Preface

I was a born juggler, and after seven years of teaching college math students how to juggle, I was disenchanted. I had begun to feel like a painting teacher who taught color wheels but never let his students paint pictures, let alone engaged them in discussions of why art is meaningful or why artists differ in their styles. My students weren't complaining, really. Some of them were also born jugglers. The rest simply shouldered the burden, as if accustomed to the idea that math is no more than a kit full of tools.

So I scrambled my calculus class to place the discovery of calculus at the *end*. I intended to teach calculus as the culmination of an intellectual pursuit that lasted two thousand years. I could not find a suitable textbook, so I taught for three years using my own notes and homework problems. Then, on sabbatical, I wrote this book.

At first, I expected the book to be a synthesis of my notes and problems, but my research led me down new paths to many pleasant surprises. I had not realized the extent to which scholars in countries like Egypt, Persia, and India had absorbed and nourished Greek geometry when the western world went dark. Nor had I fully grasped how carefully ancient thinkers treated puzzles that lurk in the infinite. I gradually learned that because of these puzzles, calculus was not *discovered* in a way that would allow me to place its discovery at the "end" of anything.

Those we credit with the discovery explained the infinite in poetic terms. Even statements within the proofs themselves sound like metaphors: this tiny number is both zero and not-zero at the same time; this solid cube is composed of infinitely many flat slices. When pressed, mathematicians defended themselves with analogies. Isaac Newton used the example of a book to suggest how a three-dimensional volume could be composed of two-dimensional parts. Of course, a page is not strictly two-dimensional; it merely symbolizes such a thing. But we forgive him, not only because his calculations *work* but also because the rest of his arguments are insightful and rigorous.

Such flights of intellectual fancy do students a huge favor: the most mind-contorting technicalities are replaced by intuitive, appealing, simple arguments that are a pleasure to study. The figures alone speak eloquently about the subject. All that is required beyond algebra and basic geometry is a willingness to untether one's creativity when thinking about the infinite. And what student would be unwilling to do that in return for the chance to learn calculus as a pursuit rather than as a toolkit?

This subject is a treasure of the human intellect, pearls strung by mathematicians across both cultures and centuries. I hope this book holds a mirror up to this beauty.

Notes for professors

If you are a professor who assigns this book, I encourage you to ask your students to read a selection before class. I wrote with this model in mind. Intent and careful reading of professionally-written mathematics instructs students in their own writing. I ask my students to write in a journal as they read, so they can jot down questions, create their own examples, work out steps the author skipped, and recreate the figures as the author narrates. My students who take this seriously enjoy a noticeable upswing in the quality of their own writing. Further, I found that classes became far more dynamic, because the content of each class meeting was driven by the questions the students brought.

Each chapter concludes with a section entitled **Furthermore**. These sections introduce notable historical figures as well as a few results that are used in later chapters. The 'exercises' are designed to be read even if they are not assigned as homework. They are not meant to be difficult, but rather to be good checks on whether the reader has paid careful attention to the text.

I steered away from routine practice problems, such as lists of derivatives that require the product rule. I also elected not to weave much about the thinkers themselves into the narrative, for this information is widely available.

My thanks

Gerald Alexanderson gracefully served as managing editor from the beginning. Victor Katz's comments prompted a wholesale improvement of my discussion of the origins of the coordinate system in chapter 3. Christoph Nahr translated French and Padmini Rajagopal translated Malayalam for me. Ryan Walp gave me the aluminum cube (pictured in the frontispiece) as a gift, and Anthony Aquilina photographed it. Along with Ryan, my students Kelly, April, Shaun, Dave, and Eric read one of my drafts with me in an independent study class. Sam Fee gave me his photograph of the desert scene that appears on the front cover. I love this scene for the sand, which reminds us of the infinite, the desert, which is characteristic of the countries where calculus finds its origins, and the patterns created by the wind, which look not only like curves but also like notation. Michelle LaBarre did for this preface (and my life) what the wind did to the sand.