Abstracts of Papers Presented at MathFest 2021

August 4 – August 7, 2021

MATHEMATICAL ASSOCIATION OF AMERICA

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Alternative Assessments: Lessons from the Pandemic

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Organizers:
David Clark, *Grand Valley State University*,**Mike Janssen**, *Dordt University*, **Austin Mohr**, *Nebraska Wesleyan University*, **Mariah Birgen**, *Wartburg University*, **Beste Gucler**, *U Mass Dartmouth*, **Michael Tallman**, *Oklahoma State University*, **Jessica OShaughnessy**, *Shenandoah University*

Debra Borkovitz, *Boston University*
Trusting Students: Assessment in the Pandemic
Pre-pandemic, Jesse Stommel's four word teaching philosophy, "Start by Trusting Students," helped me rethink many of my teaching practices; when I found myself wanting to make a pedagogical decision based on my own fears, I would first ask whether students had violated my trust, and the answer was usually no. During the pandemic, this philosophy has guided me in taking risks and making changes in my assessment practices. In this talk, I'll share some examples of assessments in my remote synchronous Linear Algebra, Discrete Mathematics, Introductory Statistics, and Mathematical Explorations courses, with class sizes ranging from 22 to 122. The assessment ideas draw from a mix of proficiency grading, specifications grading, labor-based grading, and ungrading.

Mike Janssen, *Dordt University*
Pseudo-Ungraded Exams
Proponents of ungrading identify many of the usual problems with traditional assessment, and further argue that all coursework should receive meaningful instructor feedback, be supplemented with student self-reflection, and assessed by the students themselves. I will discuss an experience integrating some of these ideas into an exam structure for Calculus I and linear algebra students in Fall 2021, particularly as a way of continuing to do authentic assessment during the COVID-19 pandemic.

Anne Sinko, *College of St. Benedict/St. John's University*
Ungrading: Assessment from Beyond Mastery Grading
At its core, Mastery grading shifts the student focus to feedback by modifying the student perception of "score" to improve learning. Ungrading works to shift the student focus to feedback by removing all scores, even if the "score" is complete versus keep working. In other words, Ungrading is a no grades, only feedback approach to assessment. The pandemic offered me a unique opportunity to try out this approach in an upper division elective during the Spring 2021 term. This talk will present the ideas behind ungrading, how the pilot went, student perspectives on Ungrading, and thoughts for future.

Jessie Hamm, *Winthrop University*
A First Attempt at Mastery Based Grading
The pandemic caused us to all rethink the way we deliver content. For some of us, including myself, it also led to a change in the way we assess our students. This past year I attempted mastery based grading for the first time in multiple upper-level courses. In this talk I will discuss successes, failures, and student perception. I have not yet mastered this grading technique but am ready for a reattempt this fall!

Rachel Grotheer, *Wofford College*

New Job, New Modalities, New Assessment: Lessons from my Dive into Mastery Grading

The fall of 2020 brought a host of changes for everyone. For me, on top of wading into using hybrid teaching modalities for the first time, I also found myself starting a job at a new institution. After a lot of research into mastery and proficiency-based grading systems, but having only briefly dipped by toes in them in the past, I decided to make a fresh start of everything in the fall to rework all my classes to use these systems for assessment. This talk outlines how I decided to set up assessment for undergraduate courses at three different levels, how I adapted those assessments to a hybrid environment, the challenges I encountered, and the benefits I discovered along the way, all explained from the point of view of a pre-tenure faculty member in her first year at a new institution.

Kristen Sellke, *Saint Mary's University of Minnesota*

A Pandemic First Attempt at Mastery/Specifications Grading for a Joint Precalculus and Calculus I Course

In Fall 2020 the pandemic grading plan for this course consisted mainly of weekly online quizzes and paper and pencil homework sets collected every three weeks. Neither the students nor the instructor missed the exams, but the homework sets were not as useful in assessment as hoped. In Spring 2021 the homework sets were reimagined in a mastery approach where each student received three attempts to master each problem. The course grade itself was based on achieving that grade in all four course components (online quizzes, homework sets, daily feedback questions and two integrative assessments). Instructor successes and challenges and student survey responses to the spring grading system will be shared as well as ideas and plans for the next attempt in Fall 2021.

Mona Mocanasu, *MSU Denver*

Reconsidering Final Exams as Mastery Assignments

When we talk to our colleagues, we say that every math course is different, and that every university has its rules and methods, adapted to their student body. And then there is Calculus - our universal base line. Many of us teach some form of Calculus I, II or III - with a common core of topics that we expect all students to master, and hopefully to remember for their academic life. At my university, these Calculus courses were offered in-person only, with a common syllabus for all sections; the curriculum underscored computational methods and formula memorization, alongside applications and conceptual topics.

So, last summer when I signed up to teach Calculus II, I knew I had to reconsider and redesign most of my assessment plan – to balance our standard in-class midterms and final exam with artifacts that are better suited to the online world with open-book exams and unlimited cheating temptations.

In this presentation I describe my new grading scheme and mastery assignments, as well as the changes I implemented in the course over three online semesters, to help students stay on track and master the material in an asynchronous format, as well as to discourage cheating. I also share how I want to implement some of these changes as we return to campus, and how my own perception of final exams has transformed.

Emily McMillon, *University of Nebraska-Lincoln*; **George Nasr**, *University of Nebraska-Lincoln*

Online Versus In-Person Delivery: Exploring the Effects of Mastery Grading in a Geometry Course for Pre-Service Elementary Teachers

In Spring 2020, we designed and piloted a mastery grading scheme for a geometry course for pre-service elementary teachers. This implementation was largely successful both in student outcomes and student experiences. In Spring 2021, a new set of instructors modified the mastery grading scheme for synchronous virtual learning. In this talk, we will discuss our implementation of mastery grading and the

differences in student outcomes and perceptions between the two implementations. We will identify and explore the challenges caused in part by the move to online instruction and identify some benefits of having this grading scheme while teaching online. Further, we discuss implications for future implementations of mastery-based grading for pre-service teachers, whether in-person or virtual.

Teresa Magnus, *Rivier University*
Shaking up the Culture: Eliminating High Stakes Tests in the Midst of a Pandemic
With some students participating remotely and others in the classroom, creating and monitoring exams that would be perceived as fair to all was a challenge I chose to avoid in 2020-2021. These high stakes tests were replaced by specifications grading with Mastery assessments, final portfolios, class contributions, homework assignments, and either papers or reading assignments. Adjusting to the new system posed some challenges primarily in workload balance, record keeping, and transforming traditional expectations, but this instructor has no plans to return to the old ways. In their essays, students acknowledged how they were expected to learn the material in the class, that they learned through revision, and that moving on without mastery was not an option. Students recognized their own growth and need for continued learning. They saw value of mathematics in their lives. They realized that taking risks and making mistakes was not only welcome but encouraged. There is more to mathematics than getting the right answers!

Cassie Williams, *James Madison University*
Do All The Things! (Using Mastery Grading in Online Calculus during a Pandemic)
Last year I used Mastery Grading in two flipped calculus courses taught synchronously online but with asynchronous, unproctored assessments. I'll share the benefits of MBG for me and my students in this context, the many challenges and productive failures, and the lessons I learned about online assessment that will inform my future in-person teaching. The realities of learning and teaching in the midst of a pandemic will be conspicuous in the discussion.

Violeta Vasilevska, *Utah Valley University*
Lessons Learned During the Pandemic – The Good, the Bad, and the Ugly
In this talk, we describe several alternative assessments used during the pandemic. Some of the assessments used include objectives re-testing, pre-class assignments, reflective final, projects (with team written and oral presentation parts), etc. In addition, proctoring exams using Proctorio will be addressed. During the presentation, we discuss how these assessments were used, what worked/did not work well for each of them, challenges and struggles that occurred and overcoming them, and share some students' feedback as well as presenter's observations of the assessments' effects on students' learning.

Christopher Shaw, *Columbia College Chicago*
Assessing participation in the time of black squares
From asynchronous courses to synchronous to hybrid, the most difficult task for this teacher has been keeping students feeling engaged in a community of people doing mathematics, and then figuring out how to reward the students who contributed without penalizing the students who for one reason or another had to remain anonymous in class. In this talk I'll mention some successes (have your students write their own textbook in real-time!), some failures (beg your students to turn on their cameras), and some of the in-between.

Brian (BK) Katz, *CSU Long Beach*

Implicit Assumptions in Assessment
Students come to our courses with many beliefs about assessment based on their past experiences, and we bring our own too. My assessments focus on synthesis, which is often unfamiliar to my students, and which is built on a perspective about teaching and learning that usually that differs from my students' in fundamental ways. If left implicit, these differences in perspective can make assessment ineffective or stressful. In this session, I will attempt to make explicit some of these areas of difference in ways that I think will help students engage in synthesis assessments more effectively and comfortably.

Benjamin Braun, *University of Kentucky*
Assessment, Proficiency, and Compassion
The Covid-19 pandemic has reminded us that compassion and care must always be central to our work as teachers. At the same time, we are tasked with setting goals for student learning and with evaluating student proficiency for these goals. I believe that we can set a broad range of goals for students that include many necessary humanistic ingredients for learning, goals that represent authentic mathematical knowledge and practices. Further, we can design assessment structures for these goals that are are built on a foundation of compassion and an ethic of care. In this talk I will describe my approach to assessment and course design during the pandemic to illustrate these ideas.

Sarah Wolff, *Denison University*
Using portfolios and refection as an alternative to final exams
Over the last few years I have used a final portfolio and reflection assignment as a final exam substitute in several classes across our curriculum: linear algebra, introduction to proofs, and an upper level class. I will discuss the assignment setup, pros and cons within each different class and how I will modify these assignments in the future.

Geoff Krall, *University of Wyoming*
Teacher-designed mathematical portfolio assessments: motivations, potential benefits, and lessons learned
In order to identify the potential benefits and challenges of implementing student portfolios as quality mathematics assessment, a pilot study was conducted with teachers in various secondary school settings. The study examined factors that affected teachers' decision to institute a portfolio system and indications about students' academic dispositions as a result of using portfolios as assessment pieces. Results showed that compared to traditional multiple choice tests, teachers felt portfolios were a better reflection of their students' "extra-mathematical" skills, such as problem-solving and persistence while yielding similar results for mathematics content skills. Teacher surveys and interviews also demonstrated confidence that the use of portfolios as assessment material were superior to more traditional measures in terms of adhering to good pedagogical practices. From these results, this paper achieves a cohesive framework for the implementation of student portfolios in mathematics. The framework, which helps understand various models of mathematics portfolios and may guide future research, identifies three primary elements: (1) motivation, (2) the technical aspects, and (3) sustainability.

Joshua Bowman, *Pepperdine University*
Turning Standards into Writing Assignments
In the past year, remote instructional modalities raised questions about whether timed, proctored tests are the most appropriate tool for assessment of student learning. I have used standards-based grading for several years, but almost always within the context of traditional in-class exams, followed by scheduled

in-person reassessments. In order to engage students in rigorous assessments without the problematic aspects of remote proctoring, I converted the list of standards I had created for a business math class into a collection of writing assignments. Each of these challenged students to produce their own examples of specific types of problems and to submit solutions along with appropriate mathematical context. Student responses were overwhelmingly positive, not only because of the reduced stress from a lack of timed exams, but also from the recognition that creating and solving their own problems deepened their understanding and appreciation of the mathematical content. I will describe the implementation of this assessment method, along with its successes and difficulties, and propose ways that similar assessment methods may be used across a broad array of courses taught through in-person instruction.

Seongchun Kwon, *University of Central Florida*
Engaging students in learning in large online classes
During the pandemic, large face-to-face classes for the freshmen were transitioned to online classes at the University of Central Florida. I taught about 1200 students each semester during the pandemic. In this talk, I will discuss the changes in teaching strategies to engage students in learning and accommodate diverse learning styles, advising activities, and how I assessed their learning.

Danny Lara, *Central Washington University*; **Emilie Hancock**, *Central Washington University*
Supporting Students' Decision-Making Process during Problem Solving in Online Introductory Calculus
When the COVID pandemic moved our introductory calculus courses online, we turned to the plethora of available online resources to supplement our courses. While there exist many online resources supporting procedural fluency and explanations of key concepts, we felt there were limited video resources supporting students' decision-making during their problem-solving process. In this talk we reflect on our experiences as a mathematician and mathematics educator working together to create problem solving videos for our differential calculus courses that incorporate a problem solving framework (Carlson & Bloom, 2005) to guide students' decision making during problem solving. We will also share student work and reflections, and consider how we can incorporate these problem-solving videos into future in-person, hybrid, and/or online (synchronous and asynchronous) calculus courses.

Cristina Bacuta, *University of Delaware*
Motivational formative assessment in a synchronous online advanced calculus course
Before the pandemic, an advanced calculus course was designed to rely mostly on a summative type of assessment. After three semesters of emergency teaching online, we concluded that formative assessment must be implemented for monitoring student learning and providing immediate feedback in order to enhance, deepen and encourage learning and class participation.
We will share a group of approaches that worked and proved to be most motivational such as: Poll Everywhere® combined with group work, homework online with both automatic grading and "Show work" enabled, peer student work critique and others.

Tuyetdong Phan-Yamada, *Cal State Los Angeles*
MyOpenMath and GeoGebra - Tools for Formative and Informative Leaning Assessments
In a face-to-face classroom, instructors show an example and then ask students to do a similar problem. Instructors would go around to check if students do it correctly. These interactive tools can help instructors do the same in their virtual classrooms. Instructors can monitor individual or group work and provide feedback to students immediately. These free tools can be uses for all undergraduate Math courses including Statistics, Discrete Math, Linear Algebra and Differential Equations. Zoom in to see

how to use the following tools: GeoGebra classrooms and MyOpenMath for both informative and formative assessments. Attendees will get a copy of ready-made apps to use immediately.

LOUIS FREESE, *Rocky Mountain Region*
Using Play Posit and Nearpod as Assessment Tools in Remote Sessions
Play Posit and Nearpod provde opprtunities for instant assessment during live remote learning sessions. We will explore how we can create PlayPosit activities that can be broadcast during live sessions to engage students in interactive learning opportunities that provide the professor with immeadiate feedback as to how well the studnets are learning what is being taught. We will explore how we can layer Nearpod activities on top of PowerPoint presentations to engage students actively with slide content and provide feedback to professors on student understanding. These tools can alter the way way you teach and assess studnet understanding during live remote sessions.

Christoph Fischbacher, *University of California, Irvine*; **Alessandra Pantano**, *University of California, Irvine*
Competing for Connection: Using Virtual Trivia as a formative assessment
Remote instruction due to the pandemic very negatively impacted direct interaction and collaboration between students and the instructor, thus making it harder for the instructor to assess students' progress without relying on tests. In this talk, I will describe an innovative type of formative assessment which I utilized in an online (upper division) introductory probability course at the University of California, Irvine. During weekly virtual "Trivia Nights", students competed to solve interesting problem sets in small teams as fast as possible. This friendly timed math competition encouraged interaction and collaboration among students. The team members worked together in zoom breakout rooms, with the help of peer "learning assistants" (undergraduate students who had previously taken the class, who also received training in active learning teaching techniques). The winning team was formally recognized at the end of the Trivia Night, and praised by classmates. Participation in the competition only accounted for participation points, but provided an excellent opportunity for both the instructor and the students to gauge the teams' level of preparation. In this talk, I will narrate instructional challenges and successes that led to the creation of the "Trivia Nights." Utilizing student and Learning Assistant feedback gathered from surveys and interviews, as well as reflections upon own experiences, I will discuss the value of this formative assessment tool and strategies to improve it. This project was done in collaboration with Dr. A. Pantano.

Darren Narayan, *Rochester Institute of Technology*
Auto-grading Proofs in Discrete Mathematics
This talk will focus on experiences from teaching an online Discrete Mathematics class with 116 students. This will include strategies for creatively creating homework, quizzes, and exam problems involving proofs (including induction) that can be auto-graded. In addition problems involving modular arithmetic, algorithms, and combinatorial proofs will be presented.

Sheila Tabanli, *Rutgers University*
Redesigning Assessments for Increased Interactions, Reflections and Active Learning
Freshmen started the college in a new norm, lacking the much-needed social and academic interactions in a physical campus setting. The pandemic has forced us to design alternative assessments to foster student engagement which is negatively impacted by lockdowns and social isolations. In this presentation, we

will discuss the instructional strategies developed and implemented in a Freshman Calculus class for life sciences to build an online learning community. These instructional strategies include:
Incorporating alternative assessments including formal assessments of midterm exam corrections, and informal assessments of reflective writing, opportunities for self and peer evaluations, goal-setting activities, instructor feedback through a virtual town hall meeting enabling reasonable instructional adjustments.
For this presentation, participants are encouraged to engage in a compare and contrast activity to discuss students' weekly traditional quizzes and the bi-monthly exam corrections activities that focuses on reasoning, analysis, and incorporating writing into Math curriculum. For the future, it is important to assess current practices and develop more alternative assessments that would support student interactions, personalized learning experiences and opportunities for students to grow as critical thinkers.

Holly Attenborough, *University of Wisconsin-Platteville*
Oral Exams for College Geometry
With the transition to alternative delivery (hybrid and online courses) and the pervasiveness of online solutions, it became evident that my traditional math exams needed revision. Because of this, I integrated oral exams into my senior level geometry classes (spring 2020 and 2021). This presentation will discuss how these exams were executed, what went well, what I might change, and why I will continue to use them even after a return to "normal".

Eugene Yablonski, *University of the Fraser Valley*
Online assessment in STEM courses through student presentations
Online exams in the 1st year math courses suffer from numerous drawbacks:
- Online proctoring has privacy concerns;
- Unsupervised exams are vulnerable to cheating;
- Finding good problems for unsupervised exams is a major challenge:
  - All textbook questions have solutions online;
  - Sites like Symbolab produce step-by-step solutions for all standard computational problems;
  - Students have been typically trained on standard problems most of their life. Most of textbook examples and problems are also quite standard. So students tend to perceive non-standard exams as unfair.
- Marking and annotating hand-written PDF is a chore.

Alternative assessment techniques, such as students' in-class presentations can save time and produce a more reliable evaluation of students' abilities.
While it's not feasible to test each and every topic this way, this is the usual trade-off: Breadth vs. Depth. In this talk, I will discuss my experience of having all Calculus students presenting problems on an online whiteboard.
This turned out to be the most enjoyable part of my online teaching.

Jenna Carpenter, *Campbell University*
Using Daily Discussion Boards to (Virtually) Assess Concept Mastery
I have used Minute Papers for years as a quick end-of-class assessment. With the onset of the pandemic, my math classes pivoted to an online, synchronous format. Given the capabilities of our online course management system, I decided to replace Minute Papers with daily discussion board posts. This talk will outline how the discussion boards were used and how effective they were at assessing and engaging students.

Katarzyna Kowal, *Ramapo College of New Jersey*
Teaching, Assessment and Directing a Virtual Multi-Section Course
The author is a director of a multi-section Elementary Probability and Statistics course at a liberal arts college, which became a virtual course during the pandemic. Most sections of the course are taught by adjunct faculty. The author was also the department chair of mathematics during the time the pandemic came. At that time the author created and suggested to implement within the department a number of teaching and assessment strategies for virtual courses, and she has been updating these strategies ever since. In this paper the author will present the resulting extensive list of technology methods that can be applied to teaching any course virtually in the future, and an effective exam proctoring protocol developed by the author that prevents students from committing acts of academic dishonesty during exams that are taken by students live in WebEx. The benefits of using a small personal document camera while lecturing from home will be discussed. The author will share the successes and challenges of using in practice all the strategies mentioned above. Key points of directing a virtual multi-section course will be discussed.

Closing Wallets while Opening Minds: Adopting Open Educational Resources in Mathematics

Part A: *Saturday, August 7, 10:00 a.m. - 11:55 a.m.*
Part B: *Saturday, August 7, 1:00 p.m. - 1:40 p.m.*

Organizers: Britney Hopkins, *University of Central Oklahoma*; **Benjamin Atchison**, *Framingham State University*; **James Quinlan**, *University of New England*; **Rob Beezer**, *University of Puget Sound*; **Oscar Levin**, *University of Northern Colorado*; **Sean Laverty**, *University of Central Oklahoma*; **Emily Hendryx**, *University of Central Oklahoma*

Taylor Kilman, *Indian River State College*
Open Mathematics: How to Reduce the Cost Burden for Entry-level Mathematics Students
Textbooks, online software, and other course materials pose a significant cost burden for entry-level mathematics students. Might there be a better way? Yes. The adoption of open educational resources (OER) can reduce or eliminate such costs. The presenter of this session will detail how he uses OER textbooks, online software, and custom course notes to drastically reduce the cost burden of his and his colleagues' students.

Krassi Lazarova, *Centenary University*; **Kathy Turrisi**, *Centenary University*
OER for College STEM courses
There is a trend in higher education to reduce and eliminate textbook costs to students. Higher education institutions are looking for more affordable and easily accessible learning resources. Using such resources removes price barriers, and assist in student retention and graduation. This presentation is sharing the experience of initiating and promoting OER on campus for intro. One open resource, OpenStax, is assessed in more details. Tips for its implementation in the classroom are presented. Additional resources are also reviewed and recommended.

Md Sazib Hasan, *Dixie State University*; **Vinodh Chellamuthu**, *Dixie State University*
Advancing Student Learning through Customized Open Education Resources
The purpose of this presentation is to discuss the strategy to foster active learners through the creation of "customized" OERs (C-OERs)" curriculum that provide deeper conceptual understanding. In this

presentation, we will share how we are developing C-OERs in collaboration with our students and a sample of student experiences in developing the C-OERs, along with sample artifacts. We will also discuss the rationale, challenges, and benefits of C-OERs and how it impacted their learning process through this authentic learning environment.

Ben Vanderlei, *University of the Fraser Valley*
Jupyter Books
Jupyter is an open-source platform that allows documents to be created and shared via a web application. These documents, known as notebooks, can be a mix of text with Latex formatting, graphics, and live code to carry out computations in a variety of languages. Jupyter Book, which is part of the Executable Book Project, provides the authoring tools needed to build interactive digital books from collections of Jupyter notebooks. We will show some examples of Jupyter notebooks and Jupyter Books, as well as provide instruction on how to get started creating your own content with Jupyter Book.

Chris Oehrlein, *Oklahoma City Community College*
Using a Free Geometry App and Activity-Based Text in a Course for Elementary Ed Majors
Most of the pre-service elementary school teachers who take our Geometry and Measurement course are taking courses at multiple institutions, and are at varying places in their progress towards a degree and a job. The course is also now and in the foreseeable future being offered in a 100% remote format. It became imperative to find course materials and resources to support the needs of the students and logistics of the course. The presenter will share his experiences in pivoting from a planned hybrid format to fully remote, and will share some of his students' reactions to the free software and online textbook used in conjunction with some of the features of the college's LMS.

Yun Su, *Indiana Tech*
Use Kahoot and GeoGebra to Engage students and Enhance learning
This presentation shares using Kahoot and GeoGebra to increase student engagement, encourage collaborations, and create an active learning environment. This is a hands-on, interactive experience to share tools, tips, and lessons learned from my classrooms.
Kahoot Game is an online game-based learning platform which helps us interact with students and test for their understandings in a fun way. Sample Kahoot Games will be shared.
GeoGebra is a free interactive application. I provide students guide to create their GeoGebra projects and share their projects with the public. These dynamic visualizations can increase students' conceptual understanding.

Kyle Claassen, *Rose-Hulman Institute of Technology*
Plotting Mathematical Structures in Minetest
Minetest (minetest.net) is a open-source voxel game engine that features gameplay similar to that of Minecraft. Minetest possesses many qualities that are desirable in educational software--it is freely available, has modest system requirements, runs on a wide variety of operating systems, and is purposefully designed so that the community can contribute to it. In this talk I will demonstrate a mod that I have created for plotting parametric and implicit curves/surfaces/solids in the game's blocky world. Moreover, I will discuss how I have used this mod in my own classroom, including a (climbable!) introduction to triple integrals as well as illustrations of phase portraits for systems of differential equations. I will also showcase some interesting student projects created in Minetest for my vector calculus course.

Robert Niemeyer, *Metropolitan State University of Denver*
Rethinking video formats and content delivery in a digital, post-pandemic world.
The pandemic and teaching online (both synchronously and asynchronously) have brought to light many strengths and weaknesses in how content gets delivered. Video format typically require large amounts of storage on servers and are difficult for students to view remotely if they lack the proper resources (high-speed internet, suitable computer, etc.). While effective in conveying information using a variety of approaches, videos do have limitations. In light of this and in an attempt to make the experience more equitable for my students, I have been developing a library of interactive slide-shows. These slide-shows emulate the traditional video experience while also incorporating in the same show a fully-functional Desmos graphing calculator (not an i-frame, but an API) and automated, computer-generated hand-writing to encourage proper writing in mathematics. The latter recreates a natural 'lecture' experience by using natural hand-writing fonts in LaTeX and then animating the writing using various commands in Javascript and HTML. Using what is called the Ractive-Player (no mispelling), the whole experience can come together seamlessly to create a professional video at 1/100th of the the file size as a traditional lecture video, and have tremendous interactivity and flexibility for incorporating other technologies. Not only do high-quality, interactive slide-shows benefit our students and allow for the creation of exercises in-show, such content is light-weight and easily accessible to anyone with a basic internet connection, including inmates in the prison system. The goal of OER is not to make content free or affordable for just our students, but for anyone seeking to learn.

Computational Investigation in Undergraduate Mathematics

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Friday, August 6, 10:00 a.m. - 11:55 a.m.*
Part C: *Friday, August 6, 1:00 p.m. - 1:55 p.m.*

Organizer:
Matthew Wright, *St. Olaf College;* **Scott Zinzer**, *Aurora University*
Computation to build mathematical curiosity and wonder
Computational approaches often provide avenues for students to explore non-standard, “messy,” and even open problems in mathematics. When combined with other problem solving techniques, computational tools aid students in understanding new problems, discovering patterns, forming conjectures, and developing proofs. A discrete mathematics course often serves as a point of transition for students in their mathematical journey and frequently demands revised approaches to mathematical work. In this talk, I describe how I used programming projects in a discrete mathematics course to help students develop effective mathematical behaviors and to encourage them to explore unfamiliar and challenging questions.

Kristi Karber, *University of Central Oklahoma*
Students Utilizing Computational Tools to Enhance a New Online Course and the Resulting Inspiration for an Undergraduate Research Project
We will discuss the incorporation of student researchers and their use of computational tools to create desirable problem sets for a cost-effective online Math for General Education course. Moreover, a student researcher extended her work of creating Voting Theory problem sets by using the computational results to form conjectures regarding sufficient initial conditions that would produce a desired outcome. These conjectures ultimately led to an original theorem and proof.

Antonio Martinez, *San Diego State University*
Set theory and logic: Leveraging computing as a mediating tool for learning
A growing area of interest in undergraduate mathematics education is the need for (and influence of) computation/programming in the undergraduate mathematics curriculum. In line with this interest lies my goal of characterizing the ways in which computing can be used as a tool for the teaching and learning of mathematical set theory and logic. In particular, the focus of this contributed report is on the development of students' ways of reasoning about the logical operators `and` and `or.` Specifically, my research investigates how students can use Python in increasingly sophisticated ways that eventually lead to the use of the `and` and `or` operators in For Loops to find the union and intersection of finite sets. The data for this study draws on a teaching experiment which consisted of five, one-hour interviews with four groups of two to three students in each group. Data collection occurred in a staggered approach with Groups 1 and 2 completing the first three sessions before Groups 3 and 4 began. By incorporating a cyclical approach to data collection, slight adjustments and modifications were made to the tasks to hone in on discussions and ideas that informed the inquiry driving this research. By the end of the teaching experiment cycle, all students showed a significant proficiency in the use of the logical operators `and` and `or` with Groups 3 and 4 showing flexible use of the logical operators in For Loops to create new finite sets.

Karen Briggs, *University of North Georgia*; **Caylee Spivey**, *University of North Georgia*
Computational Explorations in Abstract Algebra
While working through an abstract algebra homework problem, the authors stumbled upon a pair of integers modulo 50 that are both multiplicative and additive inverses of one another. Naturally, we wondered whether this phenomenon was purely coincidental or whether such pairs exist for other moduli. Initially, we began exploring the problem by hand, but soon found that much more data was needed in order to arrive at any reasonable conjectures. So, the professor tasked her student with developing a MATLAB program that would output the moduli for which such pair(s) existed together with the inverse pair(s). From the computational output, the authors were able to make several conjectures regarding the existence, number, characterization, and construction of the inverse pairs. We then leveraged a few topics from the abstract algebra course such as the Chinese remainder theorem and the fundamental theorem of cyclic groups together with some more advanced topics from number theory to prove those conjectures.

Matthew Just, *University of Georgia*; **Maxwell Schneider**
On a divisor of the central binomial coefficient
Based on initial computational observations we made, we prove a new equidistribution result for a divisor of the central binomial coefficient in a variety of combinatorial contexts. We also present interesting and far-reaching applications related to sums and products of integers, and to polynomial division. This project is a result of the authors' mentor-mentee collaboration in the Undergraduate Research Program at UGA.

Jeb Collins, *University of Mary Washington*
Investigating Competitive Graph Coloring with Unity
In this talk we discuss the use of the game engine Unity to develop software to investigate questions in competitive graph coloring theory. This is an area of math in which games are played on graphs by two players, usually involving the players taking turns properly coloring the vertices or edges of the graph. This is an ideal area for undergraduate research, as there is little previous knowledge needed to start, but there is deep mathematics involved. Investigations into this area invariably involve playing this game

many times on multiple different graphs, often while trying to keep track of all the moves. While this initial get students interested, it quickly becomes difficult on paper. Unity is an ideal platform for creating software to play these games, as it is a game engine. It has a nice balance between full control of the program, and ease of handling graphical interface and user input. This offers an ideal middle ground between a low-level language like Python, and a more abstracted platform such as Sage. We will demonstrate one type of graph coloring game that has been implemented with Unity, and demonstrate how it can be modified to investigate similar games.

Robert Niemeyer, *Metropolitan State University of Denver*
Using computer simulation to understand fractals and billiards
The advent of the personal computer and graphics cards changed how mathematics could be visualized. Fractals were mere pathological examples reinforcing the necessity for certain conditions, pointing out the pitfalls in our logic early on. Dynamical systems, especially mathematical billiards, are typically studied using analytical tools, and rightly so. But with computational software such as Matlab or Mathematica, much can be gleaned through computer simulation and experiment that would be otherwise missed by typical analytical investigations. When students are able to simulate certain situations, not only can they confirm their conjecture as being true in particular situations, but also ask more questions. Rarely, if ever, does someone say "EUREKA" when doing anything in STEM. Rather, as we all know, we typically say, "That's unexpected!" Experiments and simulations beget more questions and expand our understanding of a situation, giving us a clearer path to proving rigorous statements. In addition to numerical simulations, students can also use symbolic math programs to _prove_ certain statements, as is the case many times in my own research in fractal billiards. I will talk about certain software packages and computer programming languages that allow for the building of computer simulations that both give insight and allow us to produce computer-aided proofs of conjectures.

Feryal Alayont, *Grand Valley State University*
Computational Discovery-Based Investigations in Calculus
With many computational tools available online, it is easy for students to perform experimental calculations in calculus. These experiments, often conducted outside of class, can then serve as the foundation for inductive reasoning in the classroom to create definitions, make generalizations, come up with methods, describe some general concept and more. In this talk, I will share examples of pre-class and in-class activities from calculus classes using computational experimentation to discover ideas of calculus, and describe how these activities are assessed and the student feedback on these activities.

Sedar Ngoma, *SUNY Geneseo*
Inclusion of Computational Methods in Undergraduate Mathematics
Computational methods are getting more and more popular due in part by their capabilities to help visualize and concretize abstract concepts which are not easily observable. Moreover, they may provide intuitive ideas that could lead to the proof of a mathematical topic. Finally, they permit approximations of solutions to real-world problems. It is therefore imperative to incorporate computational mathematics in undergraduate mathematics curriculum so as to initiate students to computational thinking. In this talk, we discuss the need to develop computational solutions from analytical solutions to mathematical problems, we outline the necessity to use computational methods to investigate mathematical topics, and we illustrate the substantial inclusion of computational tools in undergraduate mathematics research.

Paul Becker, *Penn State Behrend*
Matrix representations in introductory group theory.

Matrix representations provide a unifying concept across abstract algebra, linear algebra, and geometry. Almost all finite groups encountered by undergraduates admit representations as multiplicative groups of concise, block-diagonal, binary matrices. We describe a computer-lab component of an introductory group theory course. Students computationally explore the ``abstract'' concepts introduced in the course: group actions, subgroups, normality, cosets, quotient groups, homomorphisms, isomorphisms, and kernels.

Yevgeniy Galperin, *East Stroudsburg University of PA*
An Image Processing Tour of Undergraduate Math
We discuss the use of basic and advance image processing methods to provide meaningful context for reviewing key topics of the college mathematics curriculum, to help students gain confidence in using concepts and techniques of applied mathematics, to increase student awareness of recent developments in mathematical sciences, and to help students prepare for graduate studies.

Boyan Kostadinov, *New York City College of Technology, CUNY*; **Ariane Masuda**, *New York City College of Technology, CUNY*; **Nadia Kennedy**, *New York City College of Technology, CUNY*
Computational Modeling with Real-World Data for Prospective Mathematics Teachers
This presentation describes computational modeling projects, which involve using real-world data and computing with R and Python. We use dynamic R Markdown documents in RStudio for unifying plain text narrative, mathematical typesetting with LaTeX and computing and visualizations in R and Python. We will showcase a number of projects such as fitting models to astronomical data, rediscovering physics laws, modeling oil spills and tumors. We will also demonstrate how computation and visualization can help students develop their mathematical maturity and intuition in the context of purely mathematical concepts. The projects have been designed to engage prospective and in-service mathematics teachers in exploring computational modeling and problem-solving. The projects will be piloted with prospective teachers and high-school students as part of our preparation and design of a Computer Science Teaching Certificate Program for mathematics teachers.

Creating Relevance in Introductory Mathematics Courses

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Wednesday, August 4, 1:00 p.m. - 3:40 p.m.*

Organizers:
Tracii Friedman, *Colorado Mesa University*; **Lisa Driskell**, *Colorado Mesa University*

Alice Petillo, *Marymount University*
Experiencing Mathematics as Relevant: Classroom-tested Stand-Alone Activities
How should we think about the concept of relevance with diverse undergraduate students in introductory classes? What are some easy-to-implement ideas that help students use current topics to help them experience mathematics as relevant? The presenter will provide sample assignments and rubrics that were developed and improved over the course of several semesters. Student feedback related to the

assignments was positive and encouraging. Participants will have sample assignments and grading rubrics they can use as is or adapt to their particular situation or course management system.

Tracii Friedman, *Colorado Mesa University*
A Quantitative Reasoning Course Redesign
Traditional quantitative reasoning courses often tackle a variety of topics that students may find to be disconnected from and irrelevant to their future careers outside of STEM. At Colorado Mesa University, we have redesigned our entry level QR course with a focus on what this particular group of students needs in their (very likely) last mathematics course. We are employing a modified flipped classroom approach with daily class activities that engage students in relevant applications of topics as well as carefully scaffolded projects in which students investigate real-life scenarios that utilize course topics. We have implemented the redesign in all 26 of our QR sections this past academic year with very positive outcomes. In this presentation, we will describe the details of the redesign, course projects, and share preliminary results.

Jeneva Clark, *University of Tennessee, Knoxville*, **Jonathan Clark**
The Beautility of Math: A Mathematical Reasoning Course
The beauty and the utility of mathematics are two conceptions that are not commonly held simultaneously. However, attending to both aesthetics and usefulness brings greater relevance to the content. In this talk, we present a redesigned course for non-STEM majors. It deemphasizes computation, opting more for qualitative reasoning skills. As such, the textbook we have written for the course, "The Beautility of Math," presents content without overreliance on jargon or equations, instead focusing on understanding the relevant concepts. With an intentional blend of utility and beauty, we leverage both extrinsic and intrinsic motivations, giving multiple senses of relevance for students.

Jacob Duncan, *Winona State University*
Saving the World with Mathematical Modeling: An Introductory Course in Sustainability-Math
Some of the biggest challenges facing humanity today stem from issues surrounding unsustainable environmental, social, and economic practices. The need to address the ramifications of these issues from a STEM perspective is greater than ever. This talk centers around a recently developed course called Mathematics for Sustainability which applies an array of mathematical concepts and tools to quantitatively explore real-world, topical problems pertaining to sustainability. Topics are motivated by exciting hands-on experiences, e.g., experiments, demonstrations, and outdoor data collection excursions. This introductory course is designed primarily for undecided and non-STEM majors with the intent of sparking interest in the usefulness of mathematical modeling in solving the world's pressing environmental, social, and economic problems.

Brendan Sullivan, *Emmanuel College*
Suggestions for a survey course for liberal arts students that is not like "a traditional math class"
Over the past year, I have improvised the redesign of an introductory survey course in mathematics for non-science students at a small liberal arts college. In the midst of the global pandemic and a remote learning environment, I decided to throw away everything that students might traditionally associate with "math class": we had no exams, no textbook, and no formal grades (until the end of the semester). As a result, I found that students were more open to new and challenging ideas and I believe they ultimately learned more about what it means to do mathematics and who mathematicians are. In this talk, I will share my general approach to teaching the course, as well as some specific examples of class activities, perhaps including: a "deep dive" on a viral TikTok video, ways to prompt a discussion of "what mathematics

really is", and reading assignments about discrimination and other struggles in the professional mathematics community. Throughout, I will share student comments that illustrate the effectiveness and (hopefully) long-lasting impact of this course on the students' lives.

Chamila Ranaweera, *Southeast Technical College*
Enhance Students learning by introducing real life problems and examples into Quantitative Reasoning Course
Many students complete mathematics courses to fulfil the program requirements rather than seeing any use of the content in their daily life. MATH 103 Quantitative reasoning course was designed to addresses this concern by teaching mathematics as a life skill.
Course was designed by carefully choosing the book chapters that discuss real life applications such as financial mathematics, probability, statistics, and mathematics in sports. Excel was introduced as a learning tool to create charts, graphs, mathematical calculations, and monthly budget sheets. Students were taught to create their monthly budget sheet, weekly schedule for study time as a time management tool, calculate the semester GPA using Excel during the first half of the semester as lab activities. Student feedback was recorded as lab reflections regularly.

Students positively commented that they can use Excel for their daily life such as budget management sheet, keeping record of personal activities. Students were motivated in the class to learn the subject material by seeing the connection to their life applications rather than being anxious about learning new technological tool to complete course requirements.
Introducing real life problems and applications to mathematics curriculum is useful improving students motivation, course performance and knowledge retention.

Erika Ward, *Jacksonville University*
Abstract Mathematics can be Relevant: I Used it to Paint My Bathroom!
When I selected the topics the first time I taught our then-new core level, non-algebra based math course (Discover, Decode, Decide), I was hoping to grab my students attention, share mathematics I thought was engaging, and chose topics I had worked with at least a bit before. I ended up with what felt to me like some of the most abstract math I could teach at the freshman level: a combination of graph theory, voting theory, and cryptography. Yet my students routinely tell me that it's the most useful and relevant math class they've ever taken and describe their real-life applications of the ideas. Through the lens of student reflections, I share some of the assignments and materials my students have tagged as relevant, and describe the approach that seems to elicit that response.

Lydia Kennedy, *Virginia Wesleyan University*
Using the card game SET in a General Education math class
In January 2020, the book The Joy of SET was used to teach a general education math class typically targeted at math-phobic students. This talk will outline the mathematics taught using SET, experiences teaching the class and the response of the students to the course.

Lisa Driskell, *Colorado Mesa University*
Modeling Ebola Spread in Introductory Courses
Modeling across the curriculum can begin in the lowest level math courses. Students in introductory courses engage more deeply with course activities that relate to their lived experiences. We describe a modeling activity that began as a project for differential equations and has since been modified to help bring relevance and incorporate data into introductory courses. While the data used comes from the 2014

Ebola outbreak in West Africa, the the activity structure of building and testing different growth models can be used to investigate various scenarios.

Erin Williams, *University of Central Oklahoma*
Math for Gen Ed: Car Loan Exercise
Students who need one mathematics course can typically take Mathematics for General Education. In this course we touch on a variety of interesting topics: number theory, different number systems, statistics, voting, and finances. During the finances section I found many students were both engaged and confused. Many textbook questions try to be realistic: "John purchased a car for $20,000." Who is John? Is this car expensive? Cheap? Would I buy this car? Because of this and the importance of financial literacy I created a Financial Worksheet where the students would go through the numbers of purchasing a realistic car. They looked for a car they were interested in purchasing as well as trade in value of their current car, interest rate percent, loan length, monthly payments, etc. The students also reflected on how the assignment changed their past views and any financial habits that might change, if at all. In this talk I'll show the two assignments, describe grading, as well as give a broad view of their personal reflections.

Holly Attenborough, *University of Wisconsin-Platteville*
Discussion Boards and a Math in Culture Assignment in a Mathematics General Education Course
In the fall of 2020, I taught a math for the liberal arts class at UW-Platteville called Mathematical Explorations. The course material was delivered synchronously over Zoom. For community building and student engagement, discussion boards on our LMS (Canvas) were integrated into the course design. For the first few discussions, the students watched growth-mindset videos from Youcubed.org (created by Jo Boaler of Stanford University), wrote response posts and replied to each other. The discussions then shifted to preparing for a presentation on "Math in Culture," giving each student an opportunity to explore a mathematical topic that they found interesting. The students then uploaded their "Math in Culture" presentation to a discussion board, as a 5 to 12-minute video (or PowerPoint with voice), to which other students replied. The online discussions were introduced to fill a community-building need created by the pandemic, but I plan to continue using these discussions after a return to in-person instruction due to their valuable contribution to the class.
These discussion boards and presentations could be easily incorporated into other courses. Adopting a growth mindset is beneficial for any student's success in college. In the past, I've done a poster presentation on applications in a higher-level math class (Linear Algebra). This and other types of presentations could easily be adapted to a presentation in an integrated LMS Discussion Board.
During this talk, I will give an overview of my implementation of these discussion boards and the "Math in Culture" assignment.

Cross-Curricular Applications for Pure Mathematics Courses

Thursday, August 5, 11:00 a.m. - 11:55 a.m.
Organizers:
Elizabeth Donovan, *Murray State University*

Lucas Hoots, *Centre College*
Lesley Wiglesworth, *Centre College*

Vladimir Minev, *retired*
An Introduction To Dialectic Mathematics
Creation of an dialectic triangle

Donna Beers, *Simmons University*
Renewing Elementary Linear Algebra Courses with Activities in Data Science
The increasing presence of data science and computer science students in elementary linear algebra has inspired us to give a fresh look at this course, from finding relevant applications, to thinking about how to order and motivate topics to build and sustain student interest. Nearly all linear algebra students are familiar with recommender systems. Many data science and computer science students are curious about machine learning and have heard of singular value decomposition. They have enrolled in the course to learn the linear algebra behind these topics of current interest.
Traditional elementary linear algebra textbooks cover many applications, including population biology, Markov chains, and cryptography. Our goal is to update and enlarge this application set. We will describe group work activities that we have developed where students apply essential linear algebra theory to analyze data and to reduce the dimension of a set of data. Students use vectors and matrices to represent data, e.g., user movie ratings; they measure the Euclidean or Cosine distance between vectors (or measure the distance between matrices with the Frobenius norm), e.g., to measure similarity of user taste; they calculate eigenvalues and eigenvectors to reduce the dimensionality of a set of data by reorienting it in directions that give maximum variance; and they perform singular value decomposition on a movie ratings matrix to build a model for a recommendation algorithm.

Joan E. DeBello, *St. John's University*; **Erald Troja**, *St. John's University*
The Importance and Impact of Mathematics and Cryptography in Cybersecurity
Mathematics and Cryptography play a crucial role in all aspects of cybersecurity. This presentation will discuss the importance and impact of mathematics and cryptography in the field of cybersecurity. It will discuss mathematics courses and topics essential to the cybersecurity undergraduate degree and techniques and tools used to engage the students in the curriculum.

Ethnomathematics: Culture Meets Mathematics in the Classroom

Part A: *Thursday, August 5, 11:00 a.m. - 11:55 a.m.*
Part B: *Thursday, August 5, 1:00 p.m. - 1:40 p.m.*
Organizers: Ximena Catepillan, *Millersville University of Pennsylvania;* **Janet Beery**, *University of Redlands*; **Cynthia Taylor**, *Millersville University of Pennsylvania*

Ann von Mehren, *University of Memphis*
Sigma: Consideration of the mathematical aspect of "the sum of things" in an ancient Greek text of rhetoric
A passage in a Greek rhetorical text, "Antidosis," by Isocrates, suggests that students at the level of philosophy should know Sigma, "the sum of things," whether for basic bookkeeping or for advanced cosmogony. Students can be invited to inquire into how Sigma is found and expressed across cultures.

Cynthia Huffman, *Pittsburg State University*; **Ximena Catepillan**, *Millersville University*; **Scott Thuong**, *Pittsburg State University*

Mathematical Mysteries of Rapa Nui with Classroom Activities

One of the most remote inhabited locations in the world, the island Rapa Nui, also known as Easter Island, is shrouded in mysteries. When was it inhabited, by whom, and how did they survive the journey? Why were the large moai statues carved and how were they transported from the Rano Raraku quarry to the ahu platforms erected throughout the island? In this presentation, after a brief introduction to Rapa Nui, we look at three mysteries: the partially undeciphered script called Rongorongo, the geometrical shape of the stone foundations of ancient houses called hare paenga, and the question of whether mathematics could have been used in laying out the foundations of those houses. We also share four classroom-ready student activities based on the mysteries surrounding the hare paenga.

Darrah Chavey, *Beloit College*

Symmetric Designs of Mirror Curves Inspired by African Sona

The Cokwe (Tchokwe, Chokwe) people of Southwest Africa developed a drawing technique that creates curves within a grid of dots with "mirror" edges, and internal walls, where the curves bounce, following tightly constrained rules. These drawings are closely related to Celtic knots and Indian Kolam, but are even more mathematical than those traditions. The Cokwe tradition includes a strong preference for monolineal circuits (Eulerian circuits) and for symmetric drawings. Inspired by these designs, we were able to construct symmetric monolineal Sona drawings for essentially any size grid, where the grid itself had central, strip, or wallpaper symmetry, while using any of the rectangular symmetry groups (central, strip, or wallpaper). These results create dense infinite families of designs which we believe would meet the aesthetic criteria of the classic Cokwe artists. We discuss the ways we have used the search for such patterns as creative exercises in an ethnomathematics classroom.

Thomas Gilsdorf, *Central Michigan University*

Mathematics within, mathematics without

This talk is about people who learn mathematics without some aspect we typically associate with school mathematics. In such cases, the people have some knowledge of the relevant mathematics within themselves, as can be observed using Bishop's Six categories of mathematics. The contexts include cultures in which people know mathematics without a lot of number words, without writing, or without having formally attended school (or very little formal schooling). Examples will be described in which a culture has only a few number words but makes use of mathematical structures equivalent to the dihedral group of symmetries of the square. Another is from a collection of artists who have little or no formal schooling but create three dimensional artwork that has complex symmetry patterns. Other examples will also be presented. This research is based in part on interviews with traditional artists in Mexico, as well as an ongoing project involving two cultural groups in India, and a planned project related to a cultural group in China. The fundamental question that arises from the study is: How can we connect the learning and knowledge of mathematics that occurs outside of typical schooling with school mathematics, especially as such connections have to do with students whose backgrounds are from a variety of cultures.

Antonia Cardwell, *Millersville University of PA*; **Erin Moss**, *Millersville University of PA*

Teaching a Project-Based Ethnomathematics Course Online

Millersville University, a mid-sized undergraduate institution, offers an introductory course in Ethnomathematics intended primarily for non-math and non-science majors as part of their General Education curriculum. Historically, this course has been taught via in-person and partially-online modalities. Recently, the course has been offered fully online. We will share our experiences teaching a

project-based course online, including how we provided opportunities for student interaction, and some mechanisms that we used for assessment. We will include a selection of topics covered and share more details about the implementation of the course. We will discuss some of the successes in recent offerings of the course and some challenges that have arisen, and will also include perspectives from students who have recently participated in the online version of the course.

Games in Math Circles

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Friday, August 6, 10:00 a.m. - 11:40 a.m.*
Organizer:
Edward C. Keppelmann, *University of Nevada Reno*

Rodi Steinig, *Talking Stick Math Circle*
A Hodgepodge of Nontraditional Games
Merriam Webster's definitions of games include the terms "rules," "strategy," "struggle," "procedure," "diversion," and "amusement." In Math Circles, how can we play games that are cooperative? How can we use role-playing? How can we entice reluctant students into competitive games? What games can we play with the youngest of students? What games can be used to help students focus and build frustration tolerance? How can we add complexity to classic games to facilitate deeper mathematical explorations? In this session, we'll play "Would You Rather," "Math Lawyer," "Odd One Out," "Knights and Liars with Puppets," "The Need for Numbers," "Mathematician Cards," "Math Red Light Green Light," "Mother May I Addition," "Opposite Simon Says," and "How Is this Math?" What do all these games have in common? Each involves strategies for success and meaningful mathematical thinking. Audience participation is encouraged but not required!

Rosa Aristy, *Bridges to Science*
Giotto - A Joyous Word Puzzle
What happens when you give your math circle students pieces of paper, pencils, and a secret word to guess? Hours of fun as students use words and logic to guess five letter secret words! Giotto is considered one of the most engaging word games of all times. The object is to find out the letters in the secret word and the secret word itself by a process of elimination. Math circle students have a fantastic time as they collaborate to decode secret words. Giotto is suitable to play in teams and just as captivating virtually as in-person. In this session we will play a brief "exhibition" game, discuss strategies, and variations of Giotto.

Skona Brittain, *Santa Barbara Math Ellipse*
Games Galore
Games have always been a big part of my life. When I homeschooled my children, the curriculum we used was - games! I play, playtest, design, create and analyze games. And we do all of those activities in my math circles. I will discuss these roles for games that are various combinations of fun, interesting, and pedagogical. And also my goal of finding a game for each math circle topic.

Philip Yasskin, *Texas A&M University*; **Carl Van Huyck**, *Julia Robinson Mathematics Festival*; **Joshua Goldstein**, *Texas A&M University*

The TAMU Math Circle Apps Website

In the fall of 2020, the TAMU Math Circle switched to an online format. We needed to find activities that worked online over Zoom. Among other activities, we decided that many classic two-person games might work. We invited Carl Van Huyck , who was already programing for Phil Yasskin, to help program these games. These have grown into the TAMU Math Circle Apps collection which are available at the MYMathApps website:
https://mymathapps.com/mymacalc-sample/MathCircleApps/MContents.html.
We will discuss the classic games: Subtraction, NIM, Hex Bridges and Square Bridges along with extensions we call Odd Subtraction, Odd NIM, Tip The Die and Pop Tac Toe. The website also includes activities for conceptualizing radian measure, counting Euler Numbers, counting colorings of the vertices of a cube and conceptualizing the right hand rule.

Theodore Alper, *Stanford Online High School, Stanford Math Circle*

The Winner's Curse

In auctions for which bidders must estimate the true value of the auctioned item, the winning bid is likely to come from an overestimate, and so the winner tends to overpay. The more people participate in the auction, the more pronounced the effect, which leads to the counterintuitive conclusion that you should lower your bid when more people enter the auction. This accessible mathematical concept with surprising consequences feels very math circle-appropriate, but how to make a simple bidding game for middle or high school students that can motivate the discussion?

Brandy Wiegers, *Central Washington University*; **Emilie Hancock**, *Central Washington University*; **Dan Zaharopol**, *BEAM*

Math Circles in Times of Physical Distancing

We all learned new ways to share math over the year of physical distancing. This talk will share how our undergraduate mentors worked to move games and play online for the Kittitas Valley Math Circle (https://www.cwu.edu/math/kittitas-valley-math-circle). The talk will also share the Journal of Math Circles upcoming special issue, "Math Circles in Times of Physical Distancing," which will capture the best practices in physically distance Math Circles (https://digitalcommons.cwu.edu/mathcirclesjournal/).

Edward Keppelmann, *University of Nevada Reno*

INVERSE

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The game Invers\'E from Family Games Inc by Niek Neuwahl 2006 is a unique 2-player NIM type game played on a grid with just 5 kinds (different colors and shapes with one for each player) of 3-dimensional block pieces and 3 simple rules:
\begin{enumerate}
  \item Identical pieces must be played in different ways (either vertically/horizontally or on different sides).
  \item Pieces of the same color or the same height cannot touch on a side.
  \item Pieces cannot be stacked (i.e. each piece must use the base grid).
\end{enumerate}
The winner is the last player to make a legal move. In this talk we will report on some computer simulations of this game and the existence(or not) of an interesting optimal strategy. We will consider how can these results be discovered/explored in a math circle setting.

Nicole Fider, *University of Arizona*
A Tale of Tic Tac Toe: a day of Student Curiosity and Exploration
Assigned to a six-student once-weekly workshop for University of Arizona students (loosely labelled as "supplementary" to the proofs course), I made it my goal to encourage the creative and social aspects of mathematical problem solving. I spent several workshops prodding them to work together but was met with
resistance. So, as an icebreaker, I decided to split my students into two teams and I pitted them against each
other in a game of Ultimate Tic-Tac-Toe. The students were so intrigued that they asked, "Next week, can we
just play this the whole time?" I compromised, and during the next meeting they explored variations of Tic-Tac-Toe with each other for over an hour. In this brief talk, I describe how that day unfolded, what my students learned from their day of play, and what I learned from my students

Inquiry-Based Learning and Teaching

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Wednesday, August 4, 1:00 p.m. - 3:40 p.m.*
Organizers: Nathaniel Miller, *University of Northern Colorado*; **Parker Glynn-Adey**, *University of Toronto*; **Mami Wentworth**, *Wentworth Institute of Technology*

Marina Tvalavadze, *University of Toronto, Mississauga*
Application of IBL in teaching advanced math classes
In this talk, I will share my experience of teaching the advanced course on Galois Theory using IBL at the University of Toronto, Mississauga. I will focus on the course structure, course materials, discuss a typical session and other notable moments. I will also talk about benefits and challenges of using active learning such as time management, students' participation, etc. Since the course is mainly based on students' presentations, we will touch on how to get students feel comfortable presenting in front of the class and the way presentations can deepen understanding of the material.

Alessandra Pantano, *University of California, Irvine*
An active learning Group Theory textbook to train students to think mathematically
Upper-division math courses require students to engage with the material at a deeper level, but students transitioning to their first proof-based courses need guidance. For example, they must be taught how to read the textbook critically, how to go through each step of a proof and how to reflect about the content they read, to check whether they truly understand the concepts presented.
In this talk, we present an active learning group theory textbook that seeks to support students in this transition. While many results are presented in the form of problems, and the reader cannot progress through the material without engaging with the activities, this is not the traditional IBL book, in the sense that it does not assume that students have the intellectual independence required to prove every result on their own. By supplementing an extensive list of investigations with videos, visuals and other resources, this textbook seeks to train students to think mathematically in the context of Group Theory.

Created with the intent of promoting student engagement during remote instruction (but suitable for in-person classes as well), the textbook includes fill-in-the-blank proofs, videos to check answers for the most difficult investigations, reading quizzes for each section, and many other interesting features. We will also discuss the context in which this textbook was utilized (synchronous online active learning classes, with a team of learning assistants) and share student feedback.

Emily Dennett, *Columbus Academy*; **Chris Bolognese**, *Columbus Academy*
Facilitating Inquiry Through Student Problem Posing Routine and Assessments
Our presentation will focus on the act of problem posing, which is important across all mathematical topics and courses (Brown & Walter, 1983). Problem posing is also an important part of the process of problem solving (Polya, 1945). Providing space within a mathematics classroom for students to pose problems allows for all students to gain agency and authority, as well as encouraging all students to become more curious mathematical learners. In our session we'll discuss our experiences incorporating problems that our students have posed in our classrooms. We'll share a warm-up classroom routine that uses student-posed problems, as well as how we've used student-posed problems to create assessments. We will discuss how this has provided an opportunity for each student to share their voice and to see that they are able to contribute to the study of mathematics for themselves and their peers. In solving the problems their peers pose, students also learn to position each of their classmates as competent and as a valuable contributor to the learning environment.

Joshua Bowman, *Pepperdine University*
Increasing Engagement and Building Community through Teams in a Remote Learning Environment
To address the simultaneous challenges of a larger-than-average enrollment and a remote instructional modality, I placed the students from a 300-level IBL math class in stable teams who worked together throughout the semester. The arrangement was informed by existing literature on team-based IBL structures, and the effectiveness of the teams was assessed throughout the semester via surveys and individual interviews. I will report on how the IBL model (using a set of notes I had previously created for the course) was adapted to the team setting, discuss student's reactions and responses, and propose ways that similar class arrangements may also be used for in-person IBL instruction to accommodate large enrollments.

Judit Kardos, *The College of New Jersey*
Exploring the tangent line
We learn in Calculus, that the tangent line is a good local approximation to the function f(x) near the point of tangency. In fact, this approximation is so good that we may replace the function by its tangent to approximate function values nearby. But why is the tangent line so special? Why do we use the tangent line and not any other line in linear approximation? How well can a line approximate a nonlinear function in general? What property of the function determines how well this linear approximation works? Through a sequence of concrete examples, I guide my students to understand how the definition of the tangent line is in fact necessitated by our quest to find the best linear approximation. I designed this lab for the prospective Calculus teacher who is taking my Real Analysis class. My ultimate goal is that future teachers taking Real Analysis see a connection between Real Analysis, secondary mathematics and pedagogical practice.

Xavier Ramos Olive, *Worcester Polytechnic Institute*; **Xavier Ramos Olive**, *Worcester Polytechnic Institute*

IBL in an online flipped classroom model for a Transition to Proofs class
Transition to Proofs classes play a crucial role in the development of math majors. For many students, it is the first time that they are being invited to produce mathematics. This characteristic makes this course particularly appropriate for an Inquiry Based Learning approach. In an IBL setting, typically, instructor lecturing is brought to a minimum, making students responsible for the acquisition of knowledge. Despite this, almost no student will be able to write properly a proof, with the mathematical rigor that they require, without having seen some examples. It is hard to imagine a setup where students will be able to produce proofs by induction, without having been introduced to this technique first. The IBL community has developed many options to address this issue. We will discuss a flipped classroom model, combined with IBL, from the first-hand experience of how this has been implemented in the Introduction to Higher Mathematics course at Worcester Polytechnic Institute, during the summer of 2021. Because of the current circumstances, this class was taught completely online, and we will also discuss the challenges and benefits that this created in our case.

Demet Yalman Ozen, *Middle Tennessee State University*; **Amanda Lake Heath**, *Middle Tennessee State University*; **Jordan Eugene Kirby**, *Middle Tennessee State University*; **Sarah Bleiler-Baxter** *Middle Tennessee State University*
Inquiry-based Instructional Practices in Remote Settings
The MAA Instructional Practices Guide offers suggestions on classroom practices for how to foster student engagement in mathematics classrooms. In this presentation, we describe our experiences promoting student engagement in inquiry-based university mathematics courses delivered remotely. We describe tools and strategies employed in synchronous remote courses corresponding to each of the Classroom Practices for Fostering Student Engagement as well as provide practical tips for future integration of these practices in remote mathematics courses.

Carolyn Luna, *University of Texas at San Antonio*; **Jonathan Brucks**, *University of Texas at San Antonio*; **Kimberly Massaro**, *University of Texas at San Antonio*
Using Inquiry Effectively in a High-Impact Virtual and Experiential Practice
The University of Texas at San Antonio instructor team for MAT 1053 Mathematics for Business is currently collaborating with an upper division instructor in the UTSA College of Business as part of a curricular innovation grant to develop content and requirements for a high-impact virtual and experiential ePortfolio. Projects included in the ePortfolio will use inquiry learning in collaborative, periodic, higher order thinking tasks spaced throughout the entire semester that require students to analyze, synthesize, and evaluate the information they're learning in the course through development of a website. By demonstrating their learning on a website, students are simultaneously developing marketable and transferrable technology, collaboration, and communication skills.
This session will discuss the short and long-term goals of the ePortfolio, as well as the project metrics. Specific inquiry learning tasks included in the portfolio will be shared, along with a discussion of how the tasks were developed.

Joseph Barrera, *Converse College*
Fractal Visualization for Undergraduates: A Project-based Approach to Generating Fractal Images in Python
We all enjoy looking at images of fractals: the Mandelbrot set, Newton fractals, etc. But have you ever tried making them for yourself, or, better yet, getting students to make them? This talk will discuss experiences from a one-month January term course in which students learned the fundamentals of complex analysis and the use of Python for fractal visualization in inquiry-based and project-based

settings, respectively. We will cover the design, implementation, and results of this undergraduate course that took students—some with little to no programming experience—from the basics of Python to capable programmers in a matter of weeks. Pretty pictures of students' results will of course be included!

Matthew Haines, *Augsburg University*
Visualization in a Linear Algebra course
Students' visualizations of solutions of linear systems in a linear algebra course can help their intuition. It can also also lead to misconceptions if left in the realm of R^2 and R^3. Here is an IBL activity for linear algebra students that is designed to encourage cognitive dissonance to motivate students to experience and appreciate the benefits and potential pitfalls of visualizations by exploring the visualizations of systems of linear equations and orthogonal complements of subspaces.

Kevin Gerstle, *Hillsdale College*
Mastery-Based Grading across the Calculus Sequence
Mastery-based grading is a method growing in popularity in which students are asked to demonstrate full mastery of many different learning standards throughout the semester as opposed to more traditional styles of grading in which students can earn credit for showing only partial understanding. It is a style of learning that lends itself well to an IBL environment in that it allows students to retest throughout the semester on specific ideas, encourages persistence, and rewards students who struggle early on but work hard to improve their understanding throughout the course. In this talk, I will discuss my implementations of mastery-based grading in three different courses throughout the calculus sequence, describe my testing and re-testing policies, and discuss both student and personal feedback on this approach.

Insights into Quantitative Literacy and Reasoning from the COVID-19 Pandemic

Part A: *Thursday, August 5, 11:00 a.m. - 11:55 a.m.*
Part B: *Thursday, August 5, 1:00 p.m. - 2:20 p.m.*
Organizers:
Luke Tunstall, *Trinity University*
Mark Branson, *Stevenson University*
Catherine Crockett, *Point Loma Nazarene University*
Gizem Karaali, *Pomona College*
Victor Piercey, *Ferris State University*

Enrique Acosta Jaramillo, *Mathematics Consortium Working Group*; **Deborah Hughes Hallett**, *University of Arizona/Harvard Kennedy School*
Seeing the Pandemic through a Spreadsheet
Engaging students in active learning is effective on and off Zoom; spreadsheets are an excellent catalyst for engagement. Fortunately, spreadsheets adapt easily to online learning! This talk will give examples in which an analysis of pandemic data allows the students to see a story in the data.

Milo Schield, *Augsburg University*
Covid19, Statistical Literacy and the Diabolical Denominator
The choice of the denominator is critical in forming comparisons involving Covid19, crime and disparate impact. This choice can change the size or direction of a comparison. For example, total Covid deaths are higher in Connecticut than in Rhode Island 'per person', but lower 'per case'. The choice of denominator is diabolical because most people don't think of it. Questions: What makes one denominator better? What

if the ideal denominator is hard to measure? Does this reversal relate to Simpson's paradox? Claim: This topic is not part of
mathematics, stochastics or statistical inference. It is in the new Statistical Literacy course offered by UNM.

John Sieben, *Texas Lutheran University*; **Reza Abbasian**, *Texas Lutheran University*
Putting Covid Data to Good Use: Projects For Undergraduate Research
In this talk, we will discuss a series of projects that demonstrate the importance of quantitative modeling and analysis using real data. The projects are all suitable for undergraduate research ;some were developed with our summer researchers, others came from discussions with our colleagues. Through these projects we will pose questions and hypothesis that will challenge students to separate myth from facts through quantitative analysis. A common feature of our projects is that they ask students to investigate the questions using real data on the Covid-19 pandemic and readily available CAS and Statistical technology. Several of our examples were created as part of the NSF funded grant titled "Mathematics and Statistics Across the Curriculum" .

Victor Piercey, *Mathematics, Ferris State University*; **Gary Huey**, *History, Ferris State University*
Quantitative Literacy vs QAnon
In the United States, the COVID-19 pandemic fueled conspiracy theories, many of which were propagated or adopted by the QAnon movement. Conspiracy theories are not new to the United States. Together with Gary Huey, professor of history, we will discuss conspiracy theories in the U.S. and how we can use education in general, and quantitative literacy in particular, to fight them.

Gizem Karaali, *Pomona College*; **Nathan Grawe**, *Carleton College*
Hot Off The Press: Quantitative Literacy Work Inspired By The COVID-19 Pandemic
Since the beginning of 2020, the COVID-19 pandemic has significantly affected education at all levels. Instructors and scholars of quantitative literacy have needed to be creative and versatile in the face of the pandemic. Several of them have already begun to share insights gained as well as instances of QL-related, pandemic-inspired content in Numeracy, the open-access, peer-reviewed, electronic journal of the National Numeracy Network. Other outlets, such as the Journal of Humanistic Mathematics and Mathematics Teacher: Learning and Teaching PK-12, have also been publishing work that directly engages with the pandemic and can inspire instructors of QL. In this talk we offer a showcase of this recent scholarship, focusing mainly on the content of Numeracy. We hope that this will inspire the audience to engage with the pandemic in their own courses and scholarship.

Greg Stanton, *Higher Math Help*; **Brendan Sullivan**, *Emmanuel College*
Eventmath: An open-access, community-built repository pairing current events and math lessons
We understand numeracy to require both quantitative skills and literacy skills. Literacy skills enable us to recognize when quantitative skills can be applied. In teaching literacy, math educators always run into the same barrier: a lack of up-to-date problems based on current events that students care about. There's no shortage of problems that demonstrate when quantitative reasoning applies. In fact, the pandemic highlights the need for a numerate citizenry that can respond to a surplus of public health and economic data. However, lesson plans and exercises based on such data are not easy to create and keep current, nor is there a convenient way to pool these materials, to share them equitably, or to get feedback. We will present existing attempts to solve these problems and outline a comprehensive new solution we call Eventmath. Building on lessons learned from the pandemic, as well as support from the Wikimedia

Foundation, we'll also demonstrate our collaborative, open-access solution remotely with help from the audience.

Marian Anton, *Central Connecticut State University*; **Karen Santoro**, *Central Connecticut State University*

Grouping & Regrouping Quantitative Literacy

Developing the foundations of mathematical thinking requires deep and challenging changes in the mindsets, habits, and strategies for students and their instructors. In this talk we present a few creative ways to offer quantitative literacy that are focused on algebraic reasoning with arithmetic models and active learning techniques.

Math In Action

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Friday, August 6, 10:00 a.m. - 11:55 a.m.*
Part C: *Saturday, August 7, 11:00 a.m. - 11:55 a.m.*
Organizers:
Janet Fierson, *La Salle University*; **Sarah Wolff**, *Denison University*; **Cassie Williams**, *James Madison University*; **Shanna Dobson**, *California State University, Los Angeles*; **Emelie Kenney**, *Siena College*

Leslie Jones, *University of Tampa*

Mathematical Measurement in Data Science

In this talk we look at various ways to measure distance between vectors and their implications for data science. Similarity or closeness of vectors that represent consumer purchases, for example, inform recommender systems. We also discuss a family of metrics dedicated to finding the difference between two sequences, to include Hamming and Levenshtein distance. Students see these metrics in action daily as the words they type on their smart phones and computers are replaced at the will of their operating system, and suggested items to buy based on recent purchases pop up on their screens. The concepts behind these algorithms that permeate and influence our society are mathematical and surprisingly accessible.

Thomas Chen, *Academy for Mathematics, Science, and Engineering*

The Role of Applied Mathematics in Artificial Intelligence Solutions for Humanitarian Action

The world faces a variety of humanitarian challenges today, from increasingly severe and frequent natural disasters due to climate change, to refugee crises and human trafficking. The application of mathematics in machine learning, deep learning, and computer vision solutions to tackle these issues has exploded in recent years. For example, deep learning-based remote sensing algorithms are trained on multitemporal satellite imagery to detect change, assess damage, and inform the timely allocation of resources in the case of natural disasters. In general, techniques range from simple linear regression, to random forest ensemble models, to convolutional neural networks, to generative adversarial networks. In essence, these various machine learning algorithms, to varying extents, require the use and understanding of linear algebra, analytic geometry, matrix decompositions, vector calculus, optimization, probability and statistics. Furthermore, mathematical methods are integral to creating more explainable/interpretable models. In this talk, we overview the interplay between the mathematical foundations of machine learning and its potential for humanitarian good in a wide range of capacities.

Grace Cook, *Bloomfield College*; **Ted Zolue**, *Bloomfield College*; **Delvon Rowley Hayes**, *Bloomfield College*; **Olivia Mercado** *Bloomfield College*

Surviving the Apocalypse and its Aftermath with Mathematics

In the fall of 2017, on a bit of a whim one afternoon, a research project was hatched concerning the mathematics required to survive on an apocalyptic/post-apocalyptic Earth. What would we really need to know in order to survive an atomic blast or a plague of locusts? Would Algebra save us from a roving band of zombies? Would we need Calculus to keep track of our stash of canned goods in our underground bunker? Then 2020 happened. Suddenly, our quirky little side project got a little more real. Our team scoured the CDC, FEMA, and WHO websites as well as Reddit and prepper and survivalist websites to find any mention of mathematics. In addition, we examined pop culture resources such as novels, mangas, movies, television shows, and video games, for mathematical survivalist tips. Our raw data was entered into Dedoose and qualitatively coded. In this presentation, we will discuss our findings and some interesting tidbits about mathematics during and after the apocalypse. As a novel way to present our results, we are in the process of creating a comic that tells the story of a group of mathematics students who are struggling to find their way after a zombie apocalypse. While this project started out as a fun way to include more students in research, we have begun to ponder the implications for mathematics in real world circumstances such as war zones, the aftermath of hurricanes, and during a pandemic.

Reza Abbasian, *Texas Lutheran University*; **Mike Czuchry**, *Texas Lutheran University*; **John Sieben**, *Texas Lutheran University*

Math-Stat Modeling in Non- STEM Disciplines: A Preliminary Report

In this talk we will discuss the NSF-funded project " Mathematics and Statistics Across the Curriculum" and the preliminary results comparing students' attitude pre and post use of the modeling projects in several disciplines. As part of a three-year grant, teams of faculty from non-STEM fields designed and developed quantitative projects which were then embedded in several courses outside of mathematics such as philosophy, social science, kinesiology and art. The goal was to utilize the modern tools of mathematics and statistics to show the non-STEM students the relevance and applicability of quantitative analysis in various fields of study and real-life scenarios. We will provide a brief description of the goals of the grant, the methodology used for the study, including instruments that were developed to measure the change in attitudes of students toward quantitative analysis prior versus after use of these projects. We will then present the results based on two semesters of data.

Heidi Hulsizer, *Benedictine College*

Student Research Ideas in the Liberal Arts

At our liberal arts college students are encouraged to present research to the campus at what we call Discovery Day. Many of these mathematics student projects are birthed in a mathematics classroom. Two student projects will be discussed to inspire other research directors. The first is one that analyzes COVID data to determine how school policies influenced COVID numbers. The other analyzes wasted meal plan swipes to push for change in food service on campus. These student research results should influence decision makers in academic administration.

Adam Case, *Drake University*

Developing First-Year College Students' Problem Solving Abilities through Game-Based Learning

In this talk, I present an overview of a first-year seminar aimed at developing the creative problem solving abilities of first-year college students. To accomplish this, the seminar makes use of digital game-based learning by having students solve puzzles from the computer game called The Witness (Thekla, Inc. 2016). This game challenges the player/student to explore an island filled with "line puzzles," which must be solved in order to make progress. An interesting aspect of these puzzles is that their rules are never explicitly stated. Instead, students must first discover these rules through experimentation and then master them. The game provides an engaging environment for non-mathematics majors to get a taste for what it is like to develop their own techniques for solving complex problems. Various teaching strategies used for fostering students' problem solving abilities will also be discussed.

Alexis Varada, *Worcester Polytechnic Institute (WPI)*

Inverse Problems in Mitosis

In this Major Qualifying Project for Worcester Polytechnic Institute (WPI), we study the effects of specific internal forces on mitosis, the process of cell division. Non-bipolar division causes cancerous mitosis, which can be prevented if healthy division is promoted instead. In order to achieve this, we utilize Stokes Flow, which is a system fluid-governing equations that are useful for objects submerged in a viscous fluid and bounded by an elastic membrane (such as centrosomes in cytoplasm surrounded by a cell cortex). We replicate the solutions to these equations, called Stokeslets, and apply these to live-cell location data in MATLAB to calculate the forces that catalyze certain cell movements during mitosis. We summarize the methods that generate maximal efficiency with minimal error when computing these forces, as well as analyze this data for behavioral trends. We discuss the results of this analysis and how we can use our data and calculations to prevent cancer in current and alternative contexts.

Francesca Bernardi, *Worcester Polytechnic Institute*; **Manuchehr Aminian**, *California State Polytechnic University Pomona*

Epidemiology and the SIR model: Historical Context to Modern Applications

The COVID-19 pandemic has awakened students' interest in epidemiology modeling and public health. We present a project to teach students about mathematical epidemiology utilizing historical documents and primary sources, as well as data and writings from recent events. Students begin by reading excerpts of an early 1900s report discussing the Indian plague epidemic and the seminal 1927 Kermack and McKendrick paper introducing the SIR model. Students then fit SIR models to available compiled data sets from the 2014-2016 Ebola outbreak in West Africa. Additionally, students explore how the outbreak is affected by context and culture, such as local attitudes towards government health recommendations. Throughout this project, participants explore mathematical, historical, and sociological aspects of the SIR model and approach data analysis and interpretation. Based on their work, students form opinions on public health decisions and related consequences. This curriculum has been assigned (in different versions) as part of a class syllabus, as an undergraduate research project, and as an extra credit assignment at multiple universities in the United States. Feedback from students has been encouraging. A manuscript describing this project is available through the 2021 special issue of the Community of Ordinary Differential Equations Educators (CODEE) Journal.

Genghmun Eng, *Self*

Modeling Early Pandemic CoVID-19 Spread: The IHME vs Me

Epidemiologists never had such high-quality real-time pandemic data. Modeling CoVID-19 became a predictive tool instead of an afterwards analysis. Early in the pandemic, the IHME (Institute for Health Metrics & Evaluation, U. Washington) issued a 4-month prediction for expected pandemic evolution,

which they revised every couple of weeks. This meant that their basic model was wrong. The standard SEIR (Susceptible, Exposed, Infected, Recovered/Removed) Model is a local interaction model. Social Distancing among non-infected people adds a new non-local dimension to pandemic evolution. I developed my own model for N(t) [total number of pandemic cases], and dN/dt [expected new daily cases] as Math-in-Action. Initially, an exponential growth was observed:

N(t) ~ [dN/dt] ~ [Constant] exp{ + K/o\ t } ; t/dbl\ =(ln 2) / K/o\ ,

with t/dbl\ being the doubling time. Early pandemic data suggested that Social Distancing made t/dbl\ a function of time. Can Social Distancing halt an exponential growth and stop the pandemic? My simplest model for t/dbl\(t), used a linear function:

N(t) = N/o\(t=0) exp{ + K/o\ t / (1 + α/s\ t) } ,

where α/s\ measures the amount of Social Distancing. Since

N(t→∞) = N/o\ exp{ + K/o\ / α/s\ } ,

the answer was: YES! This simple model fit much of the World's early pandemic data. The IHME substantially revised their models after my preprint release{∗}, saying their initial model was not "sophisticated enough".

{∗} [https:// medrxiv.org/cgi/content/short /2020.05.04.20091207v1]

Victoria Powers, *Emory University*

How Close Was The 2020 US Presidential Election?

There are several different ways to measure the closeness of a US presidential election due to the use of the Electoral College system. In this talk, we discuss a simple method that computes the minimum number of votes that would have to change to flip the election. Using this measure, we show that apart from the 2000 presidential election (which was officially decided by 537 votes in Florida), the 2020 election was the closest since at least 1976. The reason that the 2020 election was so close using this measure is related to the combinatorics of the states won by each candidate, and the fact that Nebraska is one of only two states that split their electoral votes.

Karthikeya Sameer Kumar Mamillapalle, *Daytona State College*

The gateway to richer life

Welcome to the world of investing - a gateway to the richer life. Real estate, CDs, precious metals and stocks are some of the pillars of investing. In this paper, the focus will be on analyzing the data over time and evolving an investing technique based on Calculus I that maximizes the returns, minimizes the risk and allow us to pursue the daily routine.

Zachary Beamer, *Piedmont Virginia Community College*; **Karolina Naranjo-Velasco**, *University of Virginia*

Desmos and GeoGebra3D Interactives in Calculus III: Visualizing the Bivariate Normal Distribution

Interactive visualizations created using technologies such as GeoGebra and Desmos allow students and instructors to explore concepts in calculus in an engaging and meaningful way. This talk, co-led by professor and student, introduces how such technologies can be employed in a Multivariable calculus course as a lecture tool as well as a basis for assessment and student exploration. This discussion overviews several examples of 2D and 3D visualizations that enrich class discussions along with how to assess student understanding using visualizations. It concludes with a student-created interactive visualization exploring the Bivariate Normal Distribution as an exemplary illustration of how 3D visualizations can inspire students to draw connections between Calculus and its applications to Statistics.

MathArt, ArtMath at MathFest

Part A: *Thursday, August 5, 2:40 p.m. - 3:55 p.m.*
Part B: *Friday, August 6, 10:00 a.m. - 11:55 a.m.*
Part C: *Friday, August 6, 1:00 p.m. - 2:55 p.m.*
Organizer: Douglas Norton, *Villanova University*

Jessica K. Sklar, *Pacific Lutheran University*; **Bronna Butler**, *B.A. Baroque Arts, LLC*
Collaboration in the Time of COVID
Socially distant collaboration can be difficult, and virtual collaboration on mathematical art comes with its unique set of challenges . . . and opportunities. Artist Bronna Butler and mathematician Jessica Sklar serendipitously met in person in January 2020, and have been in separate time zones ever since. In this talk, they discuss their long-distance collaboration both as a creative team of two, and as members of a twenty-four-person team working on Mathemalchemy, a large-scale mathematical art installation spearheaded by mathematician Ingrid Debauchies and artist Dominique Ehrmann. Learn what worked and what didn't, and see the fruits of these long-distance labors of love.

David Reimann, *Albion College*
Art from Langford Sequences
A Langford sequence is a permutation of the integers 1, 1, 2, 2, ..., n, n for some positive integer n where there is one number between the 1s, two numbers, between the 2's, and so on. For example, the sequence 31213 satisfies this property. While the roots of these sequences lie in arrangements of colored building blocks, they have found interesting applications in graph theory. This talk will present several artworks based on these sequences.

Anduriel Widmark, *Artist*
Hexastix Design Principles and Problems: Homogenous Non-intersecting Cylinder Arrangements
We explore infinite non-intersecting arrangements of cylinders known as hexastix and polystix. Symmetric cylinder arrangements restricted to only 3 and 4 directions can be modeled in several ways to illustrate an assortment of packing problems. This paper presents novel techniques that are useful in the design and fabrication of complex polystix sculptures. Examples are provided to emphasize the variety of geometric concepts that polystix can be used to demonstrate. Modeling polystix is an engaging hands-on way to get creative and connect mathematics with art, or art with mathematics.

Frank Farris, *Santa Clara University*
A Geometry/Art Assignment with a Non-Euclidean Kaleidoscope
In two different geometry courses (Survey of Geometry and Differential Geometry), I have asked students to use Peter Stampfli's online kaleidoscope to create their own artworks based on spherical and hyperbolic tilings. Then they open their creations in Geogebra to demonstrate their understanding of non-Euclidean lines and distances. The online kaleidoscope app

(http://geometricolor.ch/images/geometricolor/sphericalKaleidoscopeApp.html) uses reflections to create patterns with given angles between refllection axes.

Shanna Dobson, *California State University, Los Angeles*
The Literary Incarnations of Perfectoid Diamonds
In this talk, we present our exciting literary incarnations of perfectoid diamonds, in the sense of Scholze, appearing in our newest fantastical mathematical-fiction novel, Artemis Blu II: In Diamonds. The first incarnation is a new idea of a perfectoid-diamond hourglass, which measures emergent time as a Carrollian "looking-glass" of perfectoid diamonds, which are certain pro-'etale sheaves on the category of perfectoid spaces of characteristic p. The second is a reinterpretation of the Deleuzian concept of 'haecceity' as a pro-diamond, when Artemis awakens inside the diamond hourglass.

Jeff Johannes, *SUNY Geneseo*
Needlepoint topology, geometry, and beyond
As I child I enjoyed designing my own needlepoint. The last design I completed was during graduate school (about 25 years ago). About ten years ago I had an idea to create a mathematical design for my office that featured both topology and geometry. After much stalling, pandemic lockdown gave me (and everyone else) the impetus to find a place to get supplies and complete this project. In this talk I will describe the design and some mostly trivial details of completion. I will also discuss future ideas from symmetries to spirals. Join me for a relaxing visual tour.

Shemsi Alhaddad, *University of South Carolina Lancaster*
Using Embroidery to Visualize the Weather and More!
Data visualization is an evolving field of study. However, there are several established criteria for well-designed graphs. Information art is an emerging field that combines data science and visual art. Embroidery is a fiber art in which a needle is used to run decorative fibers through ground fabric to create aesthetically pleasing designs. Temperature embroidery is a form of information art. Artists assign a color to an interval of temperatures. Using a data set that contains temperature-readings over time, they embroider identical, consecutive regions, where the temperature-reading determines each region's color, and the regions are ordered by time. This gives a data visualization of temperature graphed over time. I will describe criteria for a good data display and explain how temperature embroideries meet, or don't meet, those criteria. I will also present an extension of temperature embroidery to embroidered data displays involving more than two variables.

Donald Plante, *University of New Hampshire*
3D Printing Pre-Scored Origami Sheets
In this talk, I show how 3D printers can be used to create origami crease patterns that act as living hinges embedded in a sheet of plastic. These 3D printed plastic sheets can then be folded by anyone, regardless of skill level, to construct durable origami and kirigami models. I also show recent explorations using this process to create various thick origami and mathematical models.

Jonathan Keiter, *East Stroudsburg University*
Lasercut rendered surfaces, traces, and slices
I will discuss the creation of many 3D objects used for projects in my multivariate Calculus course. Using a laser cutter, I built interesting shapes with either interlocking wood slices or stack slices. The results are beautiful and instructive works of art.

Douglas Dunham, *University of Minnesota Duluth*; **Lisa Shier**, *University of Maryland Global Campus*
Papercrafted Mathematical Art
We have used a paper-cutter/scorer/pen-plotter to create two pieces of mathematical art. The first is a pattern of shells in the Poincare model of the hyperbolic plane. It was inspired by a repeating Euclidean pattern by M.C. Escher, which has symmetry group p4, or 442 in orbifold notation. Our pattern has symmetry group 552. The plotter capability was used to draw interior details of the shells in black on different colors of paper for each kind of shell; the cutter was then used to cut out each shell.
The second art work is part of a triply periodic polyhedron based on the cubic division of Euclidean 3-space. It is decorated with different colored Escher-like fish. It was created like the shell pattern except the the scoring capability was used where the fish were bent across edges of the cubical structure. This polyhedron has 6 square faces meeting at each vertex, and thus can also considered to have a hyperbolic structure.

Steven Wilkinson, *Northern Kentucky University*; **Peter Lefkovitz**, *Northern Kentucky University*
Melodies As Curves
The curvature function of a 2D curve describes the shape of the curve regardless of how it is translated, reflected, and rotated. In music, a motif is a string of notes that appears throughout a piece in which the motif has been transposed (translated), and inverted (reflected). We establish an equivalence between music and curves by mapping motifs to curvature functions. For a given tune we can create a plane curve, and vice versa from a curve create a musical motif. By moving to space curves that have curvature and torsion functions, we can take advantage of them to model both rhythm and melody, respectively. Using these equivalences we establish ways of categorizing tunes as wells as create new motifs.

Larry Blaine, *Plymouth State University*
The Strange Story of Solresol
Of all the systems of human symbolic communication that have evolved or have been invented, perhaps the oddest is Solresol. Developed in the first half of the nineteenth century by the French musician Jean-Francois Sudre, the symbols used are the notes of a musical scale. In this talk, we sketch the history of Solresol, give a few examples of English-Solresol translations, and describe what all this has to do with mathematics.

Joshua Holden, *Rose-Hulman Institute of Technology*
Stochastic Snare Drums and Transition-Matrix Tom-Toms: Composing rock drum kit solos using stochastic processes
While the drum kit is not usually thought of as a solo instrument, there is a substantial literature of solo works for the kit. Many of these were composed for pedagogical purposes, but composers including John Cage and Frank Zappa have written "serious" solo compositions for the kit. This project investigates the composition of solo drum kit pieces using random processes. Unlike in Cage's oeuvre, the goal is to produce pieces consistent with a 4/4 rock idiom, including "keeping time" with bass, snare, and symbol, and "linear fills" with bass, snare, and tom-toms. In addition, we impose constraints intended to keep the music playable by a moderately experienced drummer at a reasonable tempo. These constraints include avoiding the most awkward hand crossings and limiting the number of consecutive strokes that need to be played by the same hand or foot. Examples will be played both as computer output and, technology permitting, live on a drum kit.

Debra Hydorn, *University of Mary Washington*
Mathematical Art Diversions - A Puzzle and a Gift
Teaching online did not leave much time for a side-matching puzzle I created a while ago, so I developed a simpler 16-piece corner-side-matching puzzle with more options for combining the pieces and coloring the results. I also started creating mathematical art pieces for colleagues and friends by asking them for their three favorite geometric shapes and their three favorite colors. Both of these activities have helped me to manage my stress level and the gifts especially help me to focus on the importance of friendship.

Roza Aceska, *Ball State University*
Honors Colloquium on Mathematics and the Arts
I present the MathArt work created by my students and myself in the three years I have taught an honors colloquium on Mathematics and the Arts. This talk illustrates our creative, mathematical explorations of the arts. I will reflect on the individual and group explorations that led to the birth of beautiful pieces of art and even student publications. While my students were not math majors, they were initiated on an exciting journey that increased their interest in maths and in art.

Karl Schaffer, *De Anza College and MoveSpeakSpin*
Dance and topology
Topological concepts arise in dance in a variety of ways: paths of performers around the stage, connections between dancers, winding and turning numbers of individual performers as they move, and even knots and links. We will delineate several such topological patterns that have interesting connections to dance and choreography, and suggest a number of associated kinesthetic classroom activities.

Mathematics and Sports

Thursday, August 5, 1:00 p.m. - 3:20 p.m.
Organizers: Hope McIlwain, *Mercer University*; **Russ Goodman**, *Central College*

Diana Cheng, *Towson University*; **Peter Coughlin**, *University of Maryland College Park*
Dancing through the weights: Dancesport Scoring and Power Values
In competitive ballroom dancing, called DanceSport in the United States, couples perform multiple dances within each judged event. This presentation illustrates how a weighted simple game can be formulated with a goal for a couple and the judges' evaluations of the couple's dance performances. We explain why couples and their coaches may consider a variety of goals. We also show how prominent power values can be used to measure the contributions of dance performances to achieving certain goals. As part of this analysis, we show visual representations that we developed to illustrate the Banzhaf and Shapley-Shubik index profiles for different thresholds.

Jim Case, *None*
May the Best Team Lose
Both MLB and the NFL are currently planning to admit more teams to post-season play. In so doing, they seem blissfully unaware that tournament design is a well-developed branch of mathematics -- one from which they could learn much.
Content Area: Sports & Math
Recommended for Students: Yes

Sarah Blanset, *Stevenson University*; **Josh Lang**, *Stevenson University*
Approaching scheduling problems through a mix of combinatorics and Python programming
North American sports leagues typically split their teams into divisions, which are incorporated when generating the league's schedule. This talk will explore how a challenge to create a fantasy baseball league's schedule incorporating divisions evolved into a surprisingly deep combinatorial problem. We'll look at what made the problem so difficult, at how graph theory unexpectedly made an appearance, and at the undergraduate research project which ultimately (mostly) solved the problem: the creation of a Python program to implement graph theory concepts and randomly generate schedules.

Rick Cleary, *Babson College*; **Steve Miller**, *Williams College*
Classifying GOATs (like Brady, Russell and Ruth) by Measuring Their Tails
Comparing extraordinary results and determining GOATs (Greatest of All Time) is a favorite discussion topic for sports fans and data analysts alike. The analysts, however, are more likely to recognize that attempts to measure greatness are always sensitive to the metric chosen. In this talk we consider the general questions that arise when trying to compute the "unlikeliness", or right tail probability, of events for both teams and individuals. Motivated by the individual success of Tom Brady and the team success of the New England Patriots, we examine the implicit assumptions that underlie arguments put forward by sportswriters and analysts, and raise our own ideas for best practices in such arguments.

Christopher Ingrassia, *Kingsborough Community College of the City University of New York*
A Probabilist's View of the Temporal Distribution of Triple Crown Winners
Since 1875, horse racing's Triple Crown has been conducted every spring with few exceptions. As of this writing, the 140th Triple Crown season is underway and, thanks to Rombauer's defeat of Kentucky Derby winner Medina Spirit in the Preakness, we already know that the illustrious list of Triple Crown winners will remain at 13 for at least another year. The temporal distribution of these champions is characterized by two lengthy droughts of 25 and 37 years, and several closely clustered triumphs, the most recent example being Justify's 2018 sweep occurring just three years after American Pharoah's in 2015. Perhaps the most surprising feature of this scattering is the back-to-back victories of Seattle Slew (1977) and Affirmed (1978). With just 13 Triple Crown winners emerging over 140 years, it seems rather unlikely that the list would include back-to-back champions. In reality, such an outcome is much more probable than not. In this talk, we will find the probability of a distribution containing at least one back-to-back pair of champions, subject to the condition that 13 champions exist over 140 years. This is a surprisingly difficult calculation due to inherent limitations of the combination formula.

Noah Baker, *Davidson College*; **Hope Anderson**, *Davidson College*; **Lucy Smith**, *Davidson College*
NCAA Basketball Win Probability Model
Finding accurate win probabilities during an NCAA basketball game is a valuable tool. Fans can assess the chances of their team winning, sometimes leaning on that small probability of squeaking out a win. Teams can use them to evaluate play calling associated with large changes in the probability of winning a game. Many professional sports analytic companies, who have their own strategies for doing this, keep the models secret. This talk will discuss a relatively simple NCAA basketball win probability model that combines initial win probabilities and Monte Carlo like simulations to predict the winner of an NCAA

basketball game throughout the game. The model matches ESPN's win probability predictions extremely well. The model can easily integrate further development, providing a springboard for future study.

Tim Zeitvogel, *Pepperdine University*; **Timothy Lucas**, *Pepperdine University*
Modeling the Influence of In-Match Dynamics on Tennis Outcomes
Tennis is different from many other sports because the goal is to achieve enough points to win the match rather than to have more points than your opponent when the clock runs out. Our research attempts to understand the impact of the unique scoring system of tennis by analyzing the influence of potential in-match factors such as momentum and the significance of points. We created an in-match stochastic model for the overall win probability based on the service and return ability of individual players. We then defined a regression model for the service point win probability based on average service win probability, pre-match player estimations, the outcome of the previous point, and the relative significance of the point. This allowed us to compare the quality of commonly used metrics for player estimation and ranking. Our analysis of the regression model includes best-of-three and best-of-five tournaments on three surfaces. In order to measure the mental toughness of players, we defined a pressure index that compares the expected service and return game win probabilities with the actual outcomes. Based on our results, we created a dynamic in-match forecast model for match win probability, which incorporates pre-match estimations, the pressure index, and in-match trends.

Mathematics and the Life Sciences: Initiatives, Programs, Curricula

Part A: *Friday, August 6, 1:00 p.m. - 2:55 p.m.*
Part B: *Saturday, August 7, 10:00 a.m. - 10:55 a.m.*
Organizers:
Tim Comar, *Benedictine University*; **Raina Robeva**, *Randolph-Macon College*; **Carrie Diaz Eaton**, *Bates College*

Tyler Hagerty, *University of Delaware*; **John R Jungck**, *University of Delaware*
Citizen Science, Big Data, and Mathematical Biology Educationological
How do we engage students in developing the skills of addressing interdisciplinary research questions by using heterogeneous data sets? Develop skills of data wrangling and scientific visualization beyond the standard scatter plots? We argue that engaging students in analyzing Citizen Science data helps them appreciate the power of crowdsourcing on important ecological and health problems and that by visualizing their data with powerful analytical tools that students can contribute to these Citizen Science communities. We will illustrate a variety of student projects ranging from 5 dimensional graphs using GapMinder, network analysis with BioGrapher, 3D polytopes of morphospaces – multiple linear regression – phase plane portraits of biological cycles and rhythms & - ternary graphs with JMP, to ecological niches with javaBENZER. Our work is part of the NSF Research Coordination Network on Citizen Science in Undergraduate Education (<https://citizenscience.org/2020/10/08/special-collection-citizen-science-in-higher-education/>).

Aaron Wootton, *University of Portland*; **Deborah Hughes-Hallet**, *University of Arizona/Harvard Kennedy School*
Calculus driven by pandemic data

Student engagement in math courses is increased when driven by real life models of which they have direct experience. In this talk we shall see how all major aspects of a Calculus course, from the introduction of families of functions, through derivatives and integrals, can be motivated and explored using data collected from the Covid-19 pandemic.

Alan Garfinkel, *UCLA*; **Erin Sanders O'Leary**, *University of Illinois, Chicago*; **Hannah Sayson**, *UCLA*; **Casey Shapiro** *UCLA*; **William Conley**, *UCLA*; **Marc Levis-Fitzgerald** (UCLA), **M. Kevin Eagan** (UCLA), **Blaire Van Valkenburgh** (UCLA)
Teaching Modeling and Dynamics to Biology Freshmen: the UCLA experience

We developed an approach to teaching math to biology students using a dynamical systems perspective. Our course has a "modeling first" approach, first demonstrating why students need math to understand living systems, and then providing quantitative and computational skills (Python), including concepts from Calculus, that students need to build and analyze mathematical models representing these systems. Students learn how to think quantitatively about biological systems and practice constructing dynamical models for problems they have not previously encountered. In this 2-quarter sequence, we also develop Linear Algebra sufficiently to understand eigenvalues and eigenvectors, and their applications to biological systems.
We have been teaching this course to UCLA Life Science freshmen and sophomores since 2012. For the past 4 years, student enrollments have been about 1500 students per year, greatly eclipsing, by 6:1, registration in the more traditional "Calculus for Life Sciences" class taught by the math department.
In a study of the results of the course, we saw highly significant differences ($p < .001$) in subsequent course grades in Introductory Chemistry and Physics between Calculus for Life Sciences students and students in our course.
We tested for a possible "math course effect" on student success, alongside other predictors such as students' sex, race/ethnicity, socioeconomic status (SES), and academic preparation. The addition of the math course (our course vs. Calculus) contributed predictive value to each model, as evidenced by the statistical significance of the positive math course coefficient as well as of the significance of change in R-squared value with the addition of the math course variable
Student interest in the subject matter was much higher after taking our math course than before. Finally, while the apparent grade gaps between sub-groups of students within each course still persist for students historically underserved and underrepresented in life science STEM fields, we were able, through this and other measures, to dramatically improve minority retention in the Life Sciences over the past 6 years.

Raina Robeva, *Randolph-Macon College*
A Systems Biology Course for Non-Majors
Have you ever wondered how birds form organized patterns? How can fireflies synchronize their flashing? What mechanisms govern the formation of biological patterns and coordinated behaviors? Systems biology is the discipline that uses mathematical and computational approaches to examine such questions. It explores how individual interactions in a complex system could lead to behaviors or properties of the system not observed in any of its components. As Linus Pauling famously said "Life is a relationship among molecules and not a property of any molecule." The talk describes an honors course in systems biology for students with no experience in mathematics or biology beyond high-school levels. The topic is introduced through the lens of finite dynamical systems and agent-based models.

Emma Turian, *Northeastern Illinois University*; **Lidia Filus**, *Northeastern Illinois University*
Initiating a Translational Bio-Mathematics Research Seminar for College Students
The aim of this talk is to illustrate the benefits and the drawbacks of an experimental process on how to develop and teach an interdisciplinary applied math course. The analysis comes from the experience gained during the development and teaching of a seminar course called: Mathematical Modeling for Cancer Risk Assessment, implemented at Northeastern Illinois University.

Timothy Comar, *Benedictine University*
Long-Term Student Research Projects Involving Modeling with Agent-Based Models and Impulsive Differential Equations
This presentation will discuss long-term student research projects, which involve modeling biological phenomena using both agent-based models and impulsive differential equations. In one of these projects, age-structured vaccination models for human papilloma virus HPV have been developed and analyzed. These models are used to investigate the consequences of a sufficient fraction of the population not completing the vaccination regimen. In another one of these projects, models were developed to analyze an age-structured integrated pest management system. We will also discuss the challenges and benefits for student researchers of approaching this complex problem via these two modeling paradigms.

John Zobitz, *Augsburg University*
Sitting at the intersection: developing a "just enough" mathematical biology curricula
Teaching a mathematical biology course is challenging because the material inherently intersects several co-requisite subjects: differential equations, linear algebra, statistics, probability theory, numerical analysis to name a few. Additionally data science and data visualization techniques also are gaining prominence in order to analyze big data in biology. Beyond the content, for several natural science students mathematical biology may be their first (and only) post-calculus course. This talk discusses the development of curriculum for a one-semester course in mathematical biology with a pre-requisite of Calculus I that synthesizes differential equations, modeling in biology, and the R software language. The curriculum is modularized that to include principles of continuous differential equations, qualitative analysis and stability theory, Markov Chain Monte Carlo parameter estimation, and stochastic differential equations. The aim is a "just enough" approach: to encourage additional exploration of co-requisite subjects and confidence in the application of mathematical and quantitative tools for biological analysis.

Alexander Novakovic, *Boston University*; **Alexander Novakovic**, *Boston University*
A Topological and Non-Euclidian Dynamical Model of Biological Membranes
Biological life originated from the formation of membranes, which are required to organize chemical reactions and form the boundary passageway of the most fundamental unit of life, the cell. This research presents novel mathematical models to evaluate the dynamic structure of both human cells and viruses, specifically a case study of the COVID-19 virus. The topological model of this work unveils the permeability and structural stability of portions of the membrane; the differential geometry model developed characterizes cell-level membrane fusion and splitting, known as fertilization and cytokinesis (F & C), respectively. The combination of both models results in a proposal for the detectability of the prevalent virus to the host cell membrane. The fluid mosaic model of continuous deformation of all membrane portions, by the fluid-solid tintermediate state of the membrane, is examined with respect to the class (cytosis) of processes for topological properties. Cytosis, as the gain and loss of membrane fragments, is studied through augmentation that progressively approximates the number of membrane

layers and of the macromolecule classes introduced. Subsequently, differential geometry, with calculus of variations, is used to explain life cycle properties of the membrane undergoing F & C. The differential geometry model evaluated by laboratory videos of F & C are analyzed computationally using Open Source Physics Tracker Software for dynamic geometrical properties of membrane, such as curvature over time. A case study, to ultimately block the cytosis infection mechanism of the novel coronavirus, unifies the two models. The detectability of COVID-19 virus to the human host cell is hypothesized to be dependent on increased thickness, volume smaller than observable in vivo, increased surface area to volume ratio, and characteristic membrane components. All such aspects of biological membrane dynamics thus are shown to start with and be advanced by math.

Modeling in Your Differential Equations Course-- Just Do It!

Part A: *Thursday, August 5, 11:00 a.m. - 11:55 a.m.*
Part B: *Thursday, August 5, 2:00 p.m. - 3:55 p.m.*
Part C: *Friday, August 6, 1:00 p.m. - 2:55 p.m.*
Part D: *Saturday, August 7, 10:00 a.m. - 11:40 a.m.*

Organizers: Brian Winkel, *SIMIODE*; **Rosemary Farley**, *Manhattan College*; **Janet Fierson**, *LaSalle University*; **Therese Shelton**, *Southwestern University*; **Patrice Tiffany**, *Manhattan College*

Satyanand Singh, *New York City College of Technology*
A Prelude to Competitive Modeling
We will present a modeling approach to a problem that was inspired by NASA. Students were required to model a rocket's path by using differential equations. This is particularly interesting as it was presented in the summer of 2020 at New York City College of Technology when we were mostly teaching from a remote perspective and the approach was quite different. We will present our project and talk about how students embraced the work, predicted the outcomes and show how it compared with real data from NASA's website. We will also discuss what worked in this remote setting, as the project evolved and how it served as a spring board for students to explore other modeling problems and make connections to other fields. Students pondered on the assumptions made and how viable their models were and how to apply their work to other modeling scenarios. Technology played a pivotal role in these explorations and modeling. This work served as a prelude for students to participate in modeling projects and competitions such as SCUDEM.

Rosemary Farley, *Manhattan College*; **Patrice Tiffany**, *Manhattan College*
Modeling in Differential Equations in Remote and Hybrid Courses
We will report on a pilot program at Manhattan College this past semester in which we attempted to bring more modeling into the differential equations class. The eight sections of differential equations offered at our college in Spring, 2021, were either fully remote or hybrid. Faculty decided to use this opportunity to abandon the common final, thus allowing more individual approaches to the differential equations courses we taught. The faculty members who participated in this pilot program have all been involved in SIMIODE workshops. Those faculty in the pilot program could opt for a final project rather than an in-class final. We will report on the SIMIODE resources that different faculty members found useful. We will report on the different paths taken during this past semester and discuss the impact these might have for the way each of us teaches differential equations in the future. Specific examples of what worked and what did not work will be discussed.

Elizabeth Carlson, *University of Victoria*
Active Learning: Perspective of Student Turned Researcher and Teacher
In this talk, I will present how active learning, especially with regards to calculus and differential equations, has shaped my career, from undergraduate to graduate to postdoc. Specifically, I will address how learning about how mathematics applies to the real world influenced my career choices as a student, my involvement with math modeling competitions as both a participant and host of preparatory sessions, and how I incorporated the active learning techniques I learned as an undergraduate as a graduate student. Finally, I will discuss what I have learned as a teacher and how my teaching style has evolved as an early career mathematician who was highly influenced by her own experiences as an undergraduate taught via active learning.

Lisa Bromberg, *Springfield College*
Course Correction: Adjusting to Meet Student Needs
The evidence of increased student performance (and presumably learning) in courses utilizing active learning is well documented. With these outcomes in mind, I committed to a modeling-first approach in the course design for a course in elementary differential equations in the Spring 2021 semester. My experience forced me to reconcile my theoretical understanding of best practices and the flexibility needed to meet students where they were at this point in time, and to adjust to their needs so they could have a meaningful learning experience. In this talk, I'll outline my planning process, implementation (and subsequent modifications) of that plan during the semester, and lessons learned.

Brynja Kohler, *Utah State University*; **Will Tidwell**, *Utah State University*
Memorization: A Data-Driven Activity for Modeling and Reflecting on Learning
In this talk, we will share a lesson developed for a mathematical modeling course for future secondary mathematics teachers that fits well into any differential equations course. Students develop models of the process of memorization after collecting data based on their own experiences. Their models are derived from several assumptions regarding the process and rate of memorization, and different assumptions lead to variations in the models. Then students engage in parameter fitting to customize their models and make comparisons among different models. We will highlight various student approaches that have come up after trials with small groups of students and potential extensions to these models that factor in other assumptions.

Boyan Kostadinov, *New York City College of Technology, CUNY*
Implementing in R a Generalized Hill-Keller Model Fitted to Usain Bolt's Olympic Data
We extend and implement in R a project suggested by Kurt Bryan in his new text "Differential Equations: A Toolbox For Modeling The World". We use a generalized Hill-Keller ODE $v'=P-kv^r$ for the velocity $v(t)$, with a fractional power of $r=3/2$ to model Usain Bolt's Olympic data. We compute a numerical solution $v(t)$ by implementing the improved Euler method from scratch. We then implement a linear interpolation of the discrete approximation $v(t)$ and integrate it numerically to find the distance function $x(t,P,k)=\int v(t)dt$. Finally, we fit this two-parameter model $x(t,P,k)$ to Bolt's Olympic data by minimizing the sum of squared residuals using the random search algorithm. We implement everything from scratch in R, using dynamic R Markdown documents in RStudio, which allow us to unify plain text narrative, mathematical typesetting with LaTeX and computing in R and Python.

L. Felipe Martins, *Cleveland State University*; **Ieda Rodrigues**, *Cleveland State University*; **Shawn Ryan**, *Cleveland State University*

Desmos and dynamics

Desmos has become an increasingly popular, web-based, free tool for graphical and interactive mathematics. In Desmos, it is possible to create interactive activities for use in the classroom or as an assigned homework/project. We will discuss the use of Desmos in a Dynamical Systems class while introducing a collection of Desmos activities introducing bifurcations of families of one-dimensional dynamical systems. The activities enhance student understanding of the standard types of one-parameter bifurcations: saddle-node, transcritical, and pitchfork. Each activity introduces a type of bifurcation, how to identify it, and an application in which it appears. As time permits, we also hope to present more advanced modeling activities within the Desmos framework.

Brian Winkel, *SIMIODE www.simiode.org*

Puff Puff, Toss Toss, Splish Splash and Fit Fit - Modeling Using Differential Equations and Data

We offer three modeling scenarios that can be used to motivate and teach differential equations while involving real data. These are sublimation of carbon dioxide, death and immigration using m&m's, and falling column of water. All three activities are from www.simiode.org and demonstrate the full modeling cycle through fitting a model and parameter estimation.

Dan Teague, *NC School of Science and Mathematics*

Predicting Network Degree Distribution with Simple Differential Equations

Many modern networks grow and change over time. We can describe the growth and decay process with simple differential equation models. This presentation will consider networks in which, when a new node is added, it creates links with the existing nodes by either choosing link-neighbors uniformly from the existing nodes or by selecting preferential attachment proportional to the current vertex degrees. How well do the solutions to the differential equations capture the degree structure of these networks? Simple Netlogo models allow us to compare actual networks created by the process described and the solutions to the differential equations.

Prihantini Titin, *Bandung Institute of Technology*

How Long COVID-19 Takes to Gain Herd Immunity after Vaccination in Indonesia

Based on data obtained from the Handling of COVID-19 and National Economic Recovery (KPCPEN), as of February 24, 2021 there were 1,306,141 confirmed positive cases in Indonesia, with a total of 35,254 deaths and 1,112,725 recoveries. To further suppress the development of COVID-19, Indonesia has implemented vaccinations. The announcement of the implementation of this vaccination was made by President Jokowi on January 13, 2021. The purpose of this paper is to find out how long it will take Indonesia to achieve herd immunity both through vaccination and natural immunity. The method used is SEVIRS modeling with the development of exposed and vaccinated compartments. From the calculation of the existing data and several assumptions. Numerical simulation in this paper is done with using MATLAB, where the results obtained are based on graphs, namely the number of susceptible populations decreased sharply from the beginning of the observation to the 40th day followed by an increase in the population that had been vaccinated against stage I (V). This has an impact on a sharp increase in the recovered population (R) from the beginning of the observation until the 150th day.

Deborah Hughes Hallett, *University of Arizona/Harvard Kennedy School*

From Curve Fitting to Differential Equations

Many students come to college knowing how to use a calculator to fit curves of a few well-known shapes to data. But what should they do if the data does not fit these shapes? Estimating derivatives from the data unlocks a range of new models, based on fitting differential equation to the data. The talk will demonstrate current data sets.

Erin Kiley, *Massachusetts College of Liberal Arts*
TILT- and GRASPS-Inspired Implementation of a Semester-Long Modelling Project in Differential Equations
Transparency in Learning and Teaching (TILT) is a project that "aims to advance equitable teaching and learning practices that reduce systemic inequities in higher education", and GRASPS is a model that encodes best practices in design of meaningful performance task assessments. In this contributed talk, the audience will learn the basic ideas of TILT and GRASPS, and will see how the presenter has applied these ideas in the design of a semester-long modelling project for Differential Equations at a small, public liberal arts institution. Examples will be shown and learning outcomes, standards, and rubrics will be discussed.

Ivan Dungan, *Francis Marion University*
Using Spring-Mass Systems to Study Financial Markets in a Differential Equations Class
We will show how a quick detour in a differential equations class can introduce students to mathematical modeling of the complex world of financial markets. Prices of different securities such as commodities, bonds or stocks have many similar qualities to spring-mass systems. We use this observation and leverage our knowledge about the physical world to model different phenomena in financial markets. We will show one benefit of these models is the ability to compare buy/sell strategies during different financial events such as a financial correction or worse, a crash.

Cole Butler, *North Carolina State University*
Using differential equations to model individual behaviors that limit disease spread
Differential equation models play an immense role in helping us understand natural systems. One relevant way this has happened in the past year is in the mathematical modeling of COVID-19. This is a recent example of how modeling helps guide control efforts, such as vaccine allocation, and predicts the spread of disease. As diseases spread, individuals sometimes take actions to limit their potential of getting infected. This can be in the form of wearing a mask, social distancing, or even just staying inside. How can we incorporate these behaviors in a mathematical model, and how do they change the resulting outbreak? Beginning with a simple susceptible-infected (SI) model, a few different cases are discussed illustrating how differential equations are used to model disease transmission in general. Some relevant mathematical concepts are introduced, such as equilibria and stability. A mathematical introduction to the reproduction number of a disease is also provided. Motivated by the ongoing COVID-19 pandemic, students will see how differential equations can be used to model disease spread, and how mathematical modeling allows us to answer questions that may arise in our study of the subject

Timothy Lucas, *Pepperdine University*
Using the Slopes app to Enhance Modeling in Differential Equations
Introducing mathematical models motivates the study of important classes of linear and nonlinear differential equations and systems. A key to understanding those models is visualizing slopefields, phase planes and solutions. Slopes is a mobile application currently available for iPad, iPhone and Android phones that has an intuitive interface designed to visualize solutions to differential equations and support

active learning in the classroom. Research based on observations of mathematics courses at Pepperdine University has demonstrated how students can use Slopes to visualize solutions, aid in discussion and collaboration, and demonstrate understanding of differential equations concepts. I will discuss how Slopes enhances group-based modeling activities as well as semester-long modeling projects. Slopes invites students to fully immerse themselves in the world of differential equations so that they can understand the models from not only algebraic, but also graphical and numerical perspectives.

Reza Ahangar, *Texas A & M University Kingsville*
HIV-AIDs Epidemics with Vaccination
The principles of the HIV-AIDS epidemics and the relations of susceptible, HIV infected, AIDS, and immunized subpopulation will be established. We would like to develop a general mathematical modeling for HIV - AIDS with Vaccination to understand the spread of the aids virus and to predict the number of infected individuals during a certain period of time in a population. Further we expand the research to special cases with no vaccinations.
We will investigate the model in special case when the removal subset of the population is empty, or there is no recovery in this epidemic.
We also can consider the total infected number is equal to the sum of the HIV infected and the number of AIDS infected. As a result, we can use SIR Monte Carlo simulation method in SIR case to verify the Validity of the HIV-AIDS model.

Michael Karls, *Ball State University*
Three Sand Tank Groundwater Flow Experiments
A Sand Tank Groundwater Model is a tabletop physical model constructed of plexiglass and filled with sand that is typically used to illustrate how groundwater water flows through an aquifer, how water wells work, and the effects of contaminants introduced into an aquifer. Mathematically, groundwater flow through an aquifer can be modeled with the heat equation. We will show how a Sand Tank Groundwater Model can be used as-is or slightly modified to simulate groundwater flow through an aquifer with three different sets of boundary conditions. Data can then be collected from the physical model and used to verify the corresponding mathematical models. We will illustrate with some specific examples for each case.

Weiqun Zhang, *Wright State University*
Logistic Function and Its Application in Machine Learning
The logistic function is a commonly used activation function in machine learning. An activation function is a decision-making component of a neural network of artificial intelligence. Since the logistic function takes any input from negative infinity to positive infinity and produces an output from 0 to 1 in addition to its nonlinearity and differentiability, it is a good candidate to solve complex problems in deep learning. In this paper, we will demonstrate the properties of the logistic function and how it works in machine learning.

Lubna Kadhim, *Morgan State University*
ANALYSIS OF A COUPLE OF DYNAMICAL SYSTEM ASSOCIATED WITH CANCER TREATMENT
In this paper, we consider two dynamical systems associated with cancer treatment. The two dynamical systems are derived from two free boundary problems modeling tumor growth and cancer treatment by combination therapy. By analyzing the fixed points and their linear stability, we study the asymptotic property of the solution and its dependence on the dose levels of the drug.

Cancer is one of the leading causes to death in many parts of the world. Most of the processes are modeled by partial differential equations (PDE), but the models in this paper are given by dynamic systems (ordinary differential equations (ODE)). The evolution of cancer in a tissue can be modeled by a system of PDEs, which describe the convective and reaction-diffusion process for cell densities and nutrient concentrations in the tumor. The interaction among cells and molecules are also represented in the PDEs. The tumor surface moves with the same velocity as cancer cells.
In this paper, the first dynamical system model has derived from two PDE models trying to answer this question: Are the T cells able to eradicate the cancer? This question was addressed in some cases where the tumor is spherical. The first drug a checkpoint inhibitor which disables checkpoint receptor on the T cells thus enables the T cells to remain active and kill cancer cells. The second drug is a dose of the oncolytic virus, a virus that is genetically programmed to invade only cancer cells, multiplies within them, and cause their death. By analyzing the fixed points and their linear stability, we study the asymptotic property of the tumor surface and the solution to the dynamical systems and its dependence on the dose levels of the drugs.

Viktoria Savatorova, *Central Connecticut State University*
Aleksei Talonov, *University of Nevada Las Vegas*
Melting ice in northern seas due to the global warming: self-similar solution for Stefan's problem
Existing data suggest that Earth's climate is now changing faster than at any point in the history of modern civilization [1]. The planet's average surface temperature has risen about 2.12 degrees Fahrenheit since the late 19th century. The ocean has shown warming of more than 0.6 degrees Fahrenheit since 1969. The Greenland and Antarctic ice sheets have been losing hundreds billion tons of ice per year. Both the extent and thickness of Arctic sea ice has also declined rapidly over the last several decades. This current global warming trend is mostly attributed to the result of human activity, and it makes it worth watching closely. Mathematical modeling of the full spectrum of the processes responsible for the global climate change can be seen as a daunting task. However, there can be found an accessible and elegant way to investigate the effect of global warming on the rate of change of melting ice in the polar seas. The idea originates from the work done by J. Stefan in 1891 [2], and nowadays boundary value problems in partial differential equations with a free boundary between different phases moving in time are called Stefan problems. In this talk we are going to consider Stefan problem to estimate the intensity of melting ice in polar seas due to global changes of Earth's climate. We use self-similarity to switch from partial differential to ordinarily differential equations. For 1D case, we obtain analytical expressions for the temperature distribution and for the evolution of the boundary between melting ice and water.
[1] https://climate.nasa.gov/evidence/
[2] J.Stefan, Uber die Theorie der Eisbildung, insbesondere uber die Eisbildung im Polarmeere, Annalen der Physik und Chemie, 42, 269-286 (1891).

Promoting Diversity, Equity, Inclusion, and Social Justice

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Wednesday, August 4, 1:00 p.m. - 3:20 p.m.*
Part C: *Friday, August 6, 2:00 p.m. - 2:55 p.m.*

Organizers:
Alex M. McAllister, *Centre College*; **Robin Cruz**, *The College of Idaho*; **Joel Kilty**, *Centre College*; **Prayat Poudel**, *Centre College*

Alex McAllister, *Centre College*; **Joel Kilty**, *Centre College*; **Prayat Poudel**, *Centre College*
DEI Problems in Mathematics and Some Possible Responses
In this talk, we share data trends that indicate a critical need to address diversity, equity, and inclusion in the mathematical sciences. We also give an overview of different approaches to promoting persistence and success in mathematics at the undergraduate level. In particular, we discuss strategies for promoting professors' critical self-reflection on their personal practices, for fostering effective mindsets among students, and our own efforts at curriculum reform with an eye toward enabling persistence.

Jeffrey Ludwig, *University of California, Irvine*
A New Mathematical Metric for Inclusive Excellence in Teaching Applied Before and During the COVID-19 Era
In this paper a novel metric for evaluating inclusive excellence in teaching is introduced and applied to students' performance in classes before and during the COVID-19 era. The novel metric, named the Inclusive Excellence Ratio (IER), is designed to simultaneously reflect the two desirable characteristics embraced by inclusive excellence teaching: strong student performance and low variation in performance across all students. The computation of the IER given student test score data is simple and straightforward: it is the statistical sample mean divided by the sample standard deviation. Consequently, the IER is high when the students' test scores are high and variance is low, suggesting it may provide a useful quantitative measure for those educational innovators seeking to experiment with new, effective teaching methodologies that boost inclusive excellence. The IER is applied to evaluate a posteriori student performance taken from cumulative aggregate data from the University of California, Irvine (UCI) undergraduate math finance classes involving 378 students over two academic years (2018 to 2020), spanning 5 quarters before and 1 quarter during the COVID-19 era of remote teaching at UCI. Conclusions are drawn and discussed comparing the quality of in-person teaching environments to remote teaching environments.

Robin Cruz, *The College of Idaho*
Engaging Middle School Students in Math and Science
Over the past 10 years, The College of Idaho has run a week-long summer camp for middle school students. The activities are designed to engage students in a science project that requires using mathematics. Most students in the camp are from low income households in a school district which is over 50% Latino. The goal of the camp is to demystify the college experience and provide a context in which the students can imagine themselves going on to college.

John Rock, *Cal Poly Pomona*; **Kimberly Seashore**, *San Francisco State University*; **Nikita Campos**, *Cal Poly Pomona*; **Alvaro Cornejo** *San Francisco State University*
BAMM! Building a Virtual Mentorship Community with Math Masters' Students
The BAMM! program (Bolstering the Advancement of Masters in Mathematics) is designed to provide scholarships and mentoring, and to build a supportive community of and for underserved and underrepresented master's students in the mathematical sciences as they make their way to PhD programs. This five-year NSF-funded project is a collaboration across math departments at three California State Universities: Fresno State, Cal Poly Pomona and San Francisco State University. The initial BAMM cohort of scholars came onboard amid the abrupt shift to remote and virtual learning due to COVID-19.

This challenging start provided unexpected benefits. Two of the PIs along with two BAMM! scholars will share the relationships fostered and the lessons we have learned about creating community and mentoring students across campuses in a virtual environment.

Judith Canner, *California State University, Monterey Bay*
Creating a Diverse Workforce in Biomedical Data Science: Implementation and Impact of Best Practices
In 2015, California State University, Monterey Bay (CSUMB) began several programs to support undergraduate training in biomedical data science. We developed new academic programs relevant to biomedical data science and professional and research training for undergraduate students and faculty. In addition, we began a collaborative summer research program with the Genomics Institute at UC Santa Cruz. We will present the evaluation of the last five years of the program and the impact of collaboration between a primarily undergraduate institution and a primarily research institution to support student success in biomedical data science. We will provide case studies and data on the impact of academic, professional, and personal support to support students from first-generation and historically excluded backgrounds to pursue graduate studies in biomedical data science. We will include experiences transitioning the summer program online during the pandemic. The institutionalization of the proposed structures will allow us to recruit more students, even those "late" in their academic career, into the data science graduate programs and careers.

Nadia Kennedy, *New York City College of Technology, CUNY*; **Ariane Masuda**, *New York City College of Technology, CUNY*; **Armando Cosme**, *Science Skills Center High School, Brooklyn*
Two-tiered Summer Programs to Promote Equity and Inclusion
This presentation describes tiered six-week enrichment math programs for diverse and under-represented middle and high school students in the Brooklyn (NY) School Districts during the months June and July. The program for middle school students will provide rich and intensive math experiences for the students, featuring activities that challenge mathematical thinking, extend knowledge of mathematics, arouse curiosity toward mathematics, and help students appreciate their applications in the real world. The program for high school students is designed for high school students eligible to take AP Calculus, and will help them prepare for taking an AP Calculus class next school year. This will be accomplished by creating a learning environment that focuses on inquiry, mathematical reasoning, and collaborative group work. The program activities will be facilitated by prospective mathematics teachers. The project includes training for the prospective teachers and opportunity to engage with diverse middle and high school students in equitable and inclusive teaching.

Terrence Blackman, *Medgar Evers College*; **David Goldberg**, *Purdue University*; **Phil Kutzko**, *University of Iowa*; **Leslie McClure** *Drexel University*; **William Vélez**, *University of Arizona*
The Math Alliance, Lessons from 15 years of Building a New American Community in the Mathematical and Statistical Sciences
Growing out of a project between the three Iowa Regents Universities and four HBCUs, the Math Alliance has become a national community of more than 1,200 faculty at over 380 U.S. campuses, who have mentored over 2,300 students, or Math Alliance Scholars, since 2006. The goal of the Math Alliance is to foster a more diverse professional workforce in quantitative science professions by facilitating pathways for underserved students to doctoral degrees in the Mathematical Sciences. At least 80% of Math Alliance Scholars self-identify as underrepresented U.S. minorities, as defined by NSF. Our growth from an NSF-funded project to an institutionalized entity within the quantitative sciences community has been characterized by several transformations. We have learned to adapt to an evolving

landscape of opportunities for students with strong quantitative training. In this talk, we discuss some of this evolution, some of the best practices we have learned, and where we think our efforts are headed. We'll also discuss ways we believe more of our professional communities, including MAA members, can participate and contribute to the effort to make ours a more open, welcoming, fair, and just profession.

Deborah Hughes Hallett, *University of Arizona/Harvard Kennedy School*
The Two Faces of Data and Algorithms
Data can reveal or obscure inequities. While appearing unbiased, algorithms can create inequities. Understanding how this can happen can empower students whose lived experience has been impacted by data and algorithms.

Carrie Muir, *Whatcom Community College*
Using Social Justice,Equity, Diversity, and Inclusion as Context for Probability
Probability provides a natural lens for examining issues of social justice, equity, diversity, and inclusion (JEDI); those same issues then provide natural and relevant context when teaching introductory probability topics. This talk will include examples used in a quantitative literacy course, and will look at different approaches to using JEDI topics as probability context.

Mark Branson, *Stevenson University*; **Whitney George**, *University of Wisconsin - La Crosse*
Math for the People: A Radical New Approach to Quantitative Literacy
Math for the People is a collaborative project which is creating an open source (CC-BY-NC-SA) textbook which teaches quantitative literacy through social justice topics. M4TP flips the classic QL textbook on it's head - starting each module with a social justice topic, like environmental racism or mental health in LGBT youth - and then introducing mathematics topics which can be used to understand, explore, and work to solve these problems. Our goal is to help QL students see the way mathematicians approach mathematics - not as a collection of arbitrary techniques to learn, but as a toolbox for solving problems - and to give students the context to start solving for change. In this talk, we'll discuss the status of the project and opportunities to support the project or use the text.

Linda Burks, *Santa Clara University*
Experiential Learning and Social Justice Math Class
Santa Clara University, a Jesuit university in Silicon Valley, requires all undergraduate students to complete an "Experiential Learning Social Justice" (ELSJ) class to graduate. These classes are typically offered in the arts and humanities, but what does an ELSJ class in mathematics look like? This presentation highlights the design of the ELSJ class, Mathematics Education and Social Justice. Focusing on problem solving, pedagogy, and precalculus, Mathematics Education and Social Justice class has three main components: (1) Undergraduate students read extensively about math specific social justice issues and discuss their perspectives. (2) Undergraduate students serve as precalculus tutors in local high schools which serve struggling students from socially and economically disadvantaged communities. (3) Undergraduate students regularly reflect, analyze, and integrate course content within the context of their immersion experience. As part of their final project, undergraduate students leave a lasting legacy with the high schools. The undergraduate students design relevant and engaging activities for the precalculus students to learn from independently when the tutors are not by their side.

Filippo Posta, *Estrella Mountain Community Collge*

Linear Algebra Group Projects to Promote Social Justice and Equity in Math Classrooms
Learning and teaching of Mathematics is often associated with the objectivity of the content. It is often easy for educators to dismiss the possibility of adding discourse about socio-political issues, because it does not fit within the course content. Recent events in the US and beyond have brought the need for social discussion to the forefront. At the same time, the pandemic has increased the need for more innovative ways to foster and assess learning. In this presentation, I discuss how Crime Forecasting, Community Policing, and the Broken Window theory were brought in a Linear Algebra classroom to provide an opportunity for social discourse and applicability of abstract mathematical concepts.

Recreational Mathematics: Puzzles, Card Tricks, Games, and Gambling

Part A: *Thursday, August 5, 11:00 a.m. -11:55 a.m.*
Part B: *Thursday, August 5, 1:00 p.m. - 3:55 p.m.*
Part C: *Saturday, August 7, 1:00 p.m. - 1:55 p.m.*
Organizers:
Paul R. Coe, *Dominican University*; **Sara B. Quinn**, *Dominican University*; **Kristen Schemmerhorn**, *Concordia University Chicago*; **Andrew Niedermaier**, *Jane Street Capital*

Ben Cote, *Western Oregon University*; **Leanne Merrill**, *Western Oregon University*
Multigraphs and crossword puzzle grid designs
We associate to American-style crossword puzzle grids a multigraph which captures the layout of the answers. For such multigraphs we establish criteria under which one would determine a legal crossword puzzle grid in order to enumerate crossword puzzle designs.

Travis Peters, *College of Saint Benedict & Saint John's University,*; **Ryan Munter**, *Saint John's University*
Lights Out on Graph Products over the Ring of Integers Modulo k
Lights Out is a game played on a finite, simple graph. The vertices of the graph are the lights, which may be on or off, and the edges of the graph determine how neighboring vertices turn on or off when a vertex is pressed. Given an initial configuration of vertices that are on, the object of the game is to turn all the lights out. The traditional game is played over $\mathbb{Z}_2$, where the vertices are either lit or unlit, but the game can be generalized to $\mathbb{Z}_k$ where the lights have different colors. Previously, the game was investigated on Cartesian product graphs over $\mathbb{Z}_2$. We extend this work to $\mathbb{Z}_k$ and investigate two other fundamental graph products, the direct (or tensor) product and the strong product. We provide conditions for which the direct product graph and the strong product graph are solvable based on the factor graphs, and we do
so using both open and closed neighborhood switching over $\mathbb{Z}_k$.

Anthony Bosman, *Andrews University*; **Adrian Negrea**, *Andrews University*
Bounds on Solvable Snake Cube Puzzle
The snake cube is a popular puzzle that has been analyzed for its computational difficulty and shown to be NP-complete. Conceiving of the puzzle as a Hamiltonian path in an nxnxn graph, we considering invariants of the graph to obtain necessary conditions for a particular snake cube to be solvable and eliminate possible puzzle configurations as unsolvable. In particular, we establish upper and lower bounds

on the possible number of straight components in solvable snake cube puzzles. We highlight several other questions that are well-suited for undergraduate research projects.

Lauren Rose, *Bard College*
EvenQuads: A SET-like game
We introduce EvenQuads, a variant of the popular card game SET® in which there are four states for each attribute instead of three. This game is featured in the AWM Notable Women in Math Playing Cards. We will provide a glimpse of the many mathematical aspects of this game.

Anne Quinn, *Edinboro University of PA*
More Adventures in the Game of SET ® -- Transformations and Simulations
This talk will be accessible to newcomers to the game, but will expand on some ideas from my previous talks on the Game of SET ® , a popular and addictive game. After a brief introduction for newcomers, the talk will discuss different versions of the game, including new dice versions and tips for contests. Coding, modular arithmetic, and simulation will be used to analyze the dice games. The coding will also be used to study the maximum number of cards with no set in the traditional Game of SET ® . Some introductory information is available at setgame.com/welcome and playmonster.com/set-teachers-corner/ .

Zhengyu Li, *University of Toronto Mississauga*; **Parker Glynn-Adey**, *University of Toronto Scarborough*
A Combinatorial Magic Trick using the SET Deck
What's more magical than mathematics? Mathemagic, of course. Our trick "In TetraCycles" is a variation on the "In Cycles" trick developed by Persi Diaconis and Ron Graham. Four volunteers draw four consecutive SET cards from the 81-card deck. Then the magician can easily predict all four cards, only given information of one arbitrary feature. How is such a trick possible? Turns out that the magic of combinatorics, graphs, and sequences are behind this trick and making it happen. The talk will extensively discuss de Bruijn sequences as they are used in "In TetraCycles" and how they connect to graph theory and combinatorics. We will also provide a walkthrough of the trick so that anyone can perform the trick with some practice! Our paper "In TetraCycles: a SET Deck Magic Trick" appeared in Math Horizons, Volume 28 (2021). We dedicate this work to the memory and legacy of Dr. Ronald Graham.

Dibyajyoti Deb, *Oregon Institute of Technology*
The 21 Card Trick and Its Generalization
The 21 card trick is a popular card trick. It was featured in an episode of the popular YouTube channel Numberphile few years ago. In this talk we will look at the mathematics behind this trick, and also look at a complete generalization of this trick, so that it can be performed with any number of cards.

Michael Nathanson, *Saint Mary's College of California*
Playing blackjack with an infinite deck
Blackjack is a well-studied game, mostly by simulation; and there is an accepted standard strategy. Standard blackjack uses between six and eight decks. If we instead presume that there are infinitely many decks, then each card dealt is independent of the previous one and the game is amenable to precise analysis.

We use a Markov process to analyze the infinite-deck blackjack game and compare our results to the standard strategy. We then use statistical estimates to quantify the differences between this and the standard eight-deck game.

Jessica Oehrlein, *Fitchburg State University*
Counting in Texas 42
The domino games 42, 80, and 88 are a family of trick-taking games originating in Texas. I will introduce key features of theses games alongside mathematical problems arising from those features that can be used in the classroom. These questions cover a range of counting and probability topics typical of discrete math, statistics, or math for liberal arts courses. Possible extension questions ask students to develop some basic strategy for the games or to generalize ideas from the existing games and propose variations.

Jason Rosenhouse, *James Madison University*
Lewis Carroll's Barbershop Puzzle
Late in his career, Lewis Carroll presented a logic puzzle in the academic journal "Mind". Though he presented the puzzle in the form of a humorous short story, his intention was to raise serious questions about the proper interpretation of conditional statements in formal logic. The puzzle led to a heated exchange of papers among several prominent logicians of the time, and even Bertrand Russell later weighed in on it. We will present the puzzle, discuss some of the replies it provoked, and consider its historical significance in the development of the theory of conditionals.

Christopher Ingrassia, *Kingsborough Community College of the City University of New York*
Why the Monty Hall paradox does not directly apply to Deal or No Deal
In 2008, a young woman named Jessica became the first $1 million winner on the popular television game show Deal or No Deal. Throughout the hour, she repeatedly rejected the lure of several lucrative offers to quit (including one exceeding $560,000), while successfully whittling down the vast mountain of dollar amounts until only two suitcases remained: one holding the $1 million top prize, while the other was guaranteed to contain $200,000. At this point, Jessica was given the option to swap the suitcase she initially chose for the remaining one. She stuck by her lucky #4 and was handsomely rewarded for her loyalty. If given the same enviable choice, many disciples of probability would have likely switched suitcases, citing Marilyn vos Savant's brilliant response to the so-called Monty Hall problem, which had recently been popularized in the movie 21 at the time of Jessica's fateful appearance on Deal or No Deal. However, in this situation, absolutely no advantage whatsoever would be gained by switching suitcases. In this presentation, we will explain the reason for this apparent discrepancy.

Arthur Berg, *Pennsylvania State University*
Statistical analysis of the International Mathematical Olympiad
A massive amount of scoring data is available on students who have participated in the International Mathematical Olympiad (IMO). Levering this dataset, we identify the hardest and easiest problems on the IMO, illustrate the country-level performance by problem type over time, assess potential biases with the competition structure, and explore gender disparities. This work was recently published in The Mathematical Intelligencer.

Edmund Lamagna, *University of Rhode Island*; **Robert Ravenscroft**, *Rhode Island College*
Sum Amusements with Fibonacci and Other Linear Recurrence Sequences

The sum of Fibonacci numbers can be expressed with a Fibonacci number, Sum(F[k],k=0..n) = F[n+2]–1. Similarly, the sum of harmonic numbers can expressed using a harmonic number, Sum(H[k],k=1..n) = (n+1)H[n]–n. The sequences <F[k]> and <H[k]> are examples of linear recurrence sequences in that each is defined by a linear recurrence relation with constant coefficients: F[k] = F[k-1] + F[k-2] with initial conditions F[0]=0 and F[1]=1 in the case of the Fibonacci numbers, and H[k] = H[k-1] + 1/k with H[1]=1 for the harmonic numbers. An algorithm is developed to express the sums of linear recurrence sequences like these in terms of the sequence name. The algorithm can evaluate the sums of all homogeneous recurrences and many inhomogeneous recurrences, and has been implemented in the Maple computer algebra system.
LaTeX:
The sum of Fibonacci numbers can be expressed with a Fibonacci number, $\sum_{k=0}^n F_k = F_{n+2}-1$. Similarly, the sum of harmonic numbers can expressed using a harmonic number, $\sum_{k=1}^n H_k = (n+1)H_n-n$. The sequences $\langle F_k \rangle$ and $\langle H_k \rangle$ are examples of \emph{linear recurrence sequences}
in that each is defined by a linear recurrence relation with constant coefficients: $F_k = F_{k-1} + F_{k-2}$ with initial conditions $F_0=0$ and $F_1=1$ in the case of the Fibonacci numbers, and $H_k = H_{k-1} + 1/k$ with $H_1=1$ for the harmonic numbers. An algorithm is developed to express the sums of linear recurrence sequences like these in terms of the sequence name. The algorithm can evaluate the sums of all homogeneous recurrences and many inhomogeneous recurrences,
and has been implemented in the Maple computer algebra system.

Alexander Atwood, *Suffolk County Community College*; **Russell Coe**, *Suffolk County Community College*

What is the Collatz Conjecture and why is it so interesting?

Proposed by Lothar Collatz in 1937, the Collatz Conjecture, also known as the 3n+1 problem, is simple to state but is one of the most difficult open problems in mathematics. We will describe the conjecture, demonstrate how it works, examine the history of the conjecture, talk about why proving it is so difficult, and describe very significant progress in 2019 by Fields medalist Terence Tao on this subject.

Jay Schiffman, *Retired as of June 30, 2021. Rowan University*

A 3 X 3 Magic Square Consisting Of Consecutive Primes

There exist a number of magic squares consisting of all prime entries including Rudolph Ordrejka's (1928-2001) consisting of the nine entries 17, 89, 71, 113, 59, 5, 47, 29 and 101 as well as another one attributed to Rudolph Ordrejka which includes the nine prime entries 1669, 199, 1249, 619, 1039, 1459, 829, 1879 and 409 where the primes form an arithmetic progression with common difference 210. This paper considers a companion prime magic square discovered by Harry Nelson and is referenced in Richard Guy's book entitled "Unsolved Problems in Number Theory", 2nd ed. New York Springer-Verlag, 1994. The entries in this fascinating magic square are as follows: 1480028159, 1480028153, 1480028201, 1480028213, 1480028171, 1480028141, 1480028189 and 14800028183. We initially explore an interesting property possessed by this magic square (the primes are consecutive) and determine the smallest constant required to add to each of the entries to obtain a new magic square consisting of from zero to nine primes respectively. The latter problem encompasses three flavors which will be discussed. Our ultimate goal is to obtain the smallest constant needed to add to each of the entries in Harry Nelson's prime magic square to obtain a prime magic square all of whose entries consist of consecutive primes. With the aid of a MATHEMATICA search, I was able to achieve my goal and show that the solution is not unique.

Research in Undergraduate Mathematics Education

Part A: *Thursday, August 5, 11:00 a.m. - 11:55 a.m.*
Part B: *Friday, August 6, 1:00 p.m. - 2:55 p.m.*
Part C: *Saturday, August 7, 10:00 a.m. - 11:55 a.m.*
Part D: *Saturday, August 7, 1:00 p.m. - 3:40 p.m.*
Organizers: Brian Katz, *CSU Long Beach*; **Nicole Infante**, *West Virginia University*; **Shiv Karunakaran**, *Michigan State University*

Younggon Bae, *University of Texas Rio Grande Valley*

Collaborative practices in virtual group work on dynamic geometry tasks

The goal of this study is to engage students in collaborative mathematical practices in an online classroom using dynamic geometry environments. In this study, preservice secondary mathematics teachers explored Desmos applets of geometric transformations and characterized ways in which those applets dynamically created figures in response to what they drew on the canvas. Groups of three or four were sent to Zoom breakout sessions where they used the screen-share feature to share their observations on the applets and discuss various aspects of transformations to understand mechanisms beyond the applets. The analysis on video recordings of group work showed emergent patterns of collaborative practices that allowed groups to collectively create written descriptions and visual demonstration of the transformations. Students drew figures on a shared screen to support mathematical communication when expressing one's mathematical idea and understanding others. Some students co-constructed figures by exchanging ideas to generate and test out conjectures in their discussion. Group submission of written description facilitated the students' discussion on formal and precise language. In addition, there was a difference in group dynamics with respect to roles of leaders and tasks for individuals in group work.

CELINA ABAR, *Pontifical catholic University Sâo Paulo – Brazil*; **Amabile Jeovana Mesquita**, *Pontifical catholic University Sâo Paulo - Brazil*

Modeling as Methodology and GeoGebra as a resource in the study of Differential Equations

This study is part of a research that is being carried out at a University of Goiás State in Brazil, with students from a Mathematics Degree course and aims to contribute to the learning of Differential Equations. Research, already carried out, indicates that students have difficulties in understanding the connection between the content of Differential Equation and a real modeled system. Thus, this research will be developed with proposals for workshops with activities in which mathematical modeling will be used as a methodological strategy and the GeoGebra software as a technological resource. The theoretical contribution used in this research, on Mathematical Modeling, indicates that the analytical, numerical and graphical approaches are important for the construction of a concept, in the case of this study, the Differential Equations, also justifying the use of a technological resource. Students will be involved in activities that provide intentional, active, constructive, cooperative and authentic learning in order to enable multiple representations of the same object. In this article we will present an design of an activity that will address the above mentioned approaches.

Annela Kelly, *Regis College*

Building Epidemic Awareness through Mathematical Modelling

Each semester during the pandemic, the project compared different scenarios of epidemic spread with various rates of contagiousness Ro and different levels of social distancing.
The classes modelled the data and compared the different rates of spread including the time to achieve the herd immunity.
. We will answer the question: "How has the awareness and the collective mindset of students changed during the pandemic?". The talk will give an overview about the mindset from each semester during the pandemic: Spring 2020, Fall 2020 and Spring 2021. To analyze the student responses we use Python programming and to illustrate the changes in these through the pandemic using word clouds.

Amanda Lake Heath, *Middle Tennessee State University*
Collaborative Creativity in Proving: Adapting a Measurement Tool for Group Use
Creativity is a widely recognized central tenant of mathematics education, yet the nature of defining and measuring mathematical creativity is complex. When seeking to understand mathematical creativity at an undergraduate level, one must consider the role of proof and proving. Proof is essential to an undergraduate mathematics education, and as university mathematics classrooms increasingly incorporate more active and collaborative learning, it is imperative to understand the relationship between collaboration and creativity in proving. This study seeks to apply the Creativity-In-Progress Rubric (CPR) on Proving (Savic et al., 2017) to two collaborative small-group proving episodes and to evaluate its ability to present a holistic image of a group's creative proving process. Findings of this evaluation led to three suggestions for use of the CPR on Proving in a collaborative setting.

Amelia Stone-Johnstone, *California State University at Fullerton*
Supporting Student Learning through Active Engagement: A Case Study of the Corequisite Model
Gateway mathematics courses serve as entry points to degrees in the Science, Technology, Engineering, and Mathematics (STEM) fields. Numerous studies (e.g., Koch, 2017; Weston et al., 2019) have highlighted how these courses often function as roadblocks for STEM-intending students. With the reduction of developmental mathematics course offerings at postsecondary institutions, many universities have adopted the corequisite model of academic support to combat this pressing issue of student attrition within the early stages of the STEM pathway. There is evidence supporting the idea that enrollment in a corequisite is beneficial for student learning and course completion (e.g., Kashyap & Mathew, 2017; Logue et al., 2016). However, there does not exist a general consensus around the nature of the design and implementation of a corequisite, nor which design leads to greater student success. This case study documents an implementation of the corequisite model and highlights how the course was structured to promote greater student learning and active engagement. In addition, this study examines the student experience and outlines the impact that the course had on their attitudes towards mathematics.

Ander Erickson, *University of Washington Tacoma*; **Zaher Kmail**, *University of Washington Tacoma*; **Bonnie Becker**, *University of Washington Tacoma*
The Road Not Taken: A Comparison of Precalculus Pathways
Precalculus can operate as a gatekeeper for STEM majors, a phenomenon that is particularly pronounced for minoritized and first-generation students. Currently, University of Washington Tacoma, an institution with a high proportion of first-generation and minoritized students, offers a two-quarter precalculus sequence and a compressed one-quarter sequence and it is important for us to know whether the two

different sequences are supporting student success for students of different backgrounds. This multimethod study uses causal structure analysis to compare the performance of the two precalculus pathways in order to study which pathway leads to greater student success and under which conditions. This is complemented by a series of semi-structured interviews with students who choose to leave the calculus sequence in order to better understand the factors that lead to such decisions. We address the following pair of research questions: (a) Does a student's choice of precalculus pathway and/or other student characteristics (e.g., first-generation status, high school or transfer school of origin, ethnicity/race, placement scores) predict their performance in calculus? (b) What motivates students to choose different pathways and what subsequently leads some students to leave the calculus pathway entirely? We use ANOVA and multiple comparison t-tests to analyze the student performance in Calculus and to determine if there is a significant difference in the performance of the Calculus students based on the aforementioned student characteristics. We also use a two-sample proportion z-test to determine whether or not the proportion of the two pathways are equal. The results of this study suggest that the two-quarter calculus sequence is a crucial alternative to the one-quarter sequence and provides insight into factors that lead students to leave the calculus sequence. These results can inform the work of our academic advisors and precalculus instructors.

Marilyn Reba, *Georgia State University*; **Diana McGinnis**, *Georgia State Univesity*

PRACTIS (Precalculus Review and Calculus Topics in Sync): A Linked Remediation Program for Calculus I

Many students in Calculus I struggle due to missing or inadequate prerequisite skills. The literature on calculus readiness shows that remediation for Calculus I students has been attempted in a variety of ways both before and after students enter Calculus I. None of these precalculus reviews, however, have been integrated with the Calculus topics. PRACTIS videos and worksheets accompany each new Calculus I topic. They decompose calculus problems to reveal embedded precalculus concepts. Students are incentivized to remediate because they must implement these precalculus skills inside their current assignment. Examining student participation results after three semesters, we share our research design going forward. This initiative to increase retention and success in Calculus I has been supported by a Dean's Fellowship and Seed Grant at Georgia State University.

Kaitlin Riegel, *University of Auckland*; **Tanya Evans**, *University of Auckland*; **Jason Stephens**, *University of Auckland*

Predicting mathematics exam-related self-efficacy as a function of prior achievement, gender, stress mindset, and achievement emotions

Addressing student affect around assessment is vital, given it is tightly interwoven with cognition. This study seeks to describe the relations between exam-specific affect and stress mindset in a university mathematics course. Participants (N = 356) completed a survey assessing their exam-related self-efficacy, achievement emotions, and stress mindset. The study demonstrated significant correlations between a stress-is-enhancing mindset with positive affect and a stress-is-debilitating mindset with negative affect. When controlling for prior achievement and gender, stress mindset was significant, and student exam-related emotions were dominant in explaining exam-related self-efficacy. The results are discussed with opportunities to adapt learners' stress mindset and the development of exam-related self-efficacy.

Hadas Moshonov-Cohavi, *Avila University*
Supporting Student Success in Virtual Corequisite Course
Corequisite models for mathematics courses have been implemented as part of the Missouri Math Pathway initiative since 2014. The structure and implementation of these models vary between academic institutes. In this talk we will explore online methods for supporting student success in corequisite courses.
In this talk I will share the structure of the corequisite model we implement at Avila university. I will discuss the teaching methods and some of the content I have used and developed over the past few years. Emphasis will be on methods and tools for virtual settings.

Joseph Antonides, *The Ohio State University*; **Jim Fowler**, *The Ohio State University*
How are Limits Commonly Introduced in Calculus? An Examination of Six Calculus Textbooks
Limits constitute a fundamental component of many subsequent concepts in differential and integral calculus, and most calculus textbooks begin with an introductory chapter or section dedicated to limits. Research on undergraduate students' reading behaviors and comprehension of mathematical texts suggests they generally focus on worked-out examples, without reading the exposition; when they do read exposition, they tend to read it verbatim rather than for its intended meaning. In light of this, and acknowledging the important role played by limits in calculus teaching and learning, we investigated the examples, definitions, and facts (collectively called "items"), generally distinguished from the main exposition, that are common to a sample of six introductory calculus textbooks, as well as the order in which these items tend to be introduced. Because one of the six textbooks contained relatively few items, we repeated our analysis with the remaining five textbooks. Results indicated that six items were common to all textbooks in our sample (with an additional four items common to the five textbooks in our second round of analysis), including a formal, ε–δ definition of limit.

Pablo Duran Oliva, *Florida International University*; **Adam Castillo**, *Florida International University*; **Edgar Fuller**, *Florida International University*; **Charity Watson** *Florida International University*; **Geoff Potvin**, *Florida International University*; **Laird Kramer**, *Florida International University*
The Impact of Attitudes on Achievement in an Active Learning Calculus Course
In this talk, we will present the results of a year-long experimental study of the impact of student attitudes on achievement in an active learning Calculus course. The study includes 535 multi-cohort college students, who completed the ATMI survey, a well-validated instrument of student attitudes toward mathematics, during the first week of classes in the Spring and Fall 2019 semesters. Regression models were constructed to examine the association of pre-semester attitudes to final grade overall, broken down by relevant demographics including gender, precalculus proficiency and class standing. We will also discuss the implications of these results on our understanding of the role of active learning in college Calculus classrooms.

Charity Watson, *Florida International University*; **Pablo Duran**, *Florida International University*; **Adam Castillo**, *Florida International University*; **Edgar Fuller** *Florida International University* **Geoff Potvin**, *Florida International University*; **Laird Kramer**, Florida International University
Development of Precalculus Proficiency During an Active Learning Calculus Course
At Florida International University, our team of educators and researchers developed the Modeling Practices in Calculus (MPC) curriculum. MPC is a student-centric first-semester calculus course in which

students are challenged to engage in the authentic practices of mathematicians by actively doing mathematics in a lecture-reduced classroom with a focus on students working with their peers. In an effort to study the impact of MPC on the skills in and understandings of foundational calculus topics - namely those topics native to precalculus courses, a validated precalculus concept inventory was administered at the beginning and end of each semester. In this talk, we will present the observed gains in precalculus proficiency for those students exposed to the MPC curriculum. Also, we will present a detailed look into the relationships observed between precalculus proficiency and calculus achievement in MPC with comparison to traditional, lecture-based calculus classrooms.

Barbara Moskal, *Texas Tech University*; **Jerry Dwyer**, *Texas Tech University*; **Levi Johnson**, *Texas Tech University*; **G. Brock Williams** *Texas Tech University*; **Jill White**, *Texas Tech University*

Revised Calculus Concept Inventory: Early Development

Revised Calculus Concept Inventory: Early Development

Calculus has been a gatekeeping course in many colleges. College students who struggle in calculus have difficulty progressing in science, technology, engineering, and mathematics (STEM) degrees. In March of 2020 and as a result of the coronavirus pandemic (COVID19), many courses moved to an online delivery. As different modes for teaching calculus emerge, there is a growing need for an accurate measure of the impact or effectiveness of instructional changes on student knowledge acquisition. This paper will examine the progress of, the Revised Calculus Concept Inventory (RCCI) project, which is funded by the National Science Foundation and is underway at Texas Tech University. The RCCI is designed to offer a pre- and post- measurement of students' understanding of fundamental Calculus I concepts. The onset of COVID19 has challenged the development of this instrument as well as increased its importance of its development for the mathematics community.

The development of this instrument is based on a solid foundation of the work that has previously been completed. In 2004, Epstein and Yang with NSF funding developed a Calculus Concept Inventory (CCI). Using the CCI, Epstein (2013), found no measurable significant effects on normalized gains with respect to instructor experience, number of hours of a college calculus course (in the standard range of 3-5), entry-level score, or whether the student had previously completed a calculus course. The only significant factor appeared to be whether active learning was employed during instruction. In 2016, Thompson and Ashbrook developed two more calculus concept inventories, one in Calculus I and the other in Calculus II. These were designed to measure the impact of the Developing and Investigating a Rigorous Approach to Conceptual Calculus (DIRACC; Thompson & Ashbrook, 2016) curriculum. These efforts were completed before COVID19 forced many calculus courses to be delivered online.

The RCCI was funded in October of 2019. The initial efforts of this research progressed as planned: a national survey of high school and college calculus instructors were completed, and a panel of experts met and reviewed the results. Based on the survey results, the panel developed questions for use on the RCCI. The next planned phases were to complete "think alouds" with students and the pilot testing of the instrument in calculus classrooms. With the onset of COVID19, the think alouds were completed through interactive video, Zoom, and pilot testing of the revised instrument is planned for the Fall of 2021. The proposed paper will describe the on-going efforts to develop the RCCI and how these efforts have adapted to the restrictions of COVID19. The resultant instrument is designed for online delivery and has the potential to support the measurement of learning in both classroom based and online courses.

References

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Zackery Reed, *Oklahoma State University*; **Michael Tallman**, *Oklahoma State University*; **Michael Oehrtman**, *Oklahoma State University*; **Marilyn Carlson** *Arizona State University*

Principles of Conceptual Assessment Design in Calculus I

Supported by the National Science Foundation, the Mathematical Association of America administered a national survey in the fall of 2010 to instructors of Calculus I courses from a stratified random sample of non-profit two- and four-year colleges and universities offering a degree in mathematics. Instructors provided their final exams as part of this survey. The resulting data set consisted of 254 final exams from single-variable calculus courses designed to prepare students majoring in mathematics, science, or engineering. We analyzed all 4,167 individual items within this sample to identify features of tasks that enable instructors to reliably infer students' meanings for central calculus topics. We first present our operational criteria for determining whether an exam item assesses students' application of understanding. We then explore salient features of exemplary tasks from our data set to reveal distinctions between exam items made apparent by our analytical framework. In doing so, we offer commentary on the often subtle features of assessment items that contribute to variation in the reliability of inferences an instructor is positioned to make about students' understanding based on their responses. We also demonstrate the characteristics of exam items that instructors can attend to when designing assessments. Our analysis offers instructors some initial reflections on the nuances of task writing, and the affordances of attending to particular meanings that align with the instructors' goals for students' learning.

Mami Wentworth, *Wentworth Institute of Technology*; **Mel Henriksen**, *Wentworth Institute of Technology*; **Caroline Merighi**, *Marquette University*

What is Mathematics all about? Getting insight into freshman Calculus students' mindset

What do students think math is all about? What does learning math mean to them? We seek to move our students from a fixed mindset to a growth mindset, to provide them with tools and methodologies for deeper learning and to have them develop an understanding of what deeper learning in mathematics means.

We conducted a written survey to understand the initial mindset of freshman calculus students. In this talk, we will present our preliminary findings from this survey and discuss ideas and strategies to affect their mindset to achieve deeper learning in Calculus courses.

Sayonita Ghosh Hajra, *Sacramento State*; **Topaz Wiscons**, *Sacramento State*

Kimberly Elce, *Sacramento State*

Examining Elementary Pre-service Teachers' Use of Visual Models in Fraction Addition and Subtraction Problems

There is a consensus in mathematics education that students should have the ability to apply multiple representations and strategies to reason through mathematical tasks. Visual representations create an integration between conceptual understanding and formulaic mathematics that allows the student to make sense of the underlying mathematics. Given the importance of visual models in elementary curriculum and advanced mathematics thinking, mathematics teacher educators (MTEs) need to understand in-coming pre-service teachers' (PSTs) prior knowledge, and how they can support PSTs to gain flexibility and conceptual understanding of mathematics through visual representations. More specifically, what types of visual models do PSTs use to make sense of fraction addition and subtraction concepts, and how

PSTs' use of visuals develop through a one semester undergraduate mathematics content course for future elementary educators. In this talk we will discuss implications of our findings and the future direction of this research.

Sepideh Stewart, *The University of Oklahoma*
Linear Algebra Curriculum Study Group 2.0: Our Vision for a Change
In this talk, I will present the vision of the Linear Algebra Curriculum Study Group (LACSG 2.0) Committee for the future of linear algebra education and teaching this important subject. As part of this vision, we will focus on the role of technology in learning the subject, paying careful attention to teaching and considering students' struggles with fundamental concepts, unveiling the core concepts to be taught in first and second courses, finding ways of connecting to the industry, teaching the subject as early as possible in the curriculum as well as considering to offer appropriate second courses.

Anna Marie Bergman, *Simon Fraser University*; **Dana Kirin**, *Portland State University*
Exploring introductory linear algebra as a course and prerequisite
It is well known that linear algebra is leveraged throughout many fields of mathematics and so an introductory linear algebra course is unsurprisingly required for most mathematics majors. However, linear algebra concepts are used throughout the sciences and therefore introductory linear algebra courses offered in mathematics departments end up serving many client disciplines. To gain a better understanding of content covered in introductory linear algebra, we conducted a direct survey using course catalogs from 252 public institutions across the United States. In this study, we explore the content covered in introductory linear algebra courses by examining the course descriptions of 267 introductory linear algebra courses. Additionally, to gain a better understanding of the client disciplines being served, we looked at undergraduate degree programs requiring introductory linear algebra in a subset of our institutions. In this presentation we will highlight commonalities and variability in the content covered across these courses as well as shed light on the diversity of programs requiring linear algebra.

Jordan Kirby, *Middle Tennessee State University*; **Sam Reed**, *Middle Tennessee State University*; **Sarah Bleiler-Baxter**, *Middle Tennessee State University*
Student Understanding of Mathematical Induction in an Online Setting
Several studies (e.g., Dubinsky, 1989; Movshovitz-Hadar, 1993; Stylianides et al.,2016) have looked at student understanding of induction in introductory proof classes. The Covid-19 pandemic presented an opportunity to investigate how students in an introduction to proof class understand mathematical induction, and further what these students value when discussing an induction proof. By using the Stylianides (2007) framework for proof, we discuss the aspects of proof students valued for mathematical induction in an online setting.

Stephen Strand II, *CSU, Chico*; **Zackery Reed**, *Oklahoma State University*; **John Paul Cook**, *Oklahoma State University*; **April Richardson** *Oklahoma State University*
Ways of Thinking about Inverses across Contexts: A Systematic Literature Review
We report on the results of a systematic literature review (Fink, 2005; Creswell, 2007; Okoli, 2015) conducted, using thematic analysis (Braun & Clarke, 2006), on studies of students' reasoning with inverses that have been published in Tier 1 journals. While students' activity with inverses has generally been well-documented, much of what is known applies only to how students reason within specific contexts, and thus in its current form provides limited insight into how students might reason with

inverses across algebraic contexts. The primary contribution of our study is a unifying, coherent model that categorizes ways of thinking (Harel, 2008) about inverses, along with affordances and constraints of each. First developed during the analysis of a teaching experiment (Steffe & Thompson, 2000) and then refined by our systematic review of the literature, we present three ways of thinking about inverses: inverse as an undoing, inverse as a manipulation of the original element, and inverse as a coordination of the set, binary operation, and/or identity. We will expound each way of thinking about inverses by drawing from the literature, and then discuss insights the model can provide into students' potential development of propitious ways of thinking about inverses within and across mathematical contexts. We will conclude by discussing the applicability of our methodological approach for future systematic literature reviews.

Rachel Rupnow, *Northern Illinois University*

Examining Language across Contexts: Connecting Instruction and Problem-Solving

Math education has long focused on students' approaches to problem solving and how teachers approach instruction. However, limited work has examined these pieces together in the same context. Using the lens of conceptual metaphors to examine patterns of language use, I examine instructors' and students' language for group isomorphism and homomorphism in an introductory abstract algebra course. Based on extended classroom observations and interviewing two instructors and eight students, I highlight language used by both instructors and students when describing and working with group isomorphism and homomorphism. Results include the centrality of sameness to describing isomorphism for instructors and students but greater variety in descriptions for homomorphism, including more focus on the formal definition and mapping language.

Joash Geteregechi, *Ithaca College*

Exploring Undergraduate Students' Mathematical Reasoning via Problem Posing

In this talk, I argue that, despite the term mathematical reasoning being so common in mathematics education, it is often ill-defined or not defined at all. This lack of a clear definition has impacted the field in at least two ways. First, it has led to uncoordinated work among researchers in students' mathematical reasoning, and second, confusion among practitioners trying to promote mathematical reasoning among their students. Thus, I introduce Lithner's mathematical reasoning framework and explore how this framework could give instructors and researchers a more coordinated lens through which students' mathematical products can be analyzed and categorized. Specifically, I focus on students' problem-posing products since this has received much less attention when compared to problem-solving.

Abigail Higgins, *California State University, Sacramento*; **Ryan Alvarado**, *Amherst College*; **Sayonita Ghosh Hajra**, *California State University, Sacramento*

Student Partnerships as a Mechanism to Elicit Student Engagement in Intro to Analysis Courses

Analysis is a challenging subject for many students majoring in mathematics and is often students' first introduction to proof-based analytical mathematics. In this talk, we will discuss the implementation of a Student Partnership program to support students taking an introduction to analysis course at a small liberal arts college in the northeastern United States in spring 2020 and at a regional public university on the west coast of the United States in the 20-21 academic year. Student Partnership programs hire students as pedagogical consultants (called Student Partners) for the instructor and these students provide

a continuous feedback loop that the instructor can leverage to make meaningful pedagogical decisions. Student Partners are tasked with collecting and communicating information to the instructor about student experiences in the course, which the instructor can use to respond dynamically to student needs. In this presentation, we will share how these student-faculty partnerships can promote student engagement and can provide the student partner(s) an early opportunity for professional development. We will also share how these partnerships can foster a democratic classroom by (1) building a humanistic liaison between the students and the instructor and (2) promoting student involvement and ownership in the course. Data supporting these results include student and faculty interviews, student survey responses, and classroom observations. We will conclude by discussing no-cost variations of this support program.

Rethinking Mathematics Placement

Part A: *Wednesday, August 4, 11:00 a.m. - 11:55 a.m.*
Part B: *Wednesday, August 4, 3:00 p.m. - 3:55 p.m.*
Part C: *Friday, August 6, 1:00 p.m. - 2:20 p.m.*
Organizers:
Alexandria Theakston Musselman, *University of Washington Bothell*' **Emily Gismervig**, *University of Washington Bothell*; **Nicole Hoover**, *University of Washington Bothell*

Alexandria Musselman, *University of Washington Bothell*; **Emily Gismervig**, *University of Washington Bothell*; **Nicole Hoover**, *University of Washington Bothell*

The Development of a Mathematics Directed Self-Placement Process: Lessons Learned and Challenges Presented

A recent study of the University of Washington Bothell's mathematics placement exam illuminated significant equity issues. Specifically, this study found that females had more than 2:1 odds at being placed into the lowest level of developmental mathematics over their male counterparts by this single high-stakes placement test. When considering underrepresented female students, this study found that they were more than four times as likely to be placed into this lowest class compared to their White or Asian male counterparts (Angotti, R., Rosenberg, K. & Akapame, R., 2019). In light of the issues with our previous placement testing, we set out to develop a Directed Self-Placement (DSP) process to more equitably support students in choosing the most appropriate course for themselves.

In this session, we provide a short summary of the research that serves as a foundation for the Directed Self-Placement process and philosophy. We then discuss our experience developing a DSP process that would work for our students and campus. Here, we describe some practical considerations and challenges that arose in that work. We then provide some preliminary findings about how student placement changed class enrollments in our precalculus and calculus courses, followed by a description of how student retention and success in future classes may be impacted as a result of this new placement process. We conclude with future considerations and remaining questions for this work.

Michael Nathanson, *Saint Mary's College California*; **Jim Sauerberg**, *Saint Mary's College California*

Math placement as an active verb

Math placement is one of the first detailed interactions that students have with academics at the institution. As such, we have an obligation to set the tone for how they interact with their courses and how they think about their own skill base. Saint Mary's College administers skills-based online tests that aim to measure a student's confidence and familiarity with prerequisite skills and course-specific material.

These assessments certainly share some of the structural biases inherent in the SAT; but we continue to use them in the absence of a better system. In doing this, we perpetuate these biases and condone them in the eyes of our students.
As we rebuild and reimagine the placement process post-pandemic, how can we use a student-focused, skills-based model that effectively uses our limited resources to establish clear, affirming expectations for students entering all of our math courses? How can we change placement from something passive that is done to our students to something empowering that is completed by them? And how can we identify students for whom the placement stakes are high and whose testing might not tell the whole story, and then target our outreach and advising to these students?
This talk will present more questions than answers, reflecting on our experiences as Saint Mary's coordinator of math placement and incorporating the insights from others who have held this role.

Samuel Tunstall, *Trinity University*; **Emma Ross**, *Trinity University*
Promoting Agency in Mathematics Placement through an Online "Buildup" Program
Over the last two years, Trinity University has made several changes to its Calculus I placement process, notably including switching to the ALEKS platform, as well as developing an invitation-based interactive summer program (called Quantitative Buildup for Incoming Tigers, or QBIT) to review core pre-calculus topics using the ALEKS platform. Though there is still much work to be done, in the first year we have seen positive results among students who would normally place into Pre-Calculus and who now place into Calculus I, in that we are eliminating the potential need for remedial coursework for such students, and connecting students with their peers before they arrive on campus. In this presentation, we will discuss our program and share preliminary data on its efficacy in promoting student success in Calculus I. We will also discuss issues that have come up over the last two years, inviting comments and discussion with our colleagues in attendance.

Mei Chen, *The Citadel*
Math Placement - A Calculus I Readiness Program
It is crucial to place STEM majors accurately in Precalculus or Calculus I as they start their college journeys. The process of the math placement for Precalculus/Calculus I at The Citadel is designed as a Calculus I readiness program. This program provides STEM majors whose programs require a sequence of calculus tools and assistance they need to review, refresh or study skills in college algebra and trigonometry in the summer to be ready for Calculus I in the fall. In this presentation, we will discuss in detail the Calculus I Readiness Program at The Citadel and results.

Michelle Rabideau, *University of Hartford*; **Andrew Starnes**, *Lirio, LLC*
Success in Calculus : Investigating the Relationships between SATScores, High School GPA, Undergraduate Precaclulus Grade and Calculus Grade
In this talk we discuss the analysis of decades of data from the University of Hartford, a private nonprofit university that does not currently have a calculus placement exam. We look at how well high school GPA, SAT scores and undergraduate Precalculus grades correlate to Calculus I grades. Should we be using any of these data points to advise our students, and if so, which? We also look at the population of students who fail or withdraw from Calculus I on their first try and repeat the course. If it takes a student two semesters to complete Calculus I, would they have been better off spending those two semesters taking Precalculus followed by Calculus I? And if so, can we identify this population of students from their high school GPA or SAT scores in order to advise them into an appropriate path?

Zaher Kmail, *University of Washington Tacoma*; **Ander Erickson**, *University of Washington Tacoma*; **Bonnie Becker**, *University of Washington Tacoma*

The Effect of Math Placement and Multiple Precalculus Pathways on Calculus I Outcomes

A combination of Math placement exams and offering multiple Precalculus pathways can help support student success by increasing positive outcomes in Calculus I. Currently, UW Tacoma (UWT) offers placement into either a two-quarter Precalculus sequence or a compressed one-quarter sequence into Calculus I. By contrast, UW Seattle (UWS) and UW Bothell (UWB) only offer placement into one Precalculus pathway. It is important for us to answer two important questions: 1) Is there a statistically significant difference between the two Precalculus pathways at UWT? 2) Does offering placement into a choice of two different Precalculus sequences into Calculus I provide an advantage over a single Precalculus pathway? This multimethod study uses analysis of variance (ANOVA) and multiple comparison t-tests to analyze student performance in Calculus I and to determine if there is a significant difference in the performance of Calculus I students based on the two Precalculus pathways at UWT. The same statistical methods are used to analyze student outcomes in Calculus I at UWS and UWB. The results of this study show there was a significant difference between the placement exam scores for the multiple Precalculus pathways at UWT, but that were no significant differences in Calculus I outcomes, which allows us to conclude that the Precalculus pathways offered at UWT prepared students as well as students who took Precalculus elsewhere. Additionally, students were prepared equally as well for Calculus I regardless of the Precalculus pathway taken at UWT. Our results also show that, in contrast to UWT, there was a significant difference between Calculus I outcomes of students who are offered only the one Precalculus pathway at UWS and those UWS students who took Precalculus elsewhere. These results were also true for UWB students. We conclude that accurate placement exams in combination with the availability of both placement pathways, the two-quarter Precalculus sequence along with the one-quarter Precalculus sequence, are important to help improve outcomes for students in Calculus I. These results can inform the work of our academic advisors and Precalculus instructors.

Daniel Jordan, *Columbia College Chicago*

Preparing for Significant Placement Revision

Columbia College Chicago is a private, nonprofit, liberal arts college of more than 6000 students with an emphasis on the arts, communications, and public information. Placement in mathematics courses has primarily been based on ACT or SAT math subject scores. Recently, motivated by racial equity concerns, we made the decision to move from a test-optional to a test-free admissions policy and will no longer accept ACT or SAT scores for any purpose, including placement. As part of preparing for this major revision to our placement process, we collected and analyzed past placement data to establish procedures and baselines that will help ensure we are able to rapidly respond to future data. In this presentation, I will explain the methods we used, which are based on Sawyer (1996), describe some features of our placement system that the analysis revealed, and discuss our plans for future placement.

Kristin Frank, *Towson University*; **Alexei Kolesnikov**, *Towson University*; **Xiaoyin Wang**, *Towson University*

Methodology to Estimate and Evaluate Error Rates for Mathematics Placement Policies

One method to evaluate a placement policy is to attach an error rate (or the complementary accuracy rate) to a placement policy. An example of the error rate is the proportion of students misclassified by the policy. A part of the error rate is inherently unobservable: we cannot observe which of the students placed into a lower course would have been successful in the higher-level course. Regression-based methods to estimate the rate of under-placement typically provide a single number, without giving an idea about the interval of the likely values of the proportion. We describe a more robust methodology using propensity

match to estimate under-placement. While a given error rate (of say 30%) can be used to compare one policy against another (the policy with a lower error rate is better), it remains an open question whether an error rate of 30% is evidence of a successful placement policy. We propose a benchmark random placement error rate, that is, the error rate that would result if placement decisions were stochastically independent from the ability of a student to successfully complete the course. A placement policy can be called successful if its error rate is significantly lower than the error rate of the random placement.

Dave Rosoff, *The College of Idaho*; **Robin Cruz**, *The College of Idaho*
Lessons from a homegrown placement test
Without commercial alternatives, the math faculty at The College of Idaho were left to create an original mechanism for placement in developmental math. We discuss the implementation of our placement tests in WeBWorK, the details of their administration, and evaluate their effectiveness over the last few years.

Sherrie Serros, *Mount Mary University*
Anti-Deficit Placement Practices
The talk will describe our test-optional placement process that utilizes an ant-deficit framework for placement and course pathways. Our model incorporates a recommended placement and a direct self-placement option.

Using Inquiry and Collaboration in Faculty Professional Development during the Pandemic and Beyond

Part A: *Thursday, August 5, 11:00 a.m. - 11:55 a.m.*
Part B: *Friday, August 6, 1:00 p.m. - 2:55 p.m.*
Part C: *Saturday, August 7, 2:00 p.m. - 2:55 p.m.*
Organizers: Nina White, *University of Michigan*; **Patrick Rault**, *University of Nebraska at Omaha*; **Amy Ksir**, *United States Naval Academy*; **Laura Watkins**, *Glendale Community College*; **Christine Von Renesse**, *Westfield State University*

Angie Hodge-Zickerman, *Northern Arizona University*; **Cindy York**, *Northern Illinois University*; **Eric Stade**, *University of Colorado - Boulder*
Using Virtual TACTivities to Model Active Learning
In this session, participants will engage in a virtual TACTivity. Then we will interactively discuss how virtual TACTivities can be used in a professional development setting to both model and teach about active learning strategies that can be used in the mathematics classroom. Benefits and challenges to virtual TACTivities will also be discussed.

Babette M. Benken, *California State University, Long Beach*
Professional Development for Graduate Student Instructors: Using Modeling and Collaborative Inquiry to Expand Beliefs
In this session participants will be introduced to best practices, including specific tasks, for supporting the development of beginning graduate student instructors (GSIs), who are the instructor of record for lower division university-level mathematics and statistics courses. The focus will be on how modeling of effective pedagogy and collaborative inquiry can assist new GSIs in examining and expanding their beliefs – beliefs about the nature of mathematics, how students best learn, and what constitutes equitable and inclusionary practices. Example tasks will include recommendations for both virtual and in-person

implementation. Question prompts and task examples will provide a ripe context for scholarly conversation about how to facilitate the ongoing development of new GSIs.

Hanna Bennett, *University of Michigan*; **Gavin Larose**, *University of Michigan*; **Angela Kubena**, *University of Michigan*; **Paul Kessenich** *University of Michigan*; **Beth Wolf**, *University of Michigan*
Scott Schneider, University of Michigan

Designing Asynchronous Sessions for New Instructor Professional Development

As part of the work to convert the University of Michigan Math Department's week-long Professional Development program online for Fall 2020, many of our sessions were converted to an asynchronous format to reduce "Zoom fatigue" and accommodate 50 participants in 9 time zones. I will discuss what we did to retain a sense of inquiry and collaborative discussion, particularly through reflection questions and discussion boards, in these asynchronous sessions.

Nathaniel Miller, *University of Northern Colorado*

Online working groups as a form of Professional Development

GeT: a Pencil is a Faculty Online Learning Community (FOLC) dedicated to the improvement of college Geometry for Teachers (GeT) courses. It is made up of mathematicians and math educators from across the US. As part of the work of the learning community, working groups were formed to investigate different problems of practice. For example, one working group was organized around developing a set of essential Student Learning Outcomes (SLOs) that the group felt should be addressed in all GeT courses. While professional development was not the original central aim of the FOLC or the working groups, participants discovered over time that participation in the working groups and in the FOLC exposed them to many different ideas and inspired them to reflect critically on their own instructional practices, and therefore ending up being a very effective form of professional development. This talk will describe the structure of the working groups and the ways in which they ended up serving as a vehicle for professional development.

Priya Prasad, *SA-COMMIT and University of Texas at San Antonio*; **Cody Patterson**, *SA-COMMIT and Texas State University*; **Oscar Garcia-Roman**, *San Antonio Independent School District*; **Melisa Walters** *CAST Network*

The SA-COMMIT and CAST-Network Mathematics Initiative

Beginning in January 2021, The San Antonio Community for Mathematics Inquiry in Teaching (SA-COMMIT) began a year-long professional development partnership with CAST-Network high schools which are part of San Antonio's inner-city school district, SAISD. As part of its "Algebra-City Initiative" to improve student success in mathematics, SA-COMMIT facilitated IBL workshops with teachers from CAST-Network schools. Initial results of the Texas high-stakes STAAR tests this spring indicate the teachers participating in the IBL workshops had the highest student passing rates in the district. This session will introduce specific activities used during the monthly meetings and include a discussion of successes and challenges encountered during the partnership.

Nermin Bayazit, *Fitchburg State University*; **Christine von Renesse**, *Westfield State University*; **Ileana Vasu**, *Holyoke Community College*

The "Faculty Fellowship and Coaching Program" of NE-COMMIT (New England Community for Mathematics Inquiry in Teaching)

In this session the leadership team of NE-COMMIT (New England Community for Mathematics Inquiry in Teaching) will describe the "Faculty Fellowship and Coaching Program" over the last two years (funded by Mathematics Learning by Inquiry and Transforming Post-Secondary Education in

Mathematics). Participants will think about the challenges and benefits of coaching, and learn about the experiences of the NE-COMMIT coaching and fellows team. Future work consists of reflecting on best practices and efficacy of various coaching models, as well as implementing these practices in NE-COMMIT. We hope that this session will start a discussion about coaching in higher education mathematics.

Benjamin Wilson, *Stevenson University*; **Sarah Loeb**, *Hampden-Sydney College*; **Michael Strayer**, *Hampden-Sydney College*

MAA IP Guide Reading Group: What We Learned

Five faculty from four colleges and universities in the MD-DC-VA Section of the MAA formed a professional learning community to read through the MAA Instructional Practices Guide (MAA IP Guide), meet to discuss the readings, and make plans to use the guide to improve teaching practices. In this talk we will discuss how we structured the reading group, take-aways from our participation in the reading group, and specific examples of how we used what we learned from the group and the MAA IP Guide to inform our teaching.

Sarah Bleiler-Baxter, *Middle Tennessee State University*; **Grant Gardner**, *Middle Tennessee State University*; **Gregory Rushton**, *Middle Tennessee State University*; **Olena James** *Middle Tennessee State University*; **Fonya Scott**, *Middle Tennessee State University*; **Amanda Lake He**ath, Middle Tennessee State University

Using Teaching TRIOs to Support Faculty Awareness and Responsiveness to Inclusivity in Mathematics and Science Classrooms

We describe the Teaching TRIOs model for faculty professional development, where intradepartmental teams of three faculty members from Mathematics, Chemistry, and Biology departments collaborated over the course of a year to observe and reflect on one another's classroom instruction. In particular, our most recent iteration of the Teaching TRIOs occurred during the 2020-2021 academic year, was implemented entirely in a remote setting (by Zoom), and had a specific focus on inclusive pedagogy. We share successes and challenges of the project with respect to our goal of supporting faculty awareness and responsiveness to inclusivity in mathematics and science classrooms.

Sarah Bockting-Conrad, *DePaul University*; **Elizabeth DeWitt**, *Trinity Christian College*; **Matthew Lee**, *Oakton Community College*; **Aliza Steurer** *Dominican University*; **Lance Vobornik**, *Northern Illinois University*

Conversations in Chicagoland: A Way to Connect with Colleagues while Preparing for Pandemic Teaching and Beyond

The Chicagoland COMMIT (ChIBL) facilitated a three-part virtual conversation series in summer 2020, with the themes of IBL and active learning in a virtual environment, assessment, and building classroom culture and a sense of belonging. The conversation series served two purposes: (1) to connect instructors in the Chicago area who are interested in teaching with inquiry and active learning, and (2) to discuss specific concerns and collaboratively develop solutions related to the themes of the conversations, in light of modified learning environments due to the pandemic. Each conversation allowed instructors to reflect on their own teaching practices and also modeled how one might use inquiry in their teaching. We will share the planning process, conversation format, benefits--some expected and some surprising--of the discussions, and challenges that emerged from the conversations. Participants will collaboratively brainstorm about these issues and ideas.

Deependra Budhathoki, *Ohio University*; **Gregory D. Foley**, *Ohio University*; **Marian Prince**, *Andrews University*; **Binod P. Pant** *Kathmandu University, Nepal*

International Online Professional Development for Mathematics Faculty and Teachers

Building on prior face-to-face professional development and established international collaboration, in the summer of 2020, we assembled a USA–Nepali team to promote student mathematical inquiry through a series of five online Zoom workshops for Nepali secondary school teachers and college instructors. Each of these main workshops was preceded by trainer-of-trainers workshop. The content of every workshop was either (a) connecting trigonometry to its geometric roots or (b) creating graphical and algebraic models for genuine data. The pedagogy addressed how to promote inquiry by engaging students in using free online mathematical action software, specifically GeoGebra and Desmos. Anticipating a large number of almost exclusively Nepali participants, we designed each main workshop to begin with an overview and launch in English, followed by hands-on activities done in multiple breakout sessions of 6–9 participants led in Nepali, followed in turn by a whole-group wrap up in English. In all, five small workshops were used to train breakout room leaders, and five large workshops were conducted to provide professional development for Nepali mathematics teachers and instructors. The breakout room leaders included 13 Nepali-speaking university professors and graduate students from the United States and Nepal. More than 250 secondary school teachers and college instructors participated in at least one workshop. Each of the five main workshops involved 60–80 participants. Workshop evaluation form revealed that the majority of the participants found that having Nepali-speaking breakout room leaders and hands-on activities provided them with opportunities to foster inquiry and collaboration in learning.

Elizabeth Thoren, *Pepperdine University*

Affordances and Challenges of Multi-Day Virtual PD

Over the summer of 2020 the 4-Day IBL Workshops shifted to virtual formats -- including a 3-week mini course model. In this session we will discuss some of the affordances and challenges inherent in this longer format professional development setting.

Sandra Laursen, *University of Colorado Boulder*; **Devan Daly**, *University of Colorado Boulder*

Putting (good) old wine in a new bottle: Adapting face-to-face workshops for online delivery

We will share a practical handbook developed as a resource for leaders of online professional development for mathematics instructors, based on the experiences of leaders of workshops for the Academy of Inquiry Based Learning. Online workshops required workshop leaders to figure out new tactics for reaching their goals of building community, modeling inquiry-based teaching, and attending to equity. They chose tools, schedules, and activity structures to accommodate participants across time zones, use synchronous meeting time wisely, and battle Zoom fatigue. Specific examples will emphasize the importance of intentional design and advance planning in adapting face-to-face professional development models for online environments.

PosterFest 2021: Scholarship by Early Career Mathematicians

Organizers: Holly Attenborough, *University of Wisconsin-Platteville*; **Lisa Driskell**, *Colorado Mesa University*

Ariana Dundon, *University of Washington | Bothell*

Creating a Data Science Program: Lessons Learned

Data Science has become such an in-demand field that many institutions are striving to create programs and curricula to meet this need. Learn how an interdisciplinary committee of faculty from all across our

campus came together to develop two programs in Data Science and Data Analytics, and what we wish we had known from the beginning.

Kristen Schemmerhorn, *Concordia University Chicago*
LEGO Duplo Activities for Calculus III
LEGO Duplo bricks add an element of play to introducing some three dimensional concepts in Calculus III. Activities for graphing functions of two variables, using Riemann sums to approximate double integrals over rectangles and general regions, and average value of a function of two variables using LEGO Duplo bricks will be described.

Francesca Gandini, *Kalamazoo College*
Noether's Degree Bound Over the Exterior Algebra
When we consider the action of a finite group on a polynomial ring, a polynomial unchanged by the action is called an invariant polynomial. A famous result of Noether states that in characteristic zero the maximal degree of a minimal invariant polynomial is bounded above by the order of the group. Our work establishes that the same bound holds for invariant skew polynomials in the exterior algebra. Our approach to the problem relies on a theorem of Derksen that connects invariant theory to the study of ideals of subspace arrangements. We adapt his proof over the polynomial ring to the exterior algebra, reducing the question to establishing a bound on the Castelnuovo-Mumford regularity of intersections of linear ideals in the exterior algebra. We prove the required regularity bound using tools from representation theory. In particular, the proof relies on the existence of a functor on the category of polynomial functors that translates resolutions of ideals of subspace arrangements over the polynomial ring to resolutions of ideals of subspace arrangements over the exterior algebra.

Stephanie Reed, *California State University, Fullerton*; **Elia Ziade**, *Palomar College*
On Transient Analysis of Delta_n Markov Chains
Consider a cyber physical system which has the purpose of maintaining the security of some physical asset. A particularly relevant scenario is one in which the system must perform its function while at the same time enduring malicious attacks for an extended amount of time without energy renewal. We will study a theoretical model analogous to this cyber physical system with an arbitrary number of nodes. The nodes in this system are able to communicate with one another and have the task of maintaining the security of the system. Initially all nodes comprising this system are in working order, however, due to attacks they can eventually become corrupted and then work against the security of the system. Once enough nodes become corrupted, the system will fail. Of particular interest is the calculation of a number of metrics related to reliability of the system including the reliability function and the mean time to failure. This model has been examined by many researchers in the past, mostly using numerical methods. The goal of this project is to exploit the underlying framework of the model which is a continuous-time Markov chain in order to analytically derive explicit expressions for these metrics. A result such as this would be stronger than previous results via numerical methods. The potential applications of this work are wide ranging and include unmanned aircraft systems, and disaster recovery among others.

Mingwei Sun, *Samford University*; **Sheng Gao**, *Samford University*
Penalized Regressions with Different Tuning Parameter Choosing Criteria and the Application in Economics

Recently a great deal of attention has been paid to modern regression methods such as penalized regressions which perform variable selection and coefficient estimation simultaneously, thereby providing new approaches to analyze complex data of high dimension. The choice of the tuning parameter is vital in penalized regression. In this paper, we studied the effect of different tuning parameter choosing criteria on the performances of some well-known penalization methods including ridge, lasso, and elastic net regressions. Specifically, we investigated the widely used information criteria in regression models such as Bayesian information criterion (BIC), Akaike's information criterion (AIC), and AIC correction (AICc) in various simulation scenarios and a real data example in economic modeling. We found that predictive performance of models selected by different information criteria is heavily dependent on the properties of a data set. It is hard to find a universal best tuning parameter choosing criterion and a best penalty function for all cases. The results in this research provide reference for the choices of different criteria for tuning parameter in penalized regressions for practitioners, which also expands the nascent field of applications of penalized regressions.

Emma Talis, *Stony Brook University*; **Heather Lynch**, *Stony Brook University*

Penguin Trail Networks: Applications of Graph Theory & Agent-Based Modeling

The gentoo penguin, Pygoscelis papua, is a colonial seabird that nests on patches of bare rock along the western Antarctic Peninsula. During the breeding season, these penguins move through the snow to travel to other bare rock patches to steal stones to bolster their own nests or to the water to forage. In doing so, gentoo penguins create trails in the snow that persist and evolve over the breeding season. Using unmanned aerial vehicles to capture these trails and the terrain underneath, we aim to characterize the trail networks in a similar manner to previously studied animal transportation networks and determine key patterns in network structure. We employ an agent-based modeling approach to simulating penguin movement throughout a colony in an effort to compare simulated and observed trail networks and answer key questions about penguin behavior. Here we present the data collection and representation process for this ambitious project, discuss obstacles in representing these trail networks as standard graph objects, and report preliminary results of an agent-based model.

Sedar Ngoma, *SUNY Geneseo*

Recovering a time-dependent source function in a parabolic equation

A time-dependent inverse source problem for a parabolic partial differential equation with a Neumann boundary condition and subject to an integral constraint is investigated. We show the existence, the uniqueness, and higher order regularity of solutions. Our proof of the existence and uniqueness of solutions yields an algorithm that we used to approximate solutions of the inverse problem using a finite element discretization in space and the backward Euler scheme in time. Due to instability in the reconstruction, Tikhonov regularization is applied. The errors resulting from our experiments show that the proposed scheme provides an accurate way to approximate solutions of this inverse problem.

Kristen Harvey, *Washington State University*

Systematic Literature Review on Interventions for Math Anxiety at the Community College

Over the past year I have done a deep dive into the existing research done on interventions for math anxiety. Much research has been done at the elementary school level to find the causes of math anxiety,

but less has been done focusing on later interventions once the anxiety is entrenched. Even less research has been done at the community college level. In this poster, I summarize my findings from a systematic literature review on three categories of interventions that have been studied: 1) Curricular Strategies, 2) Instructional Strategies, and 3) Non-instructional Strategies. Meta-analysis examination has found which strategies are effective and which strategies need further study. I then turn to examining the research that has been done at the community college level and conclude with a proposal for a future study which incorporates findings from the review of literature.

MAA Outreach Poster Session

Organizers: Rachelle DeCoste, *Tensor Women and Mathematics*; **Candice Price**, *Tensor SUMMA*; **Nancy Neudauer**, *Dolciani Mathematics Enrichment Grant*

Jillian Miller, *Roane State Community College*; **Alys Hugo**, *Everett Community College*

Breathing Life (Sciences) Into Mathematics Courses

"When am I ever going to use this?!" Math educators hear this question time and time again. Quantitative Biology at Community Colleges (QB@CC) is a 5-year NSF RCN-UBE funded project aimed at helping math instructors answer this age-old question for their students, especially their biology students who may not recognize the importance of quantitative skills in their chosen discipline. The primary objective of the project is to build a network of math and biology faculty from community colleges around the nation who work together to enhance students' quantitative skills by creating, publishing, and disseminating open-access cross-curricular materials. Small interdisciplinary teams work together to create or adapt OER materials at the interface of mathematics and biology, with the intent of providing context for mathematics concepts and improving quantitative literacy in biology classes. This, in turn, helps students transfer their knowledge to other scenarios and make cross-disciplinary connections.

Nicholas Heyer, *San Diego State University*; **Kaia Ralston**, *San Diego State University*; **Antonio Martinez**, *San Diego State University*; **Chris Rasmussen** *San Diego State University*

When Black Lives Matter enters the mathematics class: What would you do?

Consider the following scenario: Before class starts, a student walks into your classroom wearing a Black Lives Matter hat. Another student sits down and says to that student, "Hey, all lives matter." What would you do? Our research focuses on this exact scenario and how university mathematics instructors would handle such a situation. The data for our study consists of 14 interviews with instructors. For analysis, we used a set of a-priori codes from a related analysis of a microaggression scenario to help us make a direct connection between their responses and designated character archetypes. These archetypes reveal to us how mathematics instructors reflect and respond to issues of social justice. In this study, we categorized responses into five specific archetypes: the Action-Taker, the Cautionary, the Connector, the Thinker, or the Confidant. We found that most instructors' responses reflected the Cautionary archetype. There were many instances of hesitation and uncertainty of how to lead a discussion, especially in a mathematics environment. Furthermore, the abundant number of Cautionary responses tells us that mathematics instructors have limited experience in dealing with tense situations that center race in the classroom. A number of participants were also seen as an Action-Taker. While often paired with other archetypes, these Action-Takers all displayed confidence in their response and would take a clear stance on the issue. We discuss how limited experiences may contribute to the need for professional development in handling issues of social justice in the classroom and the instructor moves that might facilitate a more receptive environment for discussion.

Lillian Demarais, *Indian River State College – Student*; **Duane Chin-Quee**, *Indian River State College - Department of Mathematics*

A Mathematical Covid-19 SIR Model Using Data for the State of Florida.

We consider an epidemic SIR model involving a system of first order linear differential equations. Covid-19 data for the state of Florida is applied to the system to find suitable parameters for the model. The technique, used to find the parameters, will be presented along with the results of the model.

Jessie Hamm, *Winthrop University*

A Virtual Sonia Kovalevsky Day

I have hosted many math outreach events since I started my career as a professor in 2014. However this summer I hosted my very first virtual outreach event-a Sonia Kovalevsky Day for middle school girls. Our theme was cryptography and we ended the day with a "Break out of the Room" game! In this poster I will share my experience, what worked, what didn't work, and tips for those who are planning virtual outreach events in the future!

John Peca-Medlin, *UC Irvine*; **Kelly Isham**, *UC Irvine*; **Jesse Kreger**, *UC Irvine*

COMP: Graduate students building and promoting community and inclusivity during a pandemic

The Community, Outreach, and Mentoring Program (COMP) is a graduate student-led program designed to foster community and inclusivity within the Department of Mathematics at the University of California, Irvine (UCI). In 2020, student anxiety was exacerbated due in part to the physical isolation necessitated by the COVID-19 pandemic and larger social injustice issues, such as those addressed by the Black Lives Matter movement. The UCI math department acknowledged insufficient support systems existed for student success, especially for students from underrepresented groups. COMP was formed by graduate students with departmental support partly to address this need as well as to provide a formal and more substantive support structure for incoming students who would first experience graduate school in a virtual learning environment. Inspired by the UCI Physics & Astronomy Community Excellence Program (PACE), COMP expanded the existing mentoring program to include a weekly seminar where students can discuss topics ranging from student health and well-being to broader social issues that impact the mathematical community and beyond, in addition to organizing regular social events and activities. Each seminar also features a UCI math community member, so seminar participants (mostly first- and second-year students) can get a better idea of the individuals who comprise the department. Additionally, COMP actively participates in the recruitment of graduate students to UCI from historically excluded populations.

Bhuvaneswari Sambandham, *Dixie State University*; **Jie Liu**, *Dixie State University*; **Clare Banks**, *Dixie State University*; **Vinodh Chellamuthu** *Dixie State University*

Dixie Tensor Scholar Program (DTSP) - Maryam Day

Through the MAA Tensor Women and Mathematics Grants, we developed the Dixie Tensor Scholar program at Dixie state university to allow our undergraduate students to connect with local high school students and successful women STEM professionals to exchange ideas through a year-long mentoring program. We successfully completed a year of the DTSP program. As a result, we organized the one-day workshop named Maryam Day on April 24, 2021, where 21 local high school girls participated in the workshop. During the workshop, we created the awareness of career options, demonstrating the benefits of studying mathematics by providing them with an environment to exchange ideas, strengthen problem-

solving skills, and advance their intellectual confidence. In our presentation, we will share our experiences and the progress of our DTSP program.

Richard Wong, *University of Texas at Austin*; **Jonathan Johnson**, *University of Texas at Austin*; **Casandra Monroe**, *University of Texas at Austin*; **Luis Torres** *University of Texas at Austin*; **Hannah Turner**, *University of Texas at Austin*, **Nicolas Reyes**, University of Texas at Austin

Establishing the Mathematicians of Color Alliance at Texas

Started in August 2020 by graduate students of color in mathematics, the Mathematicians of Color Alliance at Texas (MoCAT) is a student organization at the University of Texas at Austin devoted to promoting the mathematical development and achievements of underrepresented groups in mathematics. In this poster session, we will discuss our experiences in establishing a new diversity-focused organization, share some of the activities and events that we organized, and talk about our goals for the future.

Francesca Bernardi, *Worcester Polytechnic Institute*; **Katrina Morgan**, *Northwestern University*; **Samantha Moore**, *University of North Carolina at Chapel Hill*; **Marissa C. Ashner** *University of North Carolina at Chapel Hill*

Girls Talk Math - Engaging Girls through Math Media

Girls Talk Math (GTM) is a free 2-week day camp for girls and non-binary rising 9th-12th grade students. Founded in 2016, GTM ran for the sixth time at the University of North Carolina at Chapel Hill in June 2021. The program has been funded by the Mathematical Association of America Tensor Women and Mathematics and SUMMA Grants. A sister camp at the University of Maryland at College Park has been running since 2018.

Campers work in groups to complete challenging problem sets and research the life of a mathematician or scientists from an underrepresented group in STEM. The camp curriculum is developed by graduate student volunteers within an inquiry-based learning framework. Problem sets cover topics in theoretical and applied mathematics and are available to download on GitHub (at https://github.com/girls-talk-math). During camp, each group of students writes a blog post focused on the math topic they learned and writes and records a podcast about the assigned mathematician or scientist. All camper-produced media is available at https://girlstalkmath.com, on iTunes, and SoundCloud.

GTM camps have hosted more than 300 students and have produced over 40 podcasts on mathematicians from underrepresented groups in STEM. Camp received positive feedback from the local community and interest from other Universities. The program is evaluated through pre- and post-surveys measuring campers' confidence and interest in enrolling in higher level courses in STEM. A new GTM chapter at the Worcester Polytechnic Institute will begin hosting students in the summer of 2022.

Ruth White, *Towson University*; **Nina Cliff**, *Towson University*; **Skylar Benson**, *Towson University*

Hop-Skip-Slide: Number Line Fun!

How can we use kinesthetic activities to teach elementary students? This presentation shows how students can interact with number lines while conducting various sports-based movements, with a lesson that was conducted virtually and in-person. As a result, students can gain number sense skills incorporating whole numbers and fractions.

Lauren Rose, *Bard College*; **Amanda Landi**, *Bard College at Simons Rock*; **Jazmin Zamora Flores**, *Bard College*; **Shea Roccaforte** *Bard College*; **Philip Barnet**, *Bard College*; **Julia Crager,** *Bard College*

MAGPIES: Math & Girls + Inspiration = Success

MAGPIES: Math & Girls + Inspiration = Success is a virtual outreach program for 4th-9th grade girls. Monthly sessions are presented and facilitated by faculty and students at Bard College and volunteer math educators. Our goal is to introduce young girls to the joy of mathematical exploration, create a community of girls who are enthusiastic about mathematics, and to mentor undergraduate students to facilitate and coordinate outreach activities.

Emily Hendryx, *University of Central Oklahoma*; **Matthew Parks**, *University of Central Oklahoma*; **Andrew Taylor**, *University of Central Oklahoma*

Math in the Field: An Affordable and Scalable Cross-Disciplinary Undergraduate Research Project

This poster presents an implementation of a 12-week undergraduate research project studying litter accumulation in local waterways. From data collection to subsequent analysis and differential equation modeling, students engaged with the research process to study a relevant question through a cross-disciplinary lens. The research team consisted of six students majoring in a variety of STEM fields and three faculty members: one from the Mathematics and Statistics Department and two from the Biology Department. With little required equipment and many possible research questions to adapt and explore, this project may serve as a potential model for others looking to involve undergraduate students in the research process at their institutions.

Sayonita Ghosh Hajra, *Sacramento State*; **Abigail Higgins**, *Sacramento State*

Math Tutoring Buddies Program

We will describe the Math Tutoring Buddies Program (MTB) at Sacramento State, which provides high-quality one-on-one tutoring to middle and high school students in the local community to enrich the learning opportunities for both university and secondary school students. This program aims to broaden educational opportunities for secondary students who might otherwise not have access to these resources by providing one-on-one support for math learning and academic mentoring to promote awareness of STEM higher education and associated career pathways. MTB was started in Fall 2020 with the intent to mitigate some of the educational inequities that are exacerbated by the COVID-19 pandemic.

Holly Attenborough, *University of Wisconsin-Platteville*; **Chris Frayer**, *University of Wisconsin-Platteville*

Mathematics Outreach with (Instant) Insanity!

The Instant Insanity^{TM} puzzle consists of four cubes whose faces are one of four different colors. The objective of the puzzle is to stack these cubes in a column so that each side (front, back, left, and right) of the stack shows each of the four colors. As there are over 40,000 ways to stack the cubes, it is easy to see how it got its name. We have used Instant Insanity^{TM} as an outreach activity to engage students ranging from middle school to college. The combinatorial connections, existence of a graph theoretical solution, and extensions of the problem, all lead to an accessible, hands-on activity that allows participants to explore mathematics. This poster will share our best practices and show you how to use our Instant Insanity^{TM} activity for mathematical engagement and outreach.

Jean Ciscell, *Towson University*, **Wendy Gibson**, *Towson University / Baltimore County Public Schools*; **Diana Cheng**, *Towson University*; **Kimberly Corum** *Towson University*; **Michael Krach**, *Towson University*; **Rachel Mulvaney**, Baltimore County Public Schools

STEM Model-Eliciting Activities with Baltimore County Public School Students

We describe a university-school partnership which provides opportunities for undergraduate and graduate students to develop and implement of Model-Eliciting Activities (MEAs) with public school elementary and middle school students in the Baltimore County Public School district. By being involved with the

development and implementation of these MEAs, TU students will also gain educational research experience by collecting and analyzing the middle school students' modeling work.