

An Interview with Doron Zeilberger

By Joe Gallian and Michael Pearson

To celebrate the opening of the Carriage House Conference Center at MAA headquarters, made possible by a gift from Paul and Virginia Halmos, the MAA received a grant from the National Security Agency to support a Distinguished Lecture series intended to appeal to a general audience.

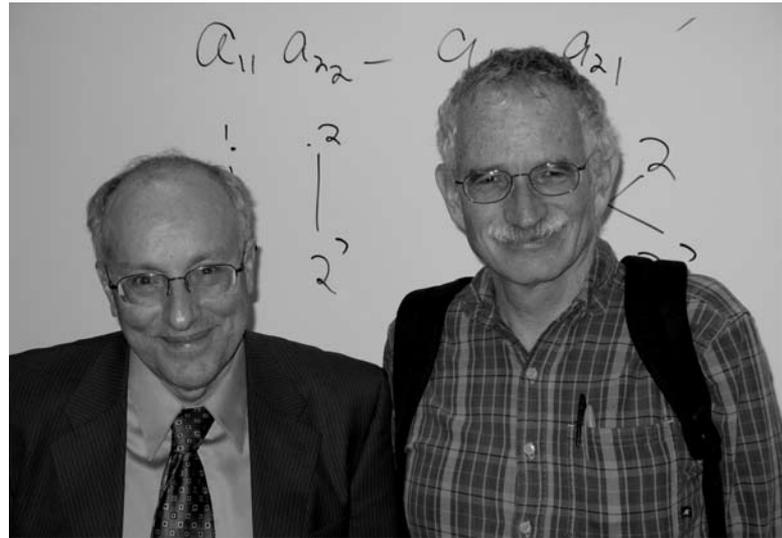
The second lecture in the series was given by Doron Zeilberger on February 20, 2007. Zeilberger is the Board of Governors Professor of Mathematics at Rutgers University. He is widely known for the development of “WZ” (Wilf-Zeilberger) Theory and Zeilberger’s algorithm which are used extensively in modern computer algebra software. Zeilberger was the first to prove the elusive result in combinatorial theory known as the alternating sign matrix conjecture. Among his honors are: the American Mathematical Society Steele Prize for seminal contributions to research (co-recipient with Herb Wilf); the Institute of Combinatorics and Its Applications Euler Medal for “Outstanding Contributions to Combinatorics”; the Laura H. Carnell Professorship at Temple University; and the MAA Lester R. Ford award for a paper in *The American Mathematical Monthly*.

The citation for the Euler Medal describes Zeilberger as “a champion of using computers and algorithms to do mathematics quickly and efficiently.” In his opinion “programming is even more fun than proving, and, more importantly it gives as much, if not more, insight and understanding.”

While at the MAA for his lecture, Professor Zeilberger talked with Joe Gallian about his research and teaching. Following are excerpts from the interview.

JG: How do you describe your research area?

DZ: I work mainly in combinatorics and the theory of special functions, but in the last 10–15 years I have considered myself mainly an experimental mathematician, and combinatorics and the theory of



Joe Gallian and Doron Zeilberger

special functions are just sources for examples and case studies of a methodology with the aim of training the computer to discover conjectures and then try to prove them all by itself, without any human intervention.

JG: Are there now a number of people who are doing experimental mathematics?

DZ: Experimental mathematics is a rapidly growing field, both explicitly and implicitly. Explicitly, there is a very good journal by that name, an Institute in Simon Fraser University, and at the recent annual meeting at New Orleans there was a special session dedicated to it. This is a good start, but still at its infancy. However, implicitly, more and more mathematicians, even pure ones, use the computer daily to formulate and test conjectures, in a mode that George Andrews calls “pencil with power-steering.” However, more often than not, the computer’s often crucial contribution is not mentioned, or grossly understated. Human beings are such ingrates.

Also, Jon Borwein and David Bailey wrote a very nice monograph on Experimental Mathematics that I highly recommend. But their emphasis is more on the traditional experimental mathematics that has been pursued by all the great,

and less-great, mathematicians through the centuries, using pencil-and-paper. Of course, with computers you can do so much more, and you can be very systematic, and the great power of today’s computers, guided wisely, can take you a very long way. However, their emphasis is still on using computers to find interesting conjectures and phenomena, but not to prove them. The proof itself (when feasible) is still done largely by human beings, although sometimes with the assistance of computers.

As for computer-generated proofs, there is a whole community of Automatic Theorem Proving, which is very successful. Their methodology is logic-based, and they try to teach the computer formal reasoning, using axioms and “laws of deduction.” My approach is more akin to what the great algebraic geometer Shreeram Abhyankar calls “high-school algebra” and physicist Richard Feynman calls “Babylonian mathematics”: using algorithmic frameworks, that I call “ansatzes.” But since the algorithms are symbolic rather than numeric, one can hope to prove genuinely new general theorems, valid for infinitely many cases. In this kind of research, we try to teach the computer to first make a conjecture, all by itself, and then automatically prove its own conjectures, also all by itself! Of course, the only way we can do it,

at present, is by focusing on narrowly-defined areas.

JG: Are you getting some resistance from the traditional math community with your approach?

DZ: Definitely. Many people are not very comfortable with this approach. First, they claim that you can't trust computers (as if humans were so trustworthy!), and they also feel that it's not fun to have a computer do your work, or that it is "cheating." For me, teaching the computer how to discover and prove new theorems is even more fun than discovering and proving them myself. As for the cheating part, this is science not sport, and besides we can always change the rules of the game.

JG: The Wilf-Zeilberger algorithmic proof theory is contained in all major computer algebra systems. What does it do?

DZ: It's a collection of algorithms that can discover, and then prove, binomial coefficient identities, and identities involving sums or integrals of special functions. As you know, the binomial coefficient " n choose k " counts the number of k -element subsets of an n -element set. Since many objects in combinatorics boil down to unions and cartesian products of such choosings, it turns out that many enumeration problems can be expressed as such binomial coefficient sums, and often we get surprising identities. Before W-Z theory, every such identity required its own ad hoc proof, and any such new identity, or even a new proof of an old identity, was publishable. But thanks to Wilf-Zeilberger theory, any such identity can now be proved automatically. No one today would submit a paper stating and proving the "theorem" that "ten plus five equals three times five." Analogously, an identity like

$$\sum_{k=0}^n (-1)^k \binom{2n}{n+k}^3 = \frac{(3n)!}{n!^3}$$

first proved by Dixon in 1904, is now completely routinely and automatically provable thanks to the Wilf-Zeilberger algorithmic proof theory.

JG: You seem to have an evangelical-like faith that computers will make obsolete the way mathematics has been done for over 2000 years. Is that a fair assessment of your view?

DZ: Yes, but people who know me well know that they should not take me too seriously (laughing). In the Talmud it says that if you have a talit (garment) and one guy says that it's all his and the other guy says that it's all his, then they each get half. But if somebody only claims a half and the other guy claims a whole, then the guy who claims a half will only get a quarter and the other guy gets three quarters. So if you want a half, you have to claim a whole. So you have to overstate your case. Then again, sometimes overstating can backfire and turn people off.

JG: Tell me about your frequent co-author Shalosh B. Ekhad.

DZ: It is a charming individual. Of course it is made of silicon, and it is not really *one* body, but it is definitely *one* soul (software). The body has just been reincarnated many times. As we know, computers are very powerful, but their life expectancy is much shorter than that of humans, since computers get better and faster so quickly, you have to get a new one every three years, but you can always upload all the software from one Shalosh to the next, thereby guaranteeing the immortality of its soul.

JG: Where did you get that particular name?

DZ: The original Shalosh B. Ekhad was actually a Hebrew translation of the first PC that I owned, called AT&T 3B1. At the time it was a very innovative machine, the first UNIX PC, that was manufactured by AT&T in the 80s. The Hebrew translation of 3B1 is Shalosh B. Ekhad.

JG: Tell me about your book $A = B$.

DZ: It was written by Marko Petkovsek, Herb Wilf, and myself, and is an elementary introduction to the so-called Wilf-Zeilberger algorithmic proof theory mentioned above. It also has a long chapter on the very important Petkovsek al-

gorithm for deciding whether a sequence is closed-form.

JG: It was quite a coup to get Donald Knuth to write the preface. How did that come about?

DZ: First, and foremost, Knuth is a good friend of Herb Wilf. Also, Knuth loves binomial coefficient identities. He dedicated quite a few pages to them in his classic *Art of Computer Programming*, which is the "bible" of computer science, and later, in much more detail, in his beautiful book with Ron Graham and Oren Patashnik, *Concrete Mathematics: A Foundation for Computer Science*. He still loves them. In the latest issue of *The American Mathematical Monthly* (February 2007) there is a problem proposed by him that could be easily done using the WZ method. I was a little taken aback. If somebody submitted a problem to prove that $11 \times 13 = 12^2 - 1$, they wouldn't accept it, since there are nowadays (and have been for the last 5000 years) algorithms that can routinely prove this. Similarly, for Knuth's *Monthly* problem there are (and have been for the last 15 years) algorithms that can routinely find a proof. But then again, Knuth is Knuth, and if he proposed as a problem to prove that $1 + 1 = 3 - 1$, it might be accepted. Of course, Knuth is very much aware of WZ, and he probably had some hidden agenda in proposing this problem. Maybe he meant some cute combinatorial argument.

JG: I notice that the preface of your book $A = B$ begins with Knuth's famous quote "Science is what we understand well enough to explain to a computer. Art is everything else we do." Was $A = B$ the first place that quote appeared in print?

DZ: Yes, I think so.

JG: I was very surprised that a commercial publisher would agree to permit the book to be downloaded free over the internet.

DZ: Herb Wilf is a great advocate of free publishing and AK Peters is not a typical publisher. Klaus Peters, and his wife Alice, are very good people, who care more about mathematics than making a quick buck. Commercially it seems to have

been a wash, some people who would have bought it, didn't, and vice versa.

JG: I notice that you prefer *Maple* over other software. Why is that?

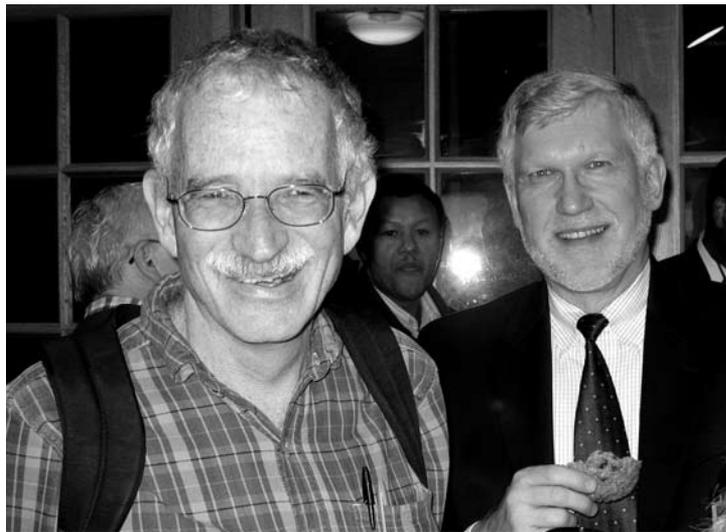
DZ: Originally, *Maple* was much cheaper and more accessible than its competitors. *Mathematica* was much more commercial. Unfortunately, now *Maple* is very commercial too, and *Mathematica*'s prices have gone down, and many people prefer the latter for its elegant syntax and beautiful graphics. So I use *Maple* for "old times sake" and mainly because I am used to it.

JG: Describe your Experimental Mathematics course.

DZ: I really enjoy teaching it. It is a graduate class (with one or two advanced undergraduate students) that is held in a "smart" classroom where everyone is connected to a terminal. It is a hands-on course where the main point is to teach students how to program in *Maple*, in order to explore new mathematics. The actual topic changes every year, so that I won't get bored, and sometimes I even learn a new subject myself, since the best way of learning a new subject is by teaching it. Of course, in this class we also learn how to program everything, and an even better way to learn a new subject is by teaching it to a computer, i.e. programming. So both my students and I learn the substance of the course very well, and at the same time my students learn how to program in *Maple*.

JG: You have Erdős number 2, Einstein number 3, Wiles number 3, and Knuth number 2 but you say on your website that you are most proud of your Garfield number 2. Why is that?

DZ: Richard Garfield is one of the most talented and creative combinatorialists alive, but he used his talents in a non-standard way. He is also a very nice guy, and I am fortunate to have known him, although only briefly. During the mid-90s, he was a teenage idol thanks to his innovative and lucrative *Magic: The Gathering* card game. He designed the game while he was Herb Wilf's PhD student at Penn in the late 80s and early



Doron Zeilberger and Ivars Peterson.

90s. He did good research, but not as much as he could have, because he was so busy developing and testing his card game. After his PhD, he got a one-year job in some small college in Washington State, with a salary of about 22K. When the year was almost up, and it was not clear what his prospect for the future in academia would be, his card game caught on, and the rest is history.

JG: There must be a story behind the "Who you gonna call" T-shirt.

DZ: This is a T-shirt that I am wearing in my picture on my website. It features a certain binomial-coefficient identity, with the caption "Who you gonna call." The back of that T-shirt has the few lines of *Macsyma* code needed to prove it, and the caption "the binomial-coefficient-identity-busters." My kids told me that it is an allusion to the film *Ghost Busters*. This cute design was made by Herb Wilf's son, David, who is a lawyer by profession, and Herb gave me one of them.

JG: Something that I found surprising is that the narratives for your grant proposals are posted at your website. That certainly helps people learn about your work.

DZ: I think that it's a waste to write something just for five or six people and for no one else to be able to see it. I think that everything should be public; I don't like secrecy. I also hate the tradition of anonymous refereeing. I think that there

should be open refereeing. Also people should post their PhD theses, especially the introduction, that often gives a very good overview of a field.

JG: Another thing that surprises me is that you slip a bit of humor in your grant proposals. One is "There is a delicate balance and trade-off between the general and the specific, the abstract and the concrete, the strategical and the tactical, the sacred and the profane." Another is the line "My particular shtick is experimental mathematics." A third is "The computer is a powerful tool, go forth and use it!" Your proposals are less formal than I would expect.

DZ: That's my style. I don't like to be too serious. But I think that it has cost me some funding. In my last grant proposal, I was hoping to get co-funded by the computer science division of the NSF. I had no problem getting funding from the mathematics division, but of course mathematics doesn't have much funds, so I was hoping to get some additional funding from computer science. So I also sent my grant proposal to the computer science division. That was a disaster. One of the panelists gave me a lecture, saying this might make a good essay, but it's not what one would expect in a grant proposal, and they refused to give me funding.

JG: You are known for your opinions on your web page, some of which are a bit over-the-top, especially those on April 1.

What changes — if any — are you hoping to encourage in the mathematics community by posting your opinions online?

DZ: I'm not trying to change people's views, at least not consciously. I just like to express my opinions. It's only the Internet, which is a free-for-all, and one doesn't have to be too uptight. So I don't really worry about whether or not my pieces always make sense. Hopefully they do most, or least some, of the time.

JG: You have been known to celebrate both Valentine's Day and April Fools Day in the classes you teach. What is the most memorable thing you have done to mathematically celebrate a holiday?

DZ: In my calculus class, I assign a homework "project" due February 14, to graph the parametric equation for a cardioid. Some people came with very strange shapes, but some people realized what was going on and just drew a regular heart shape. But they made a pointy one, which is wrong, because a cardioid is more rounded. It was nice to get 150 valentines (some rounded, some not), and I could brag to my wife how much

my students love me. In my computer algebra class, I gave my students some extremely complicated differential equations that they had to use *Maple* to solve. The solution happened to be a cardioid. Then they had to draw it (using the *Maple* plot program), and cut it out.

JG: You have been known to spontaneously hand out money to students for solving problems or pointing out a correction in class. What is the largest prize you have ever awarded, and what was it for?

DZ: For calculation errors the most I've given out is \$1. But for conceptual errors in graduate classes, I think I once gave \$10. I also offer prizes for really challenging problems.

JG: Do you have any concluding words?

DZ: Yes, I believe that teaching is at least as important as research, and I put lots of effort into my teaching and take great pride when I do a good job. Many research mathematicians dislike teaching and view it as an unavoidable chore, but

they are wrong. First, teaching is great fun, and secondly, it is very important, since this is the future!

So already today teaching is at least as important as research. But in years to come, when more and more original mathematical research will be conducted by computers, the importance of the human research mathematician will diminish, and the importance of teaching, at all levels, will increase tremendously. Also for a long time to come, we still need humans to program the computer, but what is programming? It is teaching computers, and I am sure that being a good programmer and being a good teacher (for humans) are strongly correlated. One of the reasons that I believe that I am a good teacher is that I do so much programming, and am used to spelling out each and every step. In short, the future of mathematics is in good teaching, both to machines and to humans.

Joe Gallian is President of the MAA. Michael Pearson is the MAA Director of Programs and Services.

In Memoriam

Paul Joseph Cohen died in Stanford on March 23. Born in 1934, Cohen got his graduate training at the University of Chicago and eventually became a professor at Stanford University. Though he made contributions to analysis and the theory of partial differential equations, he was best known for his development of "forcing," now a fundamental tool of set theory, and using the new technique to prove that the Continuum Hypothesis and the Axiom of Choice are independent from the axioms of set theory. This groundbreaking work won him the Fields Medal in 1966, and he received many other honors and prizes. FOCUS will have a longer article about Cohen in its August/September issue.

Frank Burk died on March 17. Born in 1942, Burk received a Ph.D. from the University of California, River-

side, in 1969. He taught at California State University, Chico, from 1968 to 2004. Burk was the author of two books, both on the theory of integration. The second, entitled *A Garden of Integrals*, will be published by the MAA in May.

James Eells died on February 14 at the age of 80. Eells graduated from Bowdoin College and then from Harvard University. His doctoral work was completed in 1954, under Hassler Whitney. After holding several prestigious positions in the U.S., he moved to the United Kingdom and became Professor of Analysis at the University of Warwick. He was later named the first head of the mathematics group at the International Center of Theoretical Physics in Trieste, Italy. His research dealt with harmonic maps, geometric evolutions, and stochastic analysis.

We are deeply grateful for the generosity of the following individual, who has made a bequest to the Mathematical Association of America.

Every bequest is a powerful expression of loyalty, their lifetime involvement, and their faith in the future of the MAA.

We remember each of them fondly and with deep personal and professional respect.

Francis D. Parker
Member for 55 years