Quantitative Literacy and Mathematical Competencies

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Quantitative Literacy and Its Relatives

When reading the material made available to Forum participants, in particular *Mathematics and Democracy: The Case for Quantitative Literacy* (Steen 2001) and the background essays prepared for the Forum, three observations came to my mind.

First, the authors all seem to be speaking of roughly the same "animal," but they give it a variety of different names such as quantitative literacy, numeracy, mathematical literacy, and mathematical competencies. We also could add the term "mathemacy" coined by Ole Skovsmose (1994). Irrespective of the labels used, what people have in mind is something other than proficiency in pure, theoretical mathematics, something that goes beyond such knowledge and skills.

Second, the same term, "quantitative literacy," is given a variety of different interpretations by different authors. The variation is mainly a matter of how narrowly the word "quantitative" is to be understood, vis à vis the involvement of numbers and numerical data. Some use the word in a much broader sense than numbers and data only.

Third, finally, and most significantly, there seems to be general consensus about the importance of making a case for the "animal," whatever it is going to be called. That consensus certainly includes me.

The first two observations suggest that we are short of a one-to-one correspondence between the terms used and the ideas these terms refer to. At best this may cause some terminological confusion in the discourse, at worst it may compromise the case itself. In other words, although terminological clarification is often tedious, dry swimming, I think some effort ought to be invested in clarifying the notions.

From my standpoint, and for a number of reasons, I prefer the term "mathematical literacy," roughly as it is defined in the Organization for Economic Cooperation and Development (OECD) Programme for International Student Assessment (PISA) project, in which I happen to be involved. In this enterprise, mathematical literacy is:

> The capacity to identify, to understand, and to engage in mathematics and to make well-founded judgments about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen. (OECD 2000)

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The main reason I prefer mathematical literacy is that the broadness of the term "mathematical" captures better than the somewhat narrower term "quantitative" what we actually seem to be after, for instance, when providing examples. Of course, we could argue on the basis of the history and epistemology of mathematics that many aspects of those mathematical topics that are of particular importance to real life, such as geometry, functions, probability, and mathematical statistics, among others, were in fact "arithmetised" in the nineteenth and twentieth centuries, so that we are not restricting the animal greatly by referring to it as quantitative literacy rather than as mathematical literacy. To see that, however, a person has to possess a fairly solid knowledge of modern mathematics and its genesis, and that is most certainly a prerequisite that we cannot and should not expect of all those with whom we want to be in dialogue.

Now, how is mathematical literacy related to mathematical knowledge and skills? Evidently, that depends on what we mean by mathematics. If we define mathematics in a restrictive way, as a pure, theoretical scientific discipline—whether perceived as a unified, structurally defined discipline or as a compound consisting of a number of subdisciplines such as algebra, geometry, analysis, topology, probability, etc.—it is quite clear that mathematical literacy cannot be reduced to mathematical knowledge and skills. Such knowledge and skills are necessary prerequisites to mathematical literacy but they are not sufficient.

This is not the only way to define mathematics, however. We may adopt a broader—partly sociological, partly epistemological—perspective and perceive mathematics as a field possessing a five-fold nature: as a pure, fundamental science; as an applied science; as a system of tools for societal and technological practice ("cultural techniques"); as an educational subject; and as a field of aesthetics (Niss 1994). Here, being a pure, fundamental science is just one of five "natures" of mathematics. If this is how we see mathematics, the mastery of mathematics goes far beyond the ability to operate within the theoretical edifice of purely mathematical topics. And then, I submit, mathematical literacy is more or less the same as the mastery of mathematics. By no means, however, does this imply that mathematical literacy can or should be cultivated only in classrooms with the label "mathematics" on their doors. There are hosts of other important sources and platforms for the fostering of mathematical literacy, including other subjects in schools and universities.

All this leaves us with a choice between two different strategies. Either we accept a restrictive definition of mathematics as being a pure, fundamental science and then establish mathematical literacy as something else, either a cross-curricular ether or a new subject. Or we insist (as I do) on perceiving mathematics as a multi-natured field of endeavor and activity. If we agree to use such a perception to define the subject to be taught and learned, that subject would have the fostering of mathematical literacy, including its narrower quantitative sense, as a major responsibility from kindergarten through to the Ph.D.

Once again, this being said, the fostering of mathematical literacy also should be the responsibility of other subjects, whenever this is appropriate, which it is much more often than agents in other subjects bother to realize or accept. Mathematical literacy is far too important to be left to mathematics educators and mathematicians (in a wide sense), but it also is far too important to be left to the users of mathematics. Mathematics educators and mathematicians have to assume a fair part of the responsibility for providing our youths and citizens with mathematical literacy.

Mathematical Literacy and Democracy

Traditionally, we tend to see the role of mathematical literacy in the shaping and maintenance of democracy as being to equip citizens with the prerequisites needed to involve themselves in issues of immediate societal significance. Such issues could be political, economic, or environmental, or they could deal with infra-structure, transportation, population forecasts, choosing locations for schools or sports facilities, and so forth. They also could deal with matters closer to the individual, such as wages and salaries, rents and mortgages, child care, insurance and pension schemes, housing and building regulations, bank rates and charges, etc.

Although all this is indeed essential to life in a democratic society, I believe that we should not confine the notion of democracy, or the role of mathematical literacy in democracy, to matters such as the ones just outlined. For democracy to prosper and flourish, we need citizens who not only are able to seek and judge information, to take a stance, to make a decision, and to act in such contexts. Democracy also needs citizens who can come to grips with how mankind perceives and understands the carrying constructions of the world, i.e., nature, society, culture, and technology, and who have insight into the foundation and justification of those perceptions and that understanding. It is a problem for democracy if large groups of people are unable to distinguish between astronomy and astrology, between scientific medicine and crystal healing, between psychology and spiritism, between descriptive and normative statements, between facts and hypotheses, between exactness and approximation, or do not know the beginnings and the ends of rationality, and so forth and so on. The ability to navigate in such waters in a thoughtful, knowledgeable, and reflective way has sometimes been termed "liberating literacy" or "popular enlightenment." As mathematical literacy often is at the center of the ways in which mankind perceives and understands the world, mathematical literacy is also an essential component in liberating literacy and popular enlightenment. We should keep that in mind when shaping education for the pursuit of mathematical literacy in service of democracy.

The Danish KOM Project

If we decide to adopt a broad, multi-natured notion of mathematics, and set out to foster mathematical literacy within mathematics education, it becomes a crucial task to find and employ new ways to define and describe mathematics curricula that focus on mathematical competence rather than on facts and techniques. I give here a brief account of current attempts in that direction being made in the Danish so-called "KOM" project. The thinking behind and underpinning of that project also has exerted some influence on the OECD PISA project, as can be seen in Jan de Lange's background essay for the Forum (de Lange, see pp. 75–89).

Traditionally, in Denmark and in many other countries, a mathematics curriculum is specified by means of three types of components:

1. Statements of the *purposes and goals* that are to be pursued in teaching and learning.

2. Determination of mathematics *content*, given in the form of a *syllabus,* i.e., lists of the mathematical topics, concepts, theories, methods, and results to be covered.

3. Forms and instruments of *assessment and testing* to judge to what extent students have achieved the goals set for the syllabus as established under (2).

Serious objections can be raised against this way of specifying a curriculum. First, on such a basis, it is very difficult to describe and explain in overarching, nontautological terms what mathematics education at a given level is all about, without relying on circular descriptions such as "the teaching and learning of mathematics at this level consist in studying the topics listed in the syllabus," which is just another way of saying that the teaching and learning of mathematics are about teaching and learning (a particular segment) of mathematics.

Second, a syllabus-based curriculum specification easily leads to identifying mathematical competence with the mastery of a syllabus, i.e., knowing the facts and being able to perform the skills tied to the topics of the syllabus. Although such mastery is certainly important, this identification tends to trivialize mathematics, reduce the notion of mathematical competence, and lead to too low a level of ambition for teaching and learning. In Denmark we often refer to this reduction as the "syllabusitis trap."

Third, if we have only syllabus-based curriculum specifications at our disposal in mathematics education, we can only make inessential, trivial comparisons between different mathematics curricula, i.e., we can only identify the differences between curricula X and Y by listing the syllabus components in $X \cap Y$, $X \setminus Y$, and $Y \setminus X$, respectively; however, the differences between two kinds of mathematics teaching and learning are typically both much more fundamental and more subtle than the differences reflected in the syllabi.

This leaves us with the following challenges and a resulting task. We wish to create a general means to specify mathematics curricula that allows us to adequately:

- Identify and characterize, in a noncircular manner, what it means to *master* (i.e., know, understand, do, use) *mathematics,* in and of itself and in contexts, irrespective of what specific mathematical content (including a syllabus) is involved;

- Validly describe *development and progression* within and between mathematics curricula;

- Characterise *different levels of mastery* to allow for describing development and progression in the individual student's mathematical competence; and

- Validly *compare* different mathematics curricula and different kinds of mathematics education at different levels or in different places.

The general idea is to deal with this task by identifying and making use of a number of overarching mathematical competencies.

This gave the stimulus (and the most important part of the brief) for the Danish KOM project, directed by the author of this paper. KOM stands for "Kompetencer Og Matematiklæring," Danish for "Competencies and Mathematical Learning." (More information is available at http://imfufa.ruc.dk/kom. By the end of August 2002 an English version of the full report of the project can be found at this site.) The project was established jointly by the Ministry of Education and the National Council for Science Education. It is not a research project but a development project to pave the way for fundamental curriculum reform in Denmark, from kindergarten to university. In fact it is a spearhead project in that similar projects are now being undertaken in Danish, physics and chemistry, and foreign languages; the natural sciences are soon to be addressed.

More specifically, the project is intended to provide inspiration by discussing and analyzing the possibility of dealing with the task just presented by means of the notion of mathematical competencies, and accordingly to propose measures and guidelines for curriculum reform. It is not the intention that the project itself shall propose detailed new curricula at all the different educational levels it addresses. Specific curriculum implementation is up to the curriculum authorities responsible for each of these levels; however, it is more than likely that the collaborators in the project will be asked to take part in that implementation in the sectors in which they work.

Mathematical Competencies and Insights

Let us begin by suggesting working definitions for two of the key words, competence and competency. It goes without saying that it is human beings that may possess competence and competencies.

To possess competence (to be competent) in some domain of personal, professional, or social life is to master (to a fair degree, appropriate to the conditions and circumstances) essential aspects of life in that domain. In some languages (such as Danish), there are two facets of the notion of competence. The first is formal competence, which is roughly the same as authorization or license, i.e., the right to do something. The second is real competence, roughly equivalent to expertise, i.e. the actual ability to do something. Here, the focus is on the latter facet. This leads us to define "mathematical competence" as the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts. Necessary, but certainly not sufficient, prerequisites for mathematical competence are extensive factual knowledge and technical skills.

A "mathematical competency" is a clearly recognizable and distinct, major constituent in mathematical competence. Competencies need be neither independent nor disjointed. Thus, the question we have to address is, What are the competencies in mathematical competence? To answer this question, let us begin by noting that mathematical competence includes two overarching sorts of capabilities. The first is to ask and answer questions about, within, and by means of mathematics. The second consists of understanding and using mathematical language and tools. A closer analysis has given rise to the following eight competencies:

1. *Thinking mathematically* (mastering mathematical modes of thought), such as:

    - Posing questions that are characteristic of mathematics and knowing the kinds of answers (not necessarily the answers themselves) that mathematics may offer;

    - Extending the scope of a concept by abstracting some of its properties and generalizing results to larger classes of objects;

    - Distinguishing between different kinds of mathematical statements (including conditioned assertions (if-then), quantifier-laden statements, assumptions, definitions, theorems, conjectures and special cases); and

    - Understanding and handling the scope and limitations of a given concept.

2. *Posing and solving mathematical problems*, such as:

    - Identifying, posing, and specifying different kinds of mathematical problems (pure or applied, open-ended or closed); and

    - Solving different kinds of mathematical problems (pure or applied, open-ended or closed), whether posed by others or by oneself, and, if appropriate, in different ways.

3. *Modelling mathematically* (i.e., analyzing and building models), such as:

    - Analysing the foundations and properties of existing models, including assessing their range and validity;

    - Decoding existing models, i.e., translating and interpreting model elements in terms of the reality modelled; and

    - Performing active modelling in a given context, i.e., structuring the field, mathematizing, working with(in) the model (including solving the problems the model gives rise to); validating the model, internally and externally; analyzing and criticizing the model (in itself and vis-à-vis possible alternatives); communicating about the model and its results; monitoring and controlling the entire modelling process.

4. *Reasoning mathematically* such as:

    - Following and assessing chains of arguments put forward by others;

    - Knowing what a mathematical proof is (is not) and how it differs from other kinds of mathematical reasoning, e.g., heuristics;

    - Uncovering the basic ideas in a given line of argument (especially a proof), including distinguishing main lines from details, and ideas from technicalities; and

    - Devising formal and informal mathematical arguments and transforming heuristic arguments to valid proofs, i.e., proving statements.

5. *Representing mathematical entities,* such as:

   - Understanding and utilizing (decoding, interpreting, and distinguishing between) different sorts of representations of mathematical objects, phenomena, and situations;
   - Understanding and utilizing the relations between different representations of the same entity, including knowing about their relative strengths and limitations; and
   - Choosing and switching between representations.

6. *Handling mathematical symbols and formalisms*, such as:

   - Decoding and interpreting symbolic and formal mathematical language and understanding its relations to natural language;
   - Understanding the nature and rules of formal mathematical systems (both syntax and semantics);
   - Translating from natural language to formal/symbolic language; and
   - Handling and manipulating statements and expressions containing symbols and formulas.

7. *Communicating in, with, and about mathematics*, such as:

   - Understanding others' written, visual, or oral "texts" (in a variety of linguistic registers) about matters having a mathematical content; and
   - Expressing oneself, at different levels of theoretical and technical precision, in oral, visual, or written form, about such matters.

8. *Making use of aids and tools* (including information technology), such as:

   - Knowing the existence and properties of various tools and aids for mathematical activity and their scope and limitations; and
   - Being able to reflectively use such aids and tools.

The first four competencies are the ones involved in asking and answering questions about, within, and by means of mathematics, whereas the last four are the ones that pertain to understanding and using mathematical language and tools. It should be kept in mind, however, that these eight competencies are meant neither to establish a partitioning of mathematical competence into disjointed segments nor to constitute independent dimensions of it. The competencies just listed are very close but not completely identical to the ones that appear in the OECD PISA framework for mathematical literacy (OECD 2000). As mentioned above, this is no accident.

These eight competencies all have to do with mental or physical processes, activities, and behavior. In other words, the focus is on what individuals can do. This makes the competencies behavioral (not to be mistaken for behavioristic).

In addition to competencies, we also have identified three important insights concerning mathematics as a discipline. These are insights into:

- The actual application of mathematics in other subjects and fields of practice that are of scientific or social significance;
- The historical development of mathematics, internally as well as externally; and
- The special nature of mathematics as a discipline.

Needless to say, these insights are closely related to the possession of the eight mathematical competencies, but they cannot be derived from them. The competencies deal with different kinds of singular mathematical activities whereas the insights deal with mathematics as a whole.

Both the competencies and the insights are comprehensive, overarching, independent of specific content, and independent of educational level. In other words, they are general to mathematics. But they are also specific to mathematics, i.e., even if other subjects come up with similar sets of competencies using similar words, those words will be interpreted completely differently from how they are interpreted in mathematics. Even though the competencies and the insights are general, they manifest themselves and play out differently at different educational levels, in different contexts, and with different kinds of mathematics subject matter.

The competencies and insights can be employed both for normative purposes, with respect to specification of a curriculum or of desired outcomes of student learning, and for descriptive purposes to describe and characterize actual teaching practice or actual student learning, or to compare curricula, and so forth.

In this paper there is room only to describe the core ideas of the KOM project. It is also a key intention of the project to specify in some detail how these competencies will actually be developed at different educational levels in schools and universities, to specify and characterize the relationships between competencies and mathematics subject matter at different levels, and to devise ways to validly and reliably assess students' possession of the mathematical competencies in a manner that allows us to describe and characterize development and progression in those competencies.

In conclusion, if we are able to meet the challenges stated above, and to complete the tasks they lead to, we will not only have done good service for mathematics education and mathematical literacy but we also may hope to be in a better position than today to engage in dialogues with quarters outside of mathematics and mathematics education about mathematical literacy and its importance for democracy.

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