# The Role of Mathematics Courses in the Development of Quantitative Literacy

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Conventional wisdom holds that quantitative literacy is developed by taking mathematics courses. Although this is often true, mathematics courses are no panacea. The views of mathematics acquired by many students during their education often hamper the development of quantitative literacy and therefore have profound implications for the strategies that we should adopt to promote it. This essay analyzes the ways in which the curriculum and pedagogy of mathematics courses could better enhance quantitative literacy.

## Quantitative Literacy: A Habit of Mind

Quantitative literacy is the ability to identify, understand, and use quantitative arguments in everyday contexts. An essential component is the ability to adapt a quantitative argument from a familiar context to an unfamiliar context. Just as verbal literacy describes fluency with new passages, so quantitative literacy describes fluency in applying quantitative arguments to new contexts.

Quantitative literacy describes a habit of mind rather than a set of topics or a list of skills. It depends on the capacity to identify mathematical structure in context; it requires a mind searching for patterns rather than following instructions. A quantitatively literate person needs to know some mathematics, but literacy is not defined by the mathematics known. For example, a person who knows calculus is not necessarily any more literate than one who knows only arithmetic. The person who knows calculus formally but cannot see the quantitative aspects of the surrounding world is probably not quantitatively literate, whereas the person who knows only arithmetic but sees quantitative arguments everywhere may be.

Adopting this definition, those who know mathematics purely as algorithms to be memorized are clearly not quantitatively literate. Quantitative literacy insists on understanding. This understanding must be flexible enough to enable its owner to apply quantitative ideas in new contexts as well as in familiar contexts. Quantitative literacy is not about how much mathematics a person knows but about how well it can be used.

## Mathematical Underpinnings of Quantitative Literacy

An alarming number of U.S. students do not become quantitatively literate on their journey through school and college. Indeed, the general level of quantitative literacy is currently sufficiently limited that it threatens the ability of citizens to make wise decisions at work and in public and private life. To rectify this, changes are needed in many areas: educational policy, pedagogy, and curriculum. Unfortunately, one of the more plausible vehicles for improvement—mathematics courses—will require significant alteration before they are helpful.

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To be able to recognize mathematical structure in context, it is of course necessary to know some mathematics. Although the knowledge of basic mathematical algorithms such as how to multiply decimals does not guarantee literacy, the absence of this knowledge makes literacy unlikely, if not impossible. It would be helpful to agree on which mathematical algorithms are necessary underpinning of quantitative literacy, but this is a matter on which reasonable people differ. The answer also may differ from country to country and from era to era; however, the fundamental description of quantitative literacy as a habit of mind is not affected by the mathematical underpinnings chosen.

In this essay, we take the mathematical underpinnings of quantitative literacy to be the topics in a strong U.S. middle school curriculum, in addition to some topics traditionally taught later, such as probability and statistics. For us, quantitative literacy includes the use of basic spreadsheets and formulas (but not, for example, the spreadsheet's built-in statistical functions). Quantitative literacy, therefore, includes some aspects of algebra, but not all. The ability to create and interpret formulas is required; their symbolic manipulation is not. Reading simple graphs is necessary; the ability to construct them is not.

Our interpretation of quantitative literacy does not involve most of traditional algebra and geometry (for example, factoring polynomials, simplifying algebraic fractions, knowing the geometric properties of circles, chords, and tangents). The reason for this choice is that most adults do not use such algebra and geometry at work, or in their private lives, or as voting citizens. This is not to deny that some traditional algebra and geometry should have a central role in the high school curriculum. The development of manipulative skill, in particular, is important in any field that makes frequent use of symbols; however, the understanding of formulas required for our definition of quantitative literacy is hardly touched on in an algebra class that focuses on the rules for symbolic manipulation.

Let us see what this definition of quantitative literacy means in practice. It includes being able to make a mental estimate of the tip in a restaurant; it includes realizing that if Dunkin' Donuts is selling donuts for 69¢ each and $3.29 a half-dozen, and if you want five, you might as well buy six. It includes reading graphs of the unemployment rate against time; it includes knowing what is meant by a report that housing starts are down by 0.2 percent over the same month last year. It includes an understanding of the implications of repeated addition (linear growth to a mathematician) and repeated percentage growth (exponential growth). Quantitative literacy does not expect, however, and in fact may not benefit from, the algebraic manipulation usually associated with these topics in a mathematics course.

To probe the boundary of quantitative literacy suggested by this definition, observe that it would not include understanding all the graphs in *The Economist* or *Scientific American* because both sometimes use logarithmic scales; however, understanding all the graphs in *USA Today* is included by this definition.

## Mathematical Literacy and Quantitative Literacy

It may surprise some readers that advanced training in mathematics does not necessarily ensure high levels of quantitative literacy. The reason is that mathematics courses focus on teaching mathematical concepts and algorithms, but often without attention to context. The word "literacy" implies the ability to use quantitative arguments in everyday contexts that are more varied and more complicated than most mathematics textbook examples. Thus, although mathematics courses teach the mathematical tools that underpin quantitative literacy, they do not necessarily develop the skill and flexibility with context required for quantitative literacy.

There is, therefore, an important distinction between mathematical and quantitative literacy. A mathematically literate person grasps a large number of mathematical concepts and can use them in mathematical contexts, but may or may not be able to apply them in a wide range of everyday contexts. A quantitatively literate person may know many fewer mathematical concepts, but can apply them widely.

## Quantitative Literacy: Who Is Responsible?

High school and college faculty may be tempted to think that because the underpinnings of quantitative literacy are middle school mathematics, they are not responsible for teaching it. Nothing could be further from the truth. Although the mathematical foundation of quantitative literacy is laid in middle school, literacy can be developed only by a continued, coordinated effort throughout high school and college.

The skill needed to apply mathematical ideas in a wide variety of contexts is not always acquired at the same time as the mathematics. Instructors in middle school, high school, and college need to join forces to deepen students' understanding of basic mathematics and to provide opportunities for students to become comfortable analyzing quantitative arguments in context.

Also key to improving quantitative literacy is the participation of many disciplines. Quantitative reasoning must be seen as playing a useful role in a wide variety of fields. The development of quantitative literacy is the responsibility of individuals throughout the education system.

## Impediments to Quantitative Literacy: Pedagogy and Testing

We start by considering the common practices in mathematical pedagogy and testing that hinder the development of quantitative literacy. Here we are concerned with the ways in which topics are taught and assessed rather than with the topics themselves. What students learn about a topic is influenced more by the activities they do than by what the instructor says. In particular, tests often determine what is learned. Teachers "teach to the test" and students "study for the test." Thus, the types of problems assigned in courses have a large effect on what is learned.

The cornerstone of quantitative literacy is the ability to apply quantitative ideas in new or unfamiliar contexts. This is very different from most students' experience of mathematics courses, in which the vast majority of problems are of types that they have seen before. Mastering a mathematics course is, for them, a matter of keeping straight how to solve each type of problem that the teacher has demonstrated. A few students, faced with the dizzying task of memorizing all these types, make sense out of the general principles instead. But a surprisingly large number of students find it easier to memorize problem types than to think in general principles. Ursula Wagener described how teaching may encourage such memorization:

> A graduate student teacher in a freshman calculus class stands at the lectern and talks with enthusiasm about how to solve a problem: "Step one is to translate the problem into mathematical terms; step two is . . ." Then she gives examples. Across the room, undergraduates memorize a set of steps. Plugging and chugging—teaching students how to put numbers in an equation and solve it—elbows out theory and understanding.[1]

In my own classroom, I have had calculus students[2] who could not imagine how to create a formula from a graph because they did not "do their graphs in that order." For these otherwise strong students from the pre-graphing-calculator era, graphs were produced by a somewhat painful memorized algorithm that started with a formula and ended with a picture. Imagining the algorithm run backward to produce a possible formula struck them as impossible. These students could not identify which features of the graph corresponded to which features of the formula. Although they had a solid mathematical background for their age, these students were not quantitatively literate.

Calculus provides other examples of how easy it is to learn procedures without being able to recognize their meaning in context. Formulas, although a small part of quantitative literacy, are central to calculus. We expect literacy in calculus to include fluency with formulas for basic concepts. Problems such as "If $f(t)$ represents the population of the United States in millions at time $t$ in years, what is the meaning of the statements $f(2000) = 281$ and $f'(2000) = 2.5$?" look as though calculus students should find them easy—there are no computations to be done, only symbols to read.[3] Yet such problems cause great difficulty to some students who are adept at calculations.

As another example, in 1996, a problem on the Advanced Placement (AP) Calculus Exam[4] gave students the rate of consumption of cola over some time interval and asked them to calculate and interpret the definite integral of the rate. All the students had learned the fundamental theorem of calculus, but many who could compute the integral did not know that it represented the total quantity of cola consumed.

These examples suggest how teaching practices in mathematics may differ from those required to develop quantitative literacy. Mathematics courses that concentrate on teaching algorithms, but not on varied applications in context, are unlikely to develop quantitative literacy. To improve quantitative literacy, we have to wrestle with the difficult task of getting students to analyze novel situations. This is seldom done in high school or in large introductory college mathematics courses. It is much, much harder than teaching a new algorithm. It is the difference between teaching a procedure and teaching insight.

Because learning to apply mathematics in unfamiliar situations is hard, both students and teachers are prone to take shortcuts. Students clamor to be shown "the method," and teachers often comply, sometimes because it is easier and sometimes out of a desire to be helpful. Learning the method may be effective in the short run—it may bring higher results on the next examination—but it is disastrous in the long run. Most students do not develop skills that are not required of them on examinations.[5] Thus if a course simply requires memorization, that is what the students do. Unfortunately, such students are not quantitatively literate.

Another obstacle to the development of quantitative literacy is the fact that U.S. mathematics texts often have worked examples of each type of problem. Most U.S. students expect to be shown how to do every type of problem that could be on an examination. They would agree with the Harvard undergraduate who praised a calculus instructor for teaching in a "cookbook fashion."[6] Both college and school teachers are rewarded for teaching practices that purposefully avoid the use of new contexts.

College mathematics faculty frequently fail to realize how carefully a course must be structured if students are to deepen their understanding. Many, many students make their way through introductory college courses without progressing beyond the memorization of problem types. Faculty and teaching assistants are not trying to encourage this, but are often blissfully unaware of the extent to which it is happening. This, of course, reinforces the students' sense that this is the way things are supposed to be, thereby making it harder for the next faculty member to challenge that belief.

K–12 teachers are more likely than college faculty to be aware of the way in which their students think. They are, however, under more pressure from students, parents, and administrators to ensure high scores on the next examination by illustrating one of every problem type. So K-12 teachers also often reinforce students' tendency to memorize.

There are strong pressures on college and K–12 mathematics instructors to use teaching practices that are diametrically opposed to those that promote quantitative literacy, and indeed much effective learning. Efforts to improve quantitative literacy must take these pressures into account.

## Teaching Mathematics in Context

One of the reasons that the level of quantitative literacy is low in the U.S. is that it is difficult to teach students to identify mathematics in context, and most mathematics teachers have no experience with this. It is much easier to teach an algorithm than the insight needed to identify quantitative structure. Most U.S. students have trouble applying the mathematics they know in "word problems" and this difficulty is greatly magnified if the context is novel. Teaching in context thus poses a tremendous challenge.

The work of Erik De Corte, in Belgium, throws some light on what helps students to think in context. De Corte investigated the circumstances under which students give unrealistic answers to mathematical questions. For example, consider the problem that asks for the number of buses needed to transport a given number of people; researchers find that a substantial number of students give a fractional answer, such 33 2/3 buses. De Corte reported that if the context was made sufficiently realistic, for example, asking the students to write a letter to the bus company to order buses, many more students gave reasonable (non-fractional) answers.[7]

De Corte's work suggests that many U.S. students think the word problems in mathematics courses are not realistic. (It is hard to disagree.) Mathematicians have a lot of work to do to convince students that they are teaching something useful. Having faculty outside mathematics include quantitative problems in their own courses is extremely important. These problems are much more likely to be considered realistic.

As another example, many calculus students are unaware that the derivative represents a rate of change, even if they know the definition.[8] Asked to find a rate, these students do not know they are being asked for a derivative; yet, this interpretation of the derivative is key to its use in a scientific context. The practical issue, then, is how to develop the intuitive understanding necessary to apply calculus in context.

Mathematicians have a natural tendency to try to help students who do not understand that the derivative is a rate by re-explaining the definition; however, the theoretical underpinning, although helping mathematicians understand a subject, often does not have the same illuminating effect for students. When students ask for "an explanation, not a proof,"[9] they are asking for an intuitive understanding of a topic. Mathematicians often become mathematicians because they find proofs illuminating. Other people, however, often develop intuitive understanding separately from proofs and formal arguments. My own experience teaching calculus suggests that the realization that a derivative is a rate comes not from the definition but by talking through the interpretation of the derivative in a wide range of concrete examples.

It is important to realize that any novel problem or context can be made "old" if students are taught a procedure to analyze it. Students' success then depends on memorizing the procedure rather than on developing their ability to apply the central mathematical idea. There is a difficult balance to be maintained between providing experience with new contexts and overwhelming students by too many new contexts. Familiar contexts should be included—they are essential for developing confidence—but if the course stops there, quantitative literacy will not be enhanced.

There is tremendous pressure on U.S. teachers to make unfamiliar contexts familiar and hence to make problems easy to do by applying memorized algorithms. Changing this will take a coordinated effort: both school and college teachers will need to be rewarded for breaking out of this mold.

## Impediments to Quantitative Literacy: Attitudes Toward Mathematics

In the course of their education, many students develop attitudes about mathematics that inhibit the development of quantitative literacy. Particularly pernicious is the belief that mathematics is memorized procedures and that mere mortals do not figure things out for themselves. Students who subscribe to this view look at an unfamiliar context and immediately give up, saying that they never were "good at math."

Because this belief concerns the nature of mathematics, rather than the most efficient way to learn the subject, it often is held with surprising tenacity. Just how sure students can be that mathematics is to be memorized was brought home to me by the student whom I asked to explain why $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$. He looked puzzled that I should ask such a question, and replied confidently "It's a rule." I tried again; he looked slightly exasperated and said emphatically "It's a law." For him, mathematics involved remembering what rules were true, not figuring out why they were true.

To further my understanding of students' attitudes toward mathematics, a few years ago I gave a questionnaire to all the students enrolled in Harvard's pre-calculus and first-semester calculus courses. Responses were collected from all of the several hundred students involved. Two of the questions were as follows:

> A well-written problem makes it clear what method should be used to solve it.
>
> If you can't do a homework problem, you should be able to find a worked example in the text to show you how.

On a scale of 1 to 5, on which 5 represented strong agreement, the pre-calculus students gave the first question 4.6 and the second question 4.7; the calculus students gave both 4.1. The numbers suggest that these students—who are among the country's brightest—still think of mathematics largely as procedures.

In teaching mathematics, we should of course give some problems that suggest the method to be used and some that should be similar to worked examples; however, if all or most of the problems we assign are of this type—as is true in many U.S. classrooms—it is not surprising that our students find it difficult to apply mathematics in novel contexts. It is equally understandable that they find it "unfair" that they should be asked to use quantitative ideas in other fields, in which the context is seldom one they have seen before. Although their indignation is understandable, it is also a clear signal that we have a problem.

Reports from students of Mercedes McGowen, who teaches at William Rainey Harper College, demonstrated similar beliefs about mathematics. For example, a pre-service elementary teacher wrote:[10]

> All throughout school, we have been taught that mathematics is simply just plugging numbers into a learned equation. The teacher would just show us the equation dealing with what we were studying and we would complete the equation given different numbers because we were shown how to do it.

Another elaborated:[11]

> When I began learning mathematics everything was so simple. As I got older there were many more rules taught to me. The more rules I learned, the easier it became to forget some of the older rules.

Unfortunately, the attitudes toward mathematics displayed in these responses are diametrically opposed to the attitudes required for quantitative literacy. In attempting to improve quantitative literacy, we ignore these attitudes at our peril.

## The Mathematics Curriculum and Quantitative Literacy

Although quantitative literacy does not require the use of many mathematical tools, two curriculum areas are sufficiently important that they should receive much more emphasis. These are estimation, and probability and statistics.

### Estimation

The ability to estimate is of great importance for many applications of mathematics. This is especially true of any application to the real world and, therefore, of quantitative literacy. Unfortunately, however, estimation is a skill that falls between the cracks. Mathematics often does not see estimation as its responsibility; teachers in other fields do not teach it because they think it is part of mathematics. Many students therefore find estimation difficult. The solution is for all of us to teach it.

Worse still, because of mathematics' emphasis on precision, students often think that estimation is dangerous, even improper. In their minds, an estimate is a wrong answer much like any other wrong answer. The skill and the willingness to estimate should be included explicitly throughout the curriculum.

Given the current concern about calculator dependence, some people claim that students would be better at estimation if they were not allowed to use calculators. It is certainly true that proficiency with a slide rule required estimation; however, even in pre-calculator days, many students could not estimate. Instead of grabbing a calculator to do their arithmetic, past students launched into a memorized algorithm. For example, some years ago I watched a student use long division to divide 0.6 by 1, then 0.06 by 1, and then 0.006 by 1, before he observed the pattern.[12] Even then, he did not recognize the general principle. He never thought to make an estimate or to see if the answer was reasonable. Because this was a graduate student, we might reasonably conclude that his education had failed to develop his quantitative literacy skills.

### Probability and Statistics

One area of quantitative reasoning that is strikingly absent from the education of many students is probability and statistics. This gap is remarkable because probabilistic and statistical ideas are so extensively used in public and private life. Like quantitative literacy, probabilistic thinking is embedded in an enormous variety of contexts. For example, probabilities are used to quantify risk ("there is a 30 percent chance of recovery from this medical procedure") and in many news reports ("DNA tests . . . showed that it [the body] was 1.9 million times more likely to be the driver than anyone else." [13]) Familiarity with statistics is essential for anyone who plans to interpret opinion polls, monitor the development of a political campaign, or understand the results of a drug test. Statistical arguments are used as evidence in court and to analyze charges of racial profiling.

Let us look at an example in which public understanding of probability is crucial. Over the next generation, the effect of AIDS will be felt worldwide. How can people in the United States understand the impact of the epidemic without understanding the data? This is not to invalidate the need to understand the human suffering that the epidemic will cause; however, understanding the data is essential to constructing, voting for, and implementing policies that will mitigate the suffering. How many people die of AIDS? (In 2000, 3 million people died worldwide, and 5.3 million were infected.[14]) How many AIDS orphans will there be? (Millions.)[15] What will be the effect on the teaching profession? (In some countries, there are more AIDS deaths than retirements, which has significant implications for teacher availability.) Do these statistics describe the United States? Not now, but it would be risky to assume that they never could.

Avoiding similar statistics in the United States depends on sound educational policies aimed at prevention. These policies must be based on a solid understanding of infection rates. What does this mean for AIDS testing? The Center for Disease Control (CDC) in Atlanta spends tax dollars to track the disease and provide rapid, accurate testing.[16] In 1996, the American Medical Association approved a recommendation mandating HIV testing for pregnant women.[17] Yet Health Education AIDS Liaison (HEAL) in Toronto provided a passionate and well-argued warning about the dangers of widespread testing.[18] Using the following two-way table, HEAL argued that, with the current infection rate of 0.05 percent, even for a test which is 99 percent accurate, "of every three women testing HIV-positive, two are certain to be false positives." (False positives are people who test positive for HIV although they are not infected.)

As things stand now, many college students could not follow these discussions. Many might wrongly conclude that the accuracy of the test is at fault.[19] The public's lack of clarity on this issue could skew efforts to rationalize policies on mandatory testing.[20]

| | Positive HIV | Negative HIV | Totals |
|---|---|---|---|
| Test Positive | 495 | 995 | 1,490 |
| Test Negative | 5 | 98,505 | 98,510 |
| Totals | 500 | 99,500 | 100,000 |

## Obstacles to Including Probability and Statistics in the Curriculum

The fact that universities teach probability and statistics in many departments (economics, business, medicine, psychology, sociology, and engineering, as well as mathematics and statistics) is evidence for their pervasive use. Yet many students pass through both school and college with no substantial exposure to these subjects. For example, applicants to medical school are more likely to be admitted without statistics than without calculus.[21] Even many mathematics and science majors are not required to take statistics.

Like estimation, the teaching of probability and statistics suffers from the fact that no one can agree on when or by whom these topics should be introduced. Each group thinks it is someone else's responsibility. Should it be in middle school? In high school? In college? It is all too reminiscent of the White Queen's proposal to Alice for "jam tomorrow and jam yesterday—but never jam today."[22]

In addition to timing, the other obstacle to the introduction of probability and statistics in the school curriculum is the question of what topics should be dropped to make room. Traditional middle school and high school curricula do not contain probability and statistics. Every topic in the traditional curriculum has its advocates, with the effect that the status quo often prevails. There are a number of notable counterexamples, such as some state's high-stakes tests[23] and the AP Statistics Examination. Many of the new school curricular materials do contain these topics, but they are usually the first to be skipped. Because probability and statistics are not required for college entrance, these topics are often considered a luxury that can be omitted if time is tight.

Many college faculty thus agree that probability and statistics are vitally important but cannot agree on what should be done about teaching them. This is a failure of leadership. The result is that most people in the United States see probabilistic and statistical arguments every day yet have no training in making sense of them.

## Relationship Between Mathematics and the Public

Mathematicians sometimes feel that the general public does not appreciate their field, and may blame this phenomenon for the low level of quantitative literacy in the United States. There is some truth to this view; the question is what to do about it. Because the media is often the interface between academics and the general public, increasing the general level of quantitative literacy will require a better relationship with the media.

Many adults' last memory of mathematics still stings many years later. Whether their last course was in school or college, some remember a teacher whom they perceived as not caring. Some blame themselves for not being able to understand. Many remember a course they perceived as having no relevance or a jungle of symbols with little meaning. Their teachers may not have realized how little their students understood, or they may have felt that it was the students' problem.[24] But now it is mathematics' problem. Whether these memories are accurate is immaterial; they make a poor base on which to build collaboration.

To alter these perceptions, mathematics courses are needed that are meaningful—to everyone—and that do not sting in memory. Notice that what does and does not sting varies greatly from culture to culture. In some societies, the threat of humiliation is used as a spur to study; however, most U.S. students do not study harder if they are humiliated. Indeed, many will drop mathematics rather than subject themselves to such treatment. To be effective, each faculty member must know which techniques inspire students to learn in his or her particular culture, and use those techniques.

As a guide as to whether we have succeeded in teaching courses that are meaningful and that do not sting, we should ask ourselves the following question: Are we comfortable having the future of mathematics and quantitative literacy determined by those to whom we gave a "C" in mathematics and who took no further courses in the subject? This is not far from the truth at the present time. If not, we may be widening the split between mathematics and the general public.

## Teaching Quantitative Literacy

Quantitative literacy requires students to have a gut feeling for mathematics. Because we desire widespread quantitative literacy, not just for those who find mathematics easy, mathematics teachers will need to diversify their teaching techniques. For example, I recently had two students who came to office hours together. One learned approximately the way I do, so writing a symbolic explanation usually worked. The second, who listened earnestly, was usually looking blank at the end of my discussion with the first student. I then had to start again and explain everything over in pictures if I wanted both students to understand. This happened numerous times throughout the semester, so it was not a function of the particular topic but of their different learning styles. To teach both of them, more than one approach was necessary.

For most students, an effective technique for improving their quantitative literacy is to be introduced to varied examples of the use of the same mathematical idea with the common theme highlighted. A student's ability to recognize quantitative structure is enhanced if faculty in many disciplines use the same techniques. The assistance ("a conspiracy," according to some students) of faculty outside mathematics is important because it sends the message that quantitative analysis is valued outside mathematics. If students see quantitative reasoning widely used, they are more likely to regard it as important.

The need to develop quantitative literacy will only be taken seriously when it is a prerequisite for college. Students and parents are rightfully skeptical that colleges think probability and statistics are important when they are not required for entrance. Making quantitative reasoning a factor in college admissions would give quantitative literacy a significant boost.

## Conclusion

Achieving a substantial improvement in quantitative literacy will require a broad-based coalition dedicated to this purpose. Higher education should lead, involving faculty in mathematics and a wide variety of fields as well as people from industry and government. Classroom teachers from across grade levels and across institutions—middle schools, high schools, and colleges—must play a significant role. The cooperation of educational administrators and policymakers is essential. To make cooperation on this scale a realistic possibility, public understanding of the need for quantitative literacy must be vastly improved. Because the media informs the public's views, success will require a new relationship with the media.

## Notes

1. Paper on "Pedagogy and the Disciplines," 1990. Written for the University of Pennsylvania.
2. For example, Harvard students who had already had some calculus in high school.
3. These statements tell us that in the year 2000, the U.S. population was 281 million and growing at a rate of 2.5 million per year.
4. AP Calculus Examination, Questions AB3 and BC3, 1996.
5. A few students will independently develop the skill to apply mathematics beyond what is asked of them in courses and on examinations.

These students are rare, however; they are the students who can learn without a teacher. If we want to increase the number of people who are quantitatively literate, we should not base our decisions about teaching practices on such students.

6. From a Harvard course evaluation questionnaire.
7. DeFranco, T. C., & Curcio, F. R. (1997). "A division problem with a remainder embedded across two contexts: Children's solutions in restrictive versus real-world settings". *Focus on Learning Problems in Mathematics*, 19(2), 58–72. Reported by Erik De Corte in a presentation entitled "Connecting Mathematics Problem Solving to the Real World" at a conference on *Mathematics for Living*, Amman, Jordan, November 18–23, 2000.
8. From David Matthews, University of Central Michigan; reported at a conference at the University of Arizona, fall 1993.
9. A student made this request in linear algebra when she wanted a picture showing why a result was true, but was not yet ready to hear the proof.
10. Published in Chick, Stacey, Vincent, and Vincent, eds., *Proceedings of the 12th ICMI Study Conference: The Future of Teaching and Learning Algebra* (University of Melbourne, Australia, 2001), vol. 2: 438–46.
11. Challenges, Issues, and Expectations of Pre-Service Teachers at CBMS/Exxon National Summit on Teacher Preparation, Washington, DC, 2000. Available at http://www.maa.org/cbms/NationalSummit/Speakers/McGowan.pdf.
12. This example may seem improbable; unfortunately, it is real.
13. ⟨http://europe.cnn.com/2001/WORLD/europe/UK/08/10/coniston.dna/index.html⟩. The article described the identification of a body in a lake in the United Kingdom as speed record breaker Donald Campbell. (CNN: August 10, 2001.)
14. ⟨http://www.unfoundation.org/campaign/aids/index.asp⟩; accessed October 31, 2001.
15. In South Africa alone, it is estimated that there will be 2.5 million orphans by the year 2010. From *Impending Catastrophe Revisited,* prepared by ABT Associates (South Africa) and distributed as a supplement to South Africa's *Sunday Times,* 24 June 2001.
16. ⟨http://www.cdc.gov/hiv/pubs/rt/rapidct.htm⟩; accessed July 22, 2001.
17. Reported in *Mass Testing: A Disaster in the Making,* by Christine Johnson at ⟨http://www.healtoronto.com/masstest.html⟩, accessed February 24, 2002.
18. High Risk of False-Positive HIV Tests at ⟨http://www.healtoronto.com/pospre.html⟩, quoting from "To Screen or Not to Screen for HIV in Pregnant Women," by Jeffrey G. Wong, M.D., in *Postgraduate Medicine* 102, 1 (July 1997); accessed February 24, 2002.
19. The word "certain" is not correct here; however, that is not what confuses many students. The central issue is that the large number of false positives is a consequence primarily of the low prevalence of the HIV virus in the U.S. population, not of inaccuracies in the test.
20. Some states apparently give all pregnant women AIDS tests whether or not they consent; others suggest an AIDS test but require consent.
21. Some medical schools explicitly require calculus for admission and many students take calculus as part of their premed program. Statistics is less frequently required for admission.
22. Lewis Carroll, *Through the Looking Glass,* ⟨http://www.literature.org/authors/carroll-lewis/through-the-looking-glass/chapter-05.html⟩; accessed September 9, 2001.
23. Arizona and Massachusetts both have included probability and data interpretation on the state-mandated high school graduation tests; however, the contents of these tests are subject to sudden changes, so there are no guarantees for the future.
24. This was quite reasonable because it was the prevailing view for most of the last century.