

What Have We Learned, ... and Have Yet to Learn?

HYMAN BASS

This remarkable Forum more than achieved its aim of stimulating a continuing national conversation on quantitative literacy. (Without expressing a preference, I shall for convenience use this term, abbreviated as QL, for what was variously referred to during this Forum as quantitative literacy, mathematical literacy, and numeracy.) I shall try to summarize some of the diverse messages that I heard here, as best I could understand them, and add some personal reflections.

Defining QL

Although we have no precise definition of QL, the case statement in *Mathematics and Democracy: The Case for Quantitative Literacy* and the background essays contributed to this Forum give us a rich, and not always consistent, set of characterizations and expressions of it. A common characterization seems to be this: QL is about knowledge and skills in *use*, so it is a kind of *applied knowledge* that is typically illustrated in particular *contexts*. But these contexts are extremely diverse, and many of them, if treated in more than a caricature fashion, are quite complex. This presents a challenge to the design of curricula for QL. What is its focus? What is its disciplinary locus?

Voices at this Forum offered a very broad perspective. In our collective minds, QL appears to be some sort of constellation of knowledge, skills, habits of mind, and dispositions that provide the resources and capacity to deal with the quantitative aspects of understanding, making sense of, participating in, and solving problems in the worlds that we inhabit, for example, the workplace, the demands of responsible citizenship in a democracy, personal concerns, and cultural enrichment.

Urgency for QL arises primarily from the effects of technology, which exposes us to vastly more quantitative information and data. Therefore, the tools of data analysis, statistics, and probabilistic reasoning (in risk assessment, for example) are becoming increasingly important. Yet there is broad agreement, with some evidence cited, that most adult Americans are substantially deficient in QL, however it may be defined. This is viewed as a serious societal problem in several respects—economic (capacity of the workforce), political (functioning of a modern industrial democracy), cultural (appreciation of the heritage and beauty of mathematics), and personal (capacity for a responsible and productive life).

I agree with the views expressed that it is neither urgent, nor even necessarily productive, to attempt to achieve a precise consensus definition of QL. At the same time, this is not an entirely benign consideration. To illustrate, one speaker proposed that university mathematicians send a collective letter to the College Board requesting more QL on the SAT and other examinations. Such a recommendation, if implemented, is not immediately actionable by the College Board without an operational interpretation of what QL should mean in that context, and that interpretation is open

Hyman Bass is the Roger Lyndon Collegiate Professor of Mathematics and Professor of Mathematics Education at the University of Michigan. His mathematical research interests cover broad areas of algebra. A member of the National Academy of Sciences, Bass is president of the American Mathematical Society and of the International Commission on Mathematics Instruction (ICMI). A former chair of the Mathematical Sciences Education Board, Bass is currently helping investigate the mathematical requirements involved in teaching mathematics at the elementary level.

to considerable license. What the College Board could end up offering in response, if it chose to respond, might not please all signers of such a letter.

Absent a definition, there is little basis for reconciling views. On the other hand, it might be an important and fruitful step not only for the College Board but also for the higher education mathematics community to conduct a negotiation of what a credible list of illustrative test items that could claim to represent a proper and balanced sampling of QL knowledge might look like, to which the professional community could subscribe. Putting such a list in a letter to the College Board would be a very different, knowledge-based gesture.

Similar cautions apply to any attempt to translate our sentiments about QL into far-reaching policy positions. Our knowledge base about QL is not sufficient to rush toward major transformations of the school curriculum, not to mention the necessary capacity-building among teachers to support such change.

Educating for QL

Remedies to the problem of QL are generally assumed to be primarily the responsibility of the education system, principally in grades 10 to 14. In fact, QL must be taught starting in the earliest grades if we are to make any headway on this problem. Nonetheless, most of the discussion at this Forum centered on ideas about QL in later grades. I note three recurrent themes:

1. The curriculum should include much more statistics and other alternatives to the calculus trajectory that are focused more on data analysis, modeling, etc.

This recommendation often has been accompanied by disparagement of the teaching of traditional mathematics topics. This reminds me of some of the debates about the teaching of computational algorithms. At root the objections were not to the skills and concepts being taught but rather to the pedagogy, to the oppressive or obscure ways in which these topics have often been taught, which the debaters could not see as distinct from the subject matter. Although a full exposure to calculus may not be appropriate for a majority of students, algebra and geometry remain fundamental to all developed uses of mathematics.

2. Mathematics instruction should be contextualized and avoid the abstraction associated with the traditional curriculum.

This common refrain of current reforms is more complex than most of its advocates appreciate. One argument, which goes back to John Dewey and others, is that learning best starts with experience, to provide both meaning and motivation for the more

general and structured ideas that will follow. Dewey's notion differs in two respects from the above recommendation. First, it does not eschew abstraction. Second, it speaks of the experience of the learner, not of the eventual context of the application of the ideas, which may be highly specialized and occur much later in adult experience.

Another argument is that mathematics is best learned in the complex contexts in which it is most significantly used. This idea has a certain appeal, provided that it is kept in balance. Authentic contexts are complex and idiosyncratic. Which contexts should we choose for a curriculum? Their very complexity often buries the mathematical ideas in other features so that, although the mathematical effects might be appreciated, there is limited opportunity to learn the underlying mathematical principles.

The main danger here, therefore, is the impulse to convert a major part of the curriculum to this form of instruction. The resulting failure to learn general (abstract) principles then may, if neglected, deprive the learner of the foundation necessary for recognizing how the same mathematics witnessed in one context in fact applies to many others.

Finally, contextualization is seen as providing early experience with the very important process of mathematical modeling. This is a laudable goal but it is often treated naïvely, in ways that violate its own purpose. Serious modeling must treat both the context and the mathematics with respect and integrity. Yet much contextualized curricular mathematics presents artificial caricatures of contexts that beg credibility. Either many of their particular features, their ambiguities, and the need for interpretation are ignored in setting up the intended mathematics, which defeats the point of the context, or else many of these features are attended to and they obscure the mathematical objectives of the lesson. Good contextualizing of mathematics is a high skill well beyond that of many of its current practitioners.

3. Quantitative knowledge and skills for QL should have a much more cross-disciplinary agenda, rather than one situated primarily in mathematics curricula.

I am generally sympathetic to this recommendation. Because mathematics is a foundational and enabling discipline for so many others, it is natural that mathematics learning in general, not just for QL, should evolve from an ongoing conversation and sometimes collaboration with client disciplines. At the same time, the historical reasons for situating the learning of QL skills in mathematics study have not lost their relevance. And I am speaking of more than the learning of basic arithmetic and measurement.

Take, for example, the learning of deductive reasoning, which most of us would count as an important component of QL. Al-

though applicable in all contexts in which mathematical ideas and methods are used, this is a practice that can most naturally be cultivated in core mathematical domains, beginning in the earliest grades. For example, it is both reasonable and educationally productive to have third graders *explain* why some algorithm for subtraction, or the multiplication of whole numbers, actually works. Or, for example, they could be asked to *prove* that they have all the possible solutions to a problem with finitely many solutions. The basic mathematics curriculum, including the primary grades, naturally affords a context for the development of the skills of disciplined mathematical reasoning, although this seems rarely to be done today. Other subject areas do not provide similar opportunities to learn this kind of deductive reasoning.

Lynn Arthur Steen recounted to me some conversations with Harvard mathematician Andrew Gleason in the early years of the Mathematical Sciences Education Board, in which Gleason argued energetically that mathematics is the only subject in which primary-grade children can gain an internal sense of truth independent of adult authority. By the power of their own minds they can, in principle, know for certain that some things are right (or wrong) even if they are different from what their teacher may say. They really cannot do this in any other area. Of course, as Steen notes, probably relatively few children have the psychological strength to adhere to their own logic in the face of contrary adult authority.

Some have argued that rigorous mathematical study develops analytical skills and qualities of mind that are of intellectual and cultural value well beyond mathematics. Although this is a fond belief of mathematicians, such broad transfer has not been established, and the public discourse of many mathematicians in non-mathematical domains, involving different evidentiary norms and warrants, calls it seriously into doubt.

Moving Forward

Where do we go from here? Has this Forum accomplished its goals? Many speakers have argued that, given the alarmingly low rates of quantitative literacy among American adults and the already lengthy discussions of this problem, we should move quickly to programs of dramatic action to improve the situation, with a strongly articulated vision of what we want to accomplish.

Although I do not want to rain on your parade, I suggest that our knowledge base about quantitative literacy is not yet adequate for designing major interventions in the school curriculum. The comprehensive agenda of providing QL to all students is one measured in decades, not years, but it is work that can productively begin in incremental ways right now.

This Forum has taken an important step. The case statement in *Mathematics and Democracy* and the collection of very interesting and provocative background essays prepared for this Forum provide a rich articulation of questions and concerns regarding QL, many analyses of the problems we face, and many stimulating but somewhat divergent suggestions for what to do about them. Together, these provide a rich resource for an ongoing, disciplined, and coordinated national (or even international) conversation about these issues.

References

- Steen, Lynn Arthur, ed. 2001. *Mathematics and Democracy: The Case for Quantitative Literacy*. Princeton, NJ: National Council on Education and the Disciplines.