11.11 No. Consider a triangle where two of the angles are close to 90°.

11.12 Use Problem 11.S3.

11.13 First show that $\overrightarrow{BC} \cdot \overrightarrow{AC} = -2c^2$.

11.14 Use that $\overrightarrow{BC}$ is perpendicular to $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ and the fact that the dot product of perpendicular vectors is 0. The answer to the last question is "no."

11.15 Inscribe triangle ABC in a circle centered at O. Now form parallelograms $AOCX$ and $XHBO$. Show H is the desired point of concurrency.

11.16 Note that for any i, $\overrightarrow{XA_i} = \overrightarrow{XO} + \overrightarrow{OA_i}$. Now expand the sum and use the result from Example 67.

11.17 The point X will be the median of the triangle.

11.18 If a, b, c, and d are the side lengths and d_1 and d_2 are the diagonal lengths, the distance between the midpoints of the diagonals will be $\sqrt{(a^2 + b^2 + c^2 + d^2 - {d_1}^2 - {d_2}^2)}/2$. To show this, let the terminal points of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$ correspond to the vertices A, B, C, and D of the quadrilateral, where these vectors all share some common initial point. Now express the desired length as a dot product and show it is equivalent to the value given above.

11.19 Let $\vec{x}$ be the vector on the left hand side of the equality. Show that $\vec{x} \cdot \overrightarrow{OA} = \vec{x} \cdot \overrightarrow{OB} = 0$.

Affine Transformations

12.1 Let h_1 and h_2 be affine transformations mapping the three given points to the three desired points, and prove $h_1 = h_2$. Compare the images of $\vec{i}$ and $\vec{j}$ under $h_1 \circ f$ and $h_2 \circ f$.

12.2 Answer: No.

12.3 Show there is an affine transformation f mapping trapezoid $ABCD$ to an isosceles trapezoid.

12.4 Use an affine transformation to map the ellipse to a circle.

12.5 Let $A_1, \ldots, A_n$ be vertices of the polygon, $\mathcal{P}$, and $A'_1, \ldots, A'_n$ be their images under an affine transformation. Find $\sum \overrightarrow{G'A'_i}$, where G' is the image of the centroid of $\mathcal{P}$.

12.6 The proof is similar to the proof of Theorem 12.13.

12.7 Consider an affine transformation mapping $\triangle ABC$ to an equilateral triangle. Show that the images of the other two triangles will also be equilateral. Finally, use the fact that centroids get mapped to centroids by affine transformations.

12.8 Map the parallelogram to a square and cite Theorem 12.7(6).

12.9 Consider the rectangle R that is tangent to the four vertices of the ellipse. Find the areas of $f(R)$ and $f(\mathcal{C})$, where f is an affine transformation sending the ellipse to $\mathcal{C}$, a circle of radius a.

12.10 Let f be an affine transformation mapping the ellipse to a circle. Note that $f(ABCD)$ will be a parallelogram circumscribing this circle.

12.11 Consider an affine transformation that maps $\triangle ABC$ to a right triangle (with right angle at the image of C) with side lengths $\alpha + 1$ and $\beta + 1$.

12.12 Consider an affine transformation that maps one of the ellipses to a circle.

12.13 Solve the problem by assuming the ellipse is a circle. For the generalization, assume that the n-gon is not regular and find a contradiction.