Defining Mathematical Literacy in France

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Two major reforms of mathematics curricula were undertaken in France during the twentieth century. The first, in 1902, viewed mathematics as an experimental science next to physics (two hours a week were dedicated to *dessin graphique*). Until 1960, no further reform occurred, for various reasons. The slogan of the 1960 reform was "mathematics for all." In that sense, democracy was a main concern for the reformers, because mathematics was supposed to carry universal values. But its results were not those expected.

In April 1999, at a time when there were a lot of controversies about mathematics, the French Minister of Education created a commission to undertake a study of the mathematics curricula from elementary school to teacher training in university. This *Commission de réflexion sur l'enseignement des mathématiques* was chaired by Jean-Pierre Kahane, member of the French Académie des sciences and former President of the International Commission on Mathematical Instruction (ICMI). I was one of the members of the commission.

This commission was neither the first nor will it be the last committee in France to think about mathematics and its teaching, but I believe it was the first time such a committee was asked, among other requests, to focus on the impact of computers on the mathematics syllabus; however, computers were not the only topic that the commission had in hand. We addressed four issues, namely: geometry, computation (or numeracy), statistics, and computers. After considerable consultation, we produced a report (Kahane 2002), available on the Internet, that is directed to teachers and to decision makers involved in all aspects of education.

Geometry

Teaching geometry from elementary to high school levels is still necessary today. We first showed the importance of geometry in order to "grasp space," to develop the vision of space, a vision that plays an essential role in our image-oriented society. Second, the report emphasized the fact that geometry is a fundamental subject for the learning of reasoning. Finally, the report recalled how important geometry is in the training of all scientists (technicians, engineers, researchers, and teachers).

We proposed to develop space geometry as an education in vision. For plane geometry, we suggested reinforcing the use of elementary invariants (such as angle and area) and reintroducing criteria for congruence and similarity of triangles. To avoid too dogmatic a teaching approach, the report also proposed to favor open problems (research on geometric loci and construction problems) and to make room for some "rich" geometries. For instance, at the end of high school, circular geometry could be taught together with complex numbers.

As regards methods, the commission focused on two main goals: teaching pupils and students to see and to think geometrically and teaching them to reason. Among the other important suggestions included in the report was to establish strong links with other disciplines, in particular with the sciences.

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Computation

The commission tried to intertwine epistemological and didactical issues by questioning the nature and role of computation in the mathematical sciences and their evolution, the cultural and educational representations of the theme, and the current curriculum from elementary to university levels. In the epistemological dimension of the report, we were especially sensitive to:

- The increasing diversity of the objects and practices that computations involve;
- The dependence of computation on computational tools;
- The relationships between exact and approximate computation, and between computation and reasoning; and
- The fundamental role that computations play in the building of mathematical concepts and theories.

We then pointed out that these epistemological characteristics of computation are deeply misunderstood in the cultural and educational representations of the theme, at least as regards three main points: the relationships between computation and reasoning, between approximate and exact computation, and between computation and computational tools.

Taking this discrepancy into account, we then developed a didactic analysis structured in three main parts: meeting with the world of computation: numbers, magnitudes, measures, and dimensions; from arithmetic computations to algebraic computations; and from algebraic computations to calculus and analysis. In each of these parts, we focused on reconstructions and breaches involved in mathematical learning processes, because these are often underestimated in teaching practices. We emphasized mental arithmetic, estimation of magnitudes, and better links between computation and reasoning and between approximate and exact computations. Finding an appropriate place for current technology is seen as a crucial transversal aim of teaching. We addressed how the curriculum can contribute to achieve these aims, from the beginning of elementary school to university. Outcomes from innovative work and educational research were used to support the analysis.

Statistics

Developing stochastic literacy for professional or individual purposes is widely understood to be part of obligatory education; nonetheless, it is surrounded by controversies and many questions set out in the commission's report are under investigation in many European countries.

We first give a brief account of the close connections between statistics (designed for analyzing, visualizing, and modeling data in contexts), probability theory, and other aspects of mathematics (such as geometry and multivariate analysis). Many statistical rules rely on common sense and specific skills (such as designing experiments). Although statistical understanding does not systematically involve probability theory or mathematical reasoning, probability and other mathematical background is required to move beyond purely descriptive statistics. We then dealt with the impact on statistical practice of the widespread use of computers and the availability of powerful statistical packages. For teaching purposes, computer random simulation yields a rapid understanding of many basic points (such as properties of sampling distributions).

Statistics are involved in other parts of the high school curricula, such as biology, social sciences, economics, and physics. All teachers in those subjects (and also in mathematics) need training because stochastic reasoning is new to them. Such training is a main theme in developing statistical literacy in any country. Teachers of different subjects start learning together in order to speak a common language and use common notations. Because statistics and probability concepts cannot be restricted to dice games or oversimplified questions, interdisciplinary working sessions have to be organized.

Teaching statistics and probability theory will also provide students with a true experience in modeling, which now is used in many professions.

Computers

In 1992, UNESCO published a report on *The Influence of Computers and Informatics on Mathematics and Its Teaching* (Cornu and Ralston 1992). (Some of the contributors to this report were participants in the Forum.) Reflecting ideas from this report, our commission examined three issues created by computers:

- Changes in life, in society, and in sciences and mathematics;
- Choices to be made for the mathematics syllabus; and
- Consequences for teacher training and for schools.

With respect to the mathematics syllabus, we wanted to focus on elementary and fundamental aspects: data structures, programs and algorithms, loops and iterations, conditional structures, and cost and complexity. These aim to define another kind of literacy. For example, data structures are crucial to understand fully algorithms and their complexity and to organize statistical data. We can expect that these topics will give meaning and reality to certain fundamental abstract notions such as variable, number, iteration, evaluation, and approximation. As a consequence, the commission report revisited some notions of classical mathematics.

We proposed that, in every lycée, there should be created a "math lab" or, better said, a "mathematical sciences lab" where pupils and teachers could meet, discuss, experiment, practice, and receive visitors.

Teacher-training issues are addressed throughout the report. We were highly concerned by the following questions: What can our teachers do? and What will they be able to do? We were convinced that the content of their training and the content of mathematics curricula for schools and high schools must evolve at the same time.

References

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