Polynomia, Maxiministan, and the Calculusian Republic

A Review by Debbie Gochenaur

Uncommon Mathematical Excursions: Polynomia and Related Realms, by Dan Kalman. MAA, 2009. Hardcover, 296 pages, $61.95 ($49.95 to MAA members).

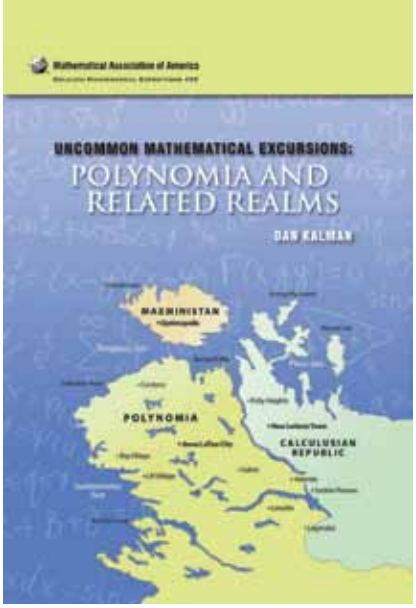
Offering an assortment of topics in the algebra, geometry, and calculus curricula, this book is intended as enrichment for those familiar with these topics at the upper-secondary or introductory college math level. The book is geared to teachers who have taught this material enough times to be thoroughly comfortable with it, but advanced students and scientists and mathematicians in general, may also find things here that will interest them.

The book is divided into three parts — *The Province of Polynomia*, *Maxiministan*, and *The Calculusian Republic*. Kalman ensures that the reader can make connections to familiar topics. He works to help the reader make extensions and perhaps understand more clearly the depth of mathematics in these seemingly elementary topics.

Uncommon Mathematical Excursions is not meant to be a textbook — it is a journey linking new ideas to familiar ones. Some topics are meant to be lingered over: in the Province of Polynomia, solving polynomial equations, including alternate solutions for cubics and quartics together with the historical reference for when each was discovered; in Maxministan, the hallway problem which deals with moving a ladder horizontally around a corner, leading to envelopes, an extension, and a discussion of duality; in the Calculusian Republic, envelopes, including boundary points, intersections and asymptotes.

Other sections are meant to be quick day jaunts into a variety of topics, each self-contained. They include palindromials, Marden's Theorem, borders on string art (where boundaries appear to be smooth curves but are in reality polygonal paths), and isoperimetric duality. While some topics may be familiar to the reader, everyone is sure to find something interesting.

The book includes descriptions of the concepts, setting them in historical context, and a variety of examples with some proofs. For example, the discussion on solutions for quartics begins with Descartes' 1637 factoring of the quartic into two quadratics. After some algebraic manipulation, one gets a sixth degree polynomial that is a cubic in u^2, which can be solved using methods that Kalman has discussed in previous sections. Kalman moves on to Euler's 1770 solution to the quartic, linking it to Euler's work on the cubic. Beginning by assuming that x is the sum of the square roots of r, s, and t, x is squared, simplified, and then squared again. Equating coefficients and substituting enables Euler to rewrite this system so that you finally find that the original r, s, and t are the roots of a cubic polynomial. After giving a third algebraic approach to solving quartics, Kalman moves on to explain the connections between solving quartics and cubics as well as Lagrange's frustrated but foundational struggle to extend this work to quintics.



There are sidebars throughout the book, including one on Lagrange and Vandermonde with a brief mention of their work with polynomial equations as well as giving a broader picture of their contributions, placing Lagrange as the greatest mathematician between Euler's and Gauss's generations. Every chapter ends with a brief history of the topics covered, with references and additional print sources if the reader wishes to explore further.

This book would make great summer reading or provide a good source for individual or group project assignments. The metaphor of traveling and exploring the back country of mathematics is used well. Kalman's sense of humor and adventure comes through from cover to cover while he takes you on the roads less traveled making connections to the mathematics you love.

Debbie Gochenaur is Assistant Professor of Mathematics at Elizabethtown College. Her interests lie in math learning disabilities and encouraging underrepresented minorities in STEM. This is a lightly edited version of a review that first appeared in MAA Reviews on May 28, 2009. For more reviews, visit http://www.maa.org/maareviews/.