, 2013). Institutions offering a Ph.D. in mathematics have the largest number of students taking mainstream calculus, but represent only 7.8% of institutions teaching the course. Two-year colleges teach mainstream calculus to a relatively smaller number of students in smaller classes. Many later transfer to 4-year institutions.

Highest Math Degree Offered	Associate's	Bachelor's	Master's	Doctoral	Total
Participating Institutions	54	60	26	73	213
# Students Participating	1006	593	1510	10856	13,965
# Students Used in Analysis	237	188	402	2278	3,105
% of Students in Study	7.6%	19.7%	12.5%	60.2%	100%
Yield of Students	23.6%	31.7%	26.6%	20.0%	22.2%
# of Institutions Contacted	207	134	60	120	521
# Institutions Agreeing	43	16	41	98	198
# Institutions Used	24	13	24	62	123
Yield of Institutions	55.8%	81.3%	58.5%	63.3%	62.1%
% of Institutions in Study	19.5%	10.6%	19.5%	50.4%	100%

Table 2. The Study Sample. Many more students and institutions participated in the study than made it into the final dataset. Causes of reduction include students or instructors not filling out a pre- or post-survey, or the institutional coordinator not filling out a survey.

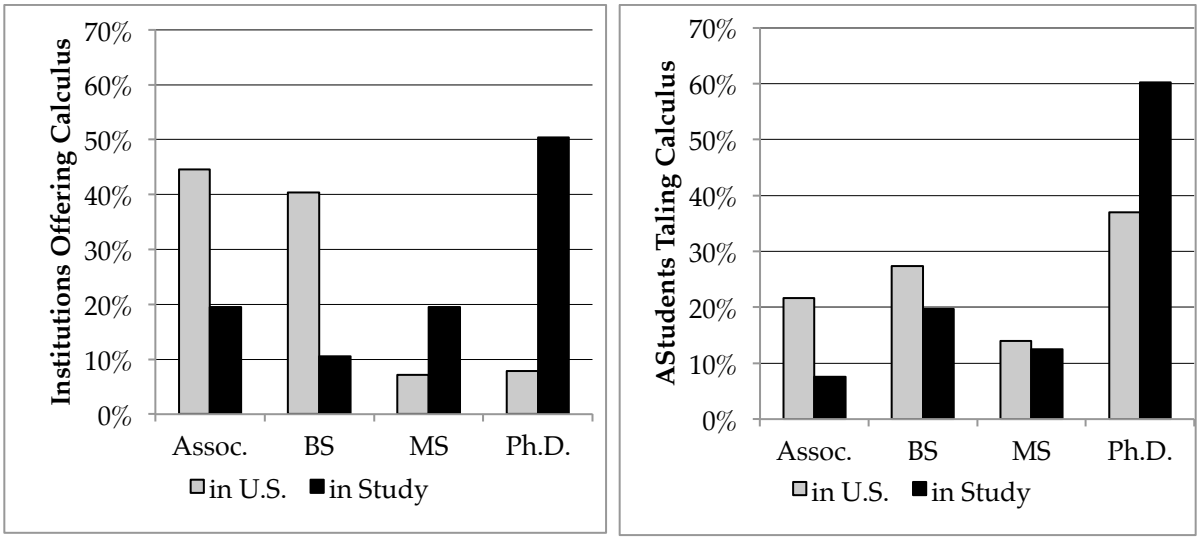

Figure 1a and 1b. Disparity in Sample Characteristics by Institution Type. The MAA-CSPCC sample over-represents mainstream calculus students at research universities and under-represents those at other institutions. We use statistical methods that control for differences at the institutional level which helps to ameliorate the differences between our sample and the population.

		FICSMath	CSPCC	
Descriptor	**Detail**	**Pre-test**	**Pre-test**	**Pre & Posttest**
Institutions	**quantity**	**135**	**164**	**123**
Instructors	**quantity**	**352**	**477**	**308**
Students	**quantity**	**10,677**	**10,185**	**3,103**
Subjects/Institution	**mean**	**79**	**62**	**25**
Subjects/Instructor	**mean**	**30**	**21**	**10**
Courses taken in HS	**Any calculus**	**40%**	**54%**	**56%**
	Precalculus/trig	**78%**	**88%**	**90%**
Courses taken in college	**Precalc**	**32%**	**12%**	**4%**
	Prior calculus	**16%**	**11%**	**11%**
Gender	**Male**	**64%**	**56%**	**57%**
Race/Ethnicity	**White**	**75%**	**76%**	**79%**
	Black	**6%**	**5%**	**4%**
	Asian	**13%**	**13%**	**13%**
	Hispanic	**9%**	**9%**	**9%**
College Year	**Freshmen**	**48%**	**73%**	**74%**
Standardized Exams	**SAT Math**	**601**	**652**	**650**
	ACT Math	**26.8**	**28.6**	**28.9**

Table 3. Comparison of FICSMath and CSPCC samples. The FICSMath study appears to capture many more students with weaker preparation, presumably since it included a larger fraction of 2-year college students. The FICSMath study also appears to have include nearly all students in a section (since it was administered in class), since mainstream calculus courses were reported to have 36 students/section in 4-year institutions. For 2-year school, class size averaged 20 students (Blair, *et. al.* 2013). While the CSPCC student captured about 2/3 of students for the pre-test and 1/3 for pre- and post-test. However, for the CSPCC study there is little differences between the pre- and post-test groups in terms of background (with the exception of those taking precalculus in college).

	Factor D1	Factor D2
Post-survey Confidence in Math	**0.600**	0.267
Post-survey Enjoyment of Math	**0.960**	0.175
Post-survey Choice to take more math	**0.645**	0.484
Intention to Take Calculus II	0.143	**0.520**
Variability Explained,%	42.928	15.148
Cumulative Variability Explained, %	42.928	58.076

Note: Values in bold correspond for each variable to the factor for which the squared cosine is largest.

Table 4. Factor pattern after Varimax rotation. Three attitude variables loaded onto factor D1. Factor D2 appears unrelated and was not included in the analysis.

Variable	Timing	Mean	SD	SE	Δ	ES	ES SE
Confidence in Math	Pre-Survey	3.89	1.01	0.02			
	Post Survey	3.42	1.18	0.02	-0.47	-0.46	.02
Enjoyment of Math	Pre-Survey	3.63	1.27	0.02			
	Post Survey	3.28	1.37	0.02	-0.35	-0.27	.02
Choice to take more Math	Pre-Survey	1.93	1.02	0.02			
	Post Survey	1.84	1.08	0.02	-0.09	-0.09	.02
Δ Attitude Composite						-0.31	.02

Table 5. Variables included in the Attitude Composite. On average, attitudes toward mathematics declined from beginning to end of a college calculus course. Effect Size (ES) is the change from pre- to post-survey in units of the pre-survey standard deviation for each variable.

Name	Description	N	Mean	SD
Student-level				
Gender	Gender (male=1)	3080	0.55	0.50
SES	Parental education (ranging 1-5)	3094	3.66	1.00
Hispanic	Hispanic (yes=1)	3078	0.09	0.29
Black	Black(yes=1)	3071	0.03	0.18
Asian	Asian (yes=1)	3071	0.12	0.33
Other	Other (yes=1)	3071	0.04	0.19
Non-AP Calc.	Took non-AP calculus (yes=1)	2961	0.19	0.39
AP AB Calc.	Took AP Calculus AB (yes=1)	2961	0.38	0.49
AP BC Calc.	Took AP Calculus BC (yes=1)	2961	0.09	0.28
Highest grade	Grade in highest math class in high school (on 4-point scale)	2961	3.56	0.72
SAT/ACT math	Mathematics SAT (ranging 200-800)	2690	653.08	75.35
College precalc	Took a pre-calculus course in college prior to this course (yes=1)	3069	0.14	0.35
College calculus	Took a calculus course in college prior to this one (yes=1)	3096	0.11	0.31
Sophomore	Sophomore (yes=1)	3090	0.12	0.32
Junior	Junior (yes=1)	3090	0.05	0.22
Senior	Senior (yes=1)	3090	0.02	0.14
Grad. stud.	Graduate student (yes=1)	3090	0.00	0.07
Special stud.	Special student (yes=1)	3090	0.02	0.15
Medicine	Career goal in medicine and health (yes=1)	2743	0.26	0.44
Science & math	Career goal in science or mathematics (yes=1)	2743	0.56	0.50
Engineering	Career goal in engineering or computing (yes=1)	2743	0.39	0.49
Pre-confidence	I am confident in my mathematics abilities (rating scale 0-5)	3092	3.89	1.01
Pre-enjoyment	I enjoy doing mathematics (rating scale 0-5)	3074	3.63	1.27
Pre-choice	Taking another math course if having a choice (rating scale 0-3)	3084	1.93	1.02

Post-confidence	I am confident in my mathematics abilities (rating scale 0-5)	3090	3.42	1.18
Post-enjoyment	I enjoy doing mathematics (rating scale 0-5)	3082	3.28	1.37
Post-choice	Taking another math course if having a choice (rating scale 0-3)	3086	1.84	1.08
Increased interest	This course has increased my interest in taking more mathematics (rating scale 0-5)	3097	2.67	1.43
Class-level				
Grad. instructor	Graduate teaching assistant (yes=1)	308	0.11	0.32
Class size	Class size (N)	308	44.14	45.67

Table 6. Variables used in Predictive Models. N for student-level variables represents the number of subjects answering the questions. Any difference from 3,105 subjects is the number of subjects for whom multiple imputation was used to impute a value for analysis.

Teaching Characteristics: Rotated Factor Pattern	Factor loading
Factor 1: Good teaching	
My Calculus Instructor provided explanations that were understandable	0.914
My Calculus Instructor listened carefully to my questions and comments	0.889
My Calculus Instructor helped me become a better problem solver	0.888
My Calculus Instructor allowed time for me to understand difficult ideas	0.862
My Calculus Instructor made me feel comfortable in asking questions during class	0.857
My Calculus Instructor presented more than one method for solving problems	0.836
My Calculus Instructor made class interesting	0.817
My Calculus Instructor asked questions to determine if I understood what was being discussed	0.803
My Calculus exams were a good assessment of what I learned	0.776
My Calculus Instructor discussed applications of calculus	0.748
My Calculus Instructor acted as if I was capable of understanding the key ideas of calculus	0.745
How frequently did your instructor ask questions	0.736
My Calculus Instructor encouraged students to seek help during office hours	0.716
My Calculus Instructor was available to make appointments outside of office hours, if needed	0.713
How frequently did your instructor prepare extra material to help students understand calculus concepts or procedures	0.709
My exams were graded fairly	0.694
My Calculus Instructor encouraged students to enroll in Calculus II	0.643
My homework was graded fairly	0.627
How frequently did your instructor show how to work specific problems	0.553
Assignments completed outside of class time were challenging but doable	0.443
My Calculus Instructor made students feel nervous during class	-0.608
My Calculus Instructor discouraged me from wanting to continue taking Calculus	-0.719
Factor 2: Technology use	
Computing technologies during your calculus class: Graphing Calculator[a]	0.802
How did your instructor use technology during your class? To find answers to problems[a]	0.779
How did you use technology during your class? To find answers to problems[a]	0.757
Indicate how often the following occurred: My instructor used technology[b]	0.751

I used a graphing calculator during class[b]	0.742
How did your instructor use technology during your class? To illustrate ideas[a]	0.741
How did your instructor use technology during your class? To check answers after we worked them out by hand[a]	0.735
The assignments completed outside of class time required that I use technology to understand ideas	0.725
My instructor demonstrated mathematics with a graphing calculator[b]	0.724
Were you allowed to use a graphing calculator during your exams? [a]	0.700
How did you use technology during your class? To understand underlying mathematical ideas[a]	0.665
How did you use technology during your class? To check written answers after I worked them out by hand[a]	0.638
Computing technologies during your calculus class: Computers[a]	0.619
My instructor demonstrated mathematics with computer algebra system[b]	0.580
How did your instructor use technology during your class? To illustrate motion/dynamic animations[a]	0.527
I used a computer algebra system[b]	0.465
Computing technologies during your calculus class: None[a]	-0.871
Factor 3: Progressive pedagogy	
How frequently did your instructor have students work with one another	0.718
Assignments completed outside of class time were submitted as a group project	0.714
The exam questions required that I solve word problems	0.695
The assignments completed outside of class time required that I solve word problems	0.640
How frequently did your instructor require you to explain your thinking on your homework	0.613
The assignments completed outside of class time required that I solve problems unlike those done in class or in the book	0.609
How frequently did your instructor ask students to explain their thinking	0.600
How frequently did your instructor hold whole-class discussion	0.577
The exam questions required that I solve problems unlike those done in class or in the book	0.575
How frequently did your instructor assign sections in your textbook for you to read before coming to class	0.563
How frequently did your instructor have students give presentations	0.555
How frequently did your instructor require you to explain your thinking on exams	0.460
Assignments completed outside of class time were returned with helpful feedback/comments	0.451
How frequently did your instructor lecture	-0.425

Notes: Item format: Unmarked items = 6-point scales; items marked [a] = dichotomous; items marked [b] =5-point scales.

Table 7. Results of Factor Analysis of Pedagogical Variables. A three-factor solution was generated from 53 out of 61 student-reported variables of their instructor's pedagogical practices and decisions. Only variables with factor loadings greater than 0.400 (or for reversed scales, -0.400) are included.

Parameter		Estimate		Estimate	
Type	**Detail**	**Model 1**		**Model 2**	
	Intercept	-1.055	***	-1.058	***
Initial State	Initial Attitude	0.647	***	0.629	***
Demographics	Gender (male)	0.104	***	0.102	***
	SES	-0.011		-0.014	
	Hispanic	-0.074		-0.080	
	Black	0.005		0.013	
	Asian	0.059		0.069	
	Other	-0.189	**	-0.181	*
Math Background	Non-AP calculus	0.145	***	0.146	***
	AP AB calculus	0.193	***	0.196	***
	AP BC calculus	0.229	***	0.227	***
	grade in most advanced HS math course	0.065	*	0.062	*
	SAT/ACT math	0.000		0.000	
	College precalc taken	-0.034		-0.023	
	College calculus taken previously	0.031		0.033	
Year in College	Sophomore	0.143	*	0.113	*
	Junior	0.112		0.087	
	Senior	-0.048		-0.102	
	Graduate student	0.417	*	0.397	*
	Special student	0.185		0.174	
Career Interest	Medicine	-0.011		-0.018	
	Science & math	0.201	***	0.201	***
	Engineering	0.069		0.064	
Course-level variables	Graduate Instructor	-0.137	*	-0.210	***
	Class size	-0.000		0.001	*
Instructor Pedagogy	Good teaching	0.250	***	0.246	***
	Technology use	0.021		0.041	*
	Progressive pedagogy	-0.093	***	-0.147	***
Interactions	Class size*progressive pedagogy			0.002	***
	Initial state*progressive pedagogy			0.037	**
	Initial state*good teaching			-0.047	**
	Graduate instructor*technology use			-0.206	**
Pseudo r^2	Variance Explained by model	50.9%		51.3%	
N		3,103		3,103	

Table 8. Hierarchical Linear Models Predicting Change in Student Attitude. Two models are presented explaining variance in the post-survey attitude composite.