Historical Modules for the Teaching and Learning of Mathematics

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To The Teacher

Introduction

The Principles and Standards for School Mathematics, released by the National Council of Teachers of Mathematics (NCTM) to begin the 21st century, set a most ambitious vision for mathematical education (p 2):

A Vision for School Mathematics

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

Furthermore, *The Principles and Standards for School Mathematics* mark the path that educators, students, and communities-at-large need to guide appropriate mathematics teaching and learning in the new millennium. In stating that, "the secondary school mathematics program must be both broad and deep" (p. 287), one can see that a central theme of the document is connections. From page 288, we see that "students develop a much richer understanding of mathematics and its applications when they can view the same phenomena from multiple mathematical perspectives." One way to have students see mathematics in this way is to use instructional materials that are intentionally designed to weave together different content strands. Another means of achieving content integration is to make sure that courses oriented toward any particular content area (such as algebra or geometry) contain many integrative problems—problems that draw on a variety of aspects of mathematics, that are solvable using a variety of methods, and that students can access in different ways."

Finally, students must be provided with meaningful activities and applications to help expand upon fundamental mathematics concepts, while using multiple perspectives

in order to contribute to students' abilities to grow as mathematical thinkers. "Mathematics is one of the greatest cultural and intellectual achievements of human kind, and citizens should develop an appreciation and understanding of that achievement." (p. 4) To that end, the information, activities, and exercises contained in these modules will provide students and teachers alike with "rich problems, a climate that supports mathematical thinking, and access to a wide variety of mathematical tools." (p. 358)

General Suggestions

Objectives of the Modules

- To enable students to develop a much richer understanding of mathematics and its applications by viewing the same phenomena from multiple mathematical perspectives.
- To enable students to understand the historical background and connections among historical ideas leading to the development of mathematics.
- To enable students to see how mathematical concepts evolved over periods of time.
- To provide students with opportunities to apply their knowledge of mathematics to various concrete situations and problems in a historical context.
- To develop in students an appreciation of the history connected with the development of different mathematical concepts.
- To enable students to recognize and use connections among mathematical ideas.
- To enable students to understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- To lead students to recognize and apply mathematics in contexts outside of mathematics.

How to Use

The modules contain a generous amount of material. The teacher needs only to review the material and pick and choose those topics that best fit teacher and students' needs. The material can be designed to fit many different types of objectives, and it can be made to fit many different lesson plans. The modules are designed to be used in a variety of mathematics classes, from middle school and prealgebra through calculus. Few classes would be able to use all the material in a particular module in one single year. On the other hand, most mathematics teachers should be able to find something in the modules that will enrich the class and help put the material of the class in a broad historical, social and scientific context.

Some of the activities may be used to introduce a topic, with further work being accomplished using exercises found in student textbooks. Other activities are better used as supplements, once the students have an initial familiarity with the subject matter.

However these activities are used, the authors believe that the history will enable students better to understand the mathematical ideas.

Most activities in the modules have Student Pages, designed to be copied and distributed to the students. The Student Pages in the modules are frequently designed with questions or hints to guide the students toward discovering the answers. General historical material may be discussed with the class as a whole or in groups as the teacher deems appropriate. Teacher notes and solutions accompany the student activities and projects along with relevant masters for transparencies. Written assignments are given in some of the activities. Many sections of the modules could be taught with an interdisciplinary approach. For example, if the teacher wishes to work with teachers from other departments such as social studies, science, or economics, they could plan lessons and activities that make connections between people, places, and topics studied in the module and also in another class. Websites are included for easy reference to relevant topics and mathematicians. Modules also include bibliographies for both students and teachers interested in further study of mathematical content from historical perspectives.

Classroom Organization

The activities are generally designed for students to work in either small groups or individually, although some may better be used in the more traditional manner with the teacher directly leading the discussion of a topic.

Time Frame

The activities developed in the modules vary in the time needed from ten-fifteen minutes up to two weeks. The time allotment depends on the mathematical level of the students and whether the materials are used to introduce concepts or as supplements. Each activity provides a general indication of the time needed.

Materials and Equipment Needed

It is important for all the activities that a world map be posted in the classroom. If the teacher has access to ancient maps, these would also be valuable.

The materials needed vary with the activities, but generally are easily available. Most of the activities are designed to be done without such technology as graphing calculators or computer programs. However, some of the activities will benefit from the use of this technology.

Module Descriptions

There are eleven modules in all, of varying lengths. Each module was written by a team of college and secondary school teachers and was field-tested around the country.

Archimedes
Activities from the work of Archimedes

Combinatorics
The elementary formulas for combinations and permutations along with an introduction to probability

Exponentials and Logarithms
The development of the ideas of the exponential and logarithmic functions with applications

Functions
The general idea of a function, with illustrations from many sources

Geometric Proof
Why do we need proofs – a historical study with numerous examples

Lengths, Areas, and Volumes
Activities from around the world dealing with the measurement of these quantities

Linear Equations
The idea of a proportion along with the solution of linear equations and systems of linear equations, illustrated with examples from throughout the centuries

Negative Numbers
How are these quantities used and why, with illustrations from many societies

Polynomials
Methods for solving quadratic and cubic equations, as well as more general polynomials

Statistics
Basic concepts of statistical reasoning, including various types of graphs

Trigonometry
From the creation of a sine table to the measurement of plane and spherical triangles

Navigation

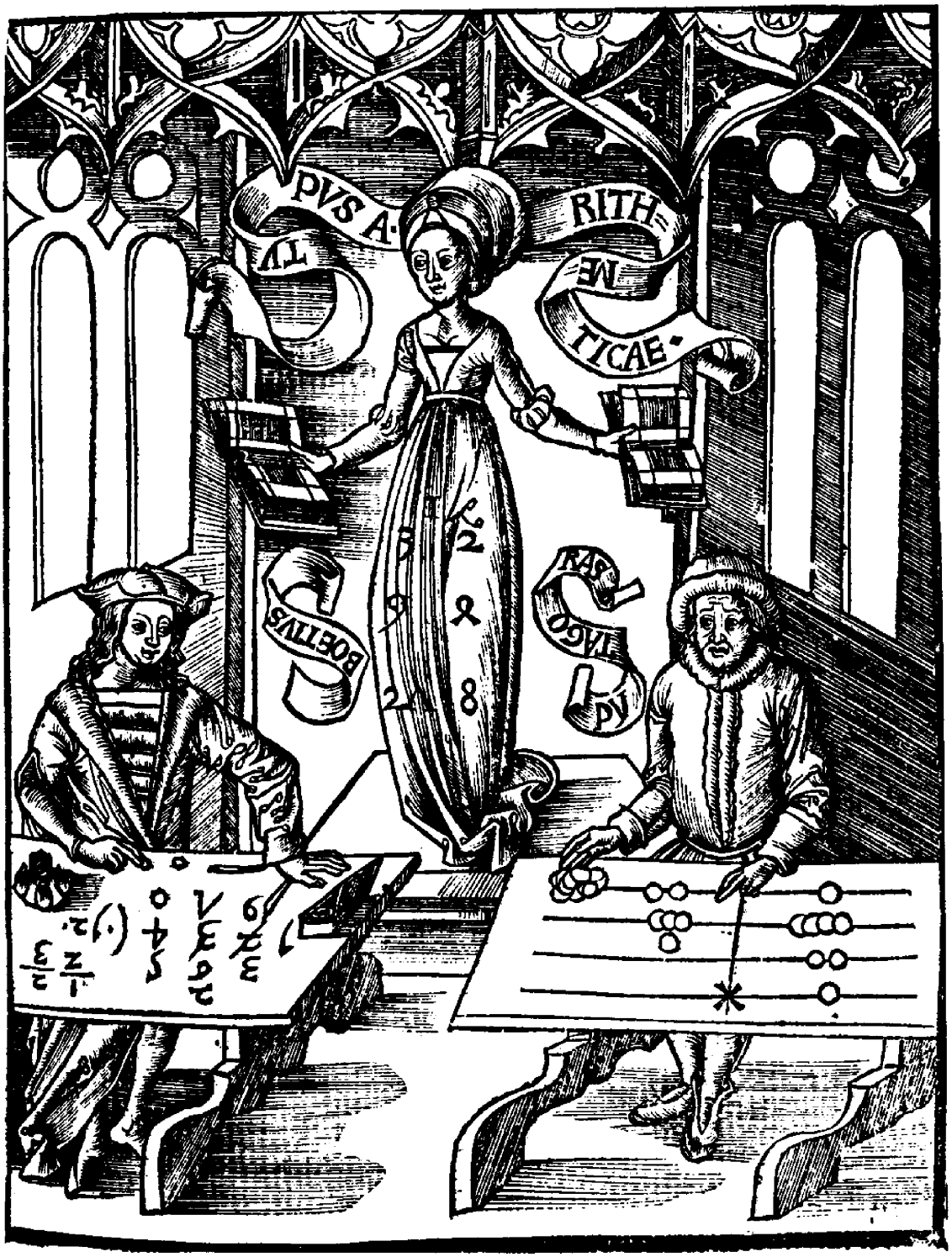
Navigation through the modules is best accomplished using the bookmarking tool of Adobe Acrobat. If you are in a module, click on the "Bookmarks" tab. You will then get the entire table of contents for that module in the bookmark list. You can easily move to a particular activity by clicking on the appropriate bookmark. If you want to move to a different module, clicking on the module's title at the bottom of the bookmark list will move you to the title page of that module.

Illustration

The illustration below is a woodcut from Gregor Reisch, *Margarita Philosophica* (Freiburg, 1503 and numerous later editions), in which Arithmetic (one of the seven liberal arts) is instructing the algorist (represented by Boethius on the left) and the abacist (represented by Pythagoras on the right). Interestingly, historically, Boethius had nothing to do with deriving or using arithmetic algorithms on paper nor did Pythagoras have anything to do with procedures for doing arithmetic on a counting board. So the woodcut probably does not symbolize the algorist winning out over the abacist, as is frequently mentioned, but simply represents aspects of the theoretical arithmetic tradition stemming originally from the classical Greek tradition.

The *Margarita Philosophica* itself, whose title, roughly translated, means "pearl of knowledge", was probably the most important encyclopedia of knowledge of the late Middle Ages and early Renaissance. It contained all the knowledge that students were required to learn and was used as the main textbook in many universities. Gregor Reisch was a monk who taught in the University of Freiburg, in southern Germany.

Note: You might ask the students to try to figure out what the calculations mean which the two men are performing.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