The Third R in Literacy

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The conventional meaning of the word "literate" (able to read and write) leaves the third of the three R's on its own, a separation that is reflected in the traditional curriculum: reading and writing are taught together, and mathematics is taught as a separate subject. The first question to ask in considering quantitative literacy and the curriculum is: Why can't we leave it that way? What is new in the world that prompts us to incorporate mathematics into the definition of literacy, or to believe that mathematics must now be spread across the curriculum rather than contained in dedicated courses? This question is answered persuasively in the opening section of *Mathematics and Democracy: The Case for Quantitative Literacy* (Steen 2001): what is new in the world is the pervasiveness of quantitative information and the necessity of acting on it. Thus the list of things a literate person must be able to read and write has greatly expanded to include the many forms in which quantitative information is represented in everyday life: graphs, charts, tables, maps, diagrams, and algorithms.

Furthermore, acting on quantitative information requires more than the basic level of comprehension that comes from listening to a story or looking at a picture; it requires the ability to extract the relevant pieces from a possibly confusing abundance of data and perform appropriate mathematical operations and reasoning on those pieces. The explosion in both the amount and variety of quantitative information, and the necessity of using such information in daily decisions, make the need for quantitative literacy both new and urgent.

In considering how the school and college curriculum can lead students to achieve quantitative literacy, it is crucial to keep in mind two aspects of quantitative literacy that are captured in a definition of conventional literacy put forth by UNESCO (Fox and Powell 1991): "A literate is a person who, with understanding, can both read and write a short simple statement on his everyday life."

This definition says that simply being able to read and write is not enough for literacy: understanding and engagement with context (everyday life) also are required. The same applies to quantitative literacy, and we might well adopt the modified definition: "A quantitatively literate person is a person who, with understanding, can both read and represent quantitative information arising in his or her everyday life."

Although it might seem unnecessary to mention the criteria of understanding and context explicitly, the fact is that in the traditional curriculum neither goes without saying. Therefore, we start by considering how each might be applied in judging new curricula.

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The Role of Understanding

The ability to perform some of the basic operations of mathematics is necessary for quantitative literacy, but even the ability to perform many of them is not sufficient. Anyone who has taught mathematics, or who has taught a subject requiring the mathematics that students have learned in previous courses, is aware of this fact. Many students are technically capable but unable to make reasonable decisions about which techniques to apply and how to apply them. Mathematicians and their colleagues in other departments share frustration at the fragility of what students learn in their mathematics classes. We hesitate to call literate a person who reads haltingly, picking out words one at a time with no appearance of understanding what is being read; yet too many of the students leaving mathematics courses use mathematics haltingly if at all. Too many appear to be lacking in conceptual understanding.

We make a distinction here between conceptual understanding and formal mathematical understanding. The latter refers to an ability to formulate precise mathematical arguments that are universal in the sense that they work for all numbers or all polygons. Conceptual understanding is understanding of a less formal nature, more like what mathematicians sometimes call intuitive understanding. It refers to an ability to recognize underlying concepts in a variety of different representations and applications. For example, a student who understands the concept of rate knows that the velocity of a moving object, the slope of its position graph, and the coefficient of t in a formula giving its position as a function of time t are all manifestations of the same underlying concept, and knows how to translate between them.

Even though recent efforts to reform mathematics education have paid attention to conceptual understanding, it is often neglected in mathematics classes. Sometimes, in the K–12 environment, this is the result of drifting curricula that, in the absence of firm guidance, gravitate inevitably toward convenient arrangements between teacher and student, and teacher and parent that concentrate mostly on the correct performance of procedures. At other times, it is a consequence of conscious decisions by curriculum designers who believe in division of labor between mathematics classes, which provide technical skills, and classes in mathematically intensive disciplines, which provide context and understanding.

It is worth listening to the voices of teachers in those disciplines. The Mathematical Association of America (MAA) recently conducted a series of workshops with faculty from different disciplines as part of a project to develop recommendations for the mathematics curriculum in the first two years of postsecondary education. Again and again, the authors of these workshop reports—engineers, physicists, biologists, chemists, computer scientists, statisticians, and mathematicians—explicitly mentioned *understanding* as a key goal of the mathematics curriculum, and they made it clear that they thought that it was a proper role of mathematics classes to teach it (*Curriculum Foundations Project* 2001):

Physics: Students need conceptual understanding *first,* and some comfort in using basic skills; then a deeper approach and more sophisticated skills become meaningful.

Life Sciences: Throughout these recommendations, the definition of *mastery* of a mathematical concept recognizes the importance of both conceptual understanding at the level of definition and understanding in terms of use/implementation/computation.

Chemical Engineering: . . . the "solution" to a math problem is often in the *understanding* of the behavior of the process described by the mathematics, rather than the specific closed form (or numerical) result.

Civil Engineering: Introductory math content should focus on developing a sound understanding of key fundamental concepts and their relevance to applied problems.

Business: Mathematics departments can help prepare business students by stressing conceptual understanding of quantitative reasoning and enhancing critical thinking skills.

Statistics: Focus on conceptual understanding of key ideas of calculus and linear algebra, including function, derivative, integral, approximation, and transformation.

What is striking about these reports is that so many science, mathematics, engineering, and technology (SMET) disciplines feel the need to explicitly request conceptual understanding from mathematics courses preparing their students. All the more must we worry about the state of conceptual understanding in students who are not preparing for SMET disciplines but simply need quantitative literacy as a basic life skill. Thus, our first criterion:

> *A curriculum for quantitative literacy must go beyond the basic ability to "read and write" mathematics and develop conceptual understanding.*

The Role of Context

The UNESCO definition specifies that a literate can read and write a "statement . . . on his everyday life." The term "everyday life" is open to interpretation: everyday life in the examples above from the SMET disciplines might include chemical reactions in a laboratory. The vast majority of students, however, are not headed for SMET careers. For most people, everyday life might include telephone rate plans, nutrition information on packages, or the relative risks of a serious car accident versus being struck by lightning, for example. A quantitatively literate person must be able to think mathematically in context. This requires a dual duty, marrying the mathematical meaning of symbols and operations to their contextual meaning, and thinking simultaneously about both. It is considerably more difficult than the ability to perform the underlying mathematical operations, stripped of their contextual meaning. Nor is it sufficient simply to clothe the mathematics in a superficial layer of contextual meaning. The mathematics must be engaged with the context and be providing power, not an engine idling in neutral. Too many attempts at teaching mathematics in context amount to little more than teaching students to sit in a car with the engine on, but not in gear. The "everyday life" test provides a measure of engagement; everyday life that moves forward must have an engine that is in gear.

We share two local examples outside of the training for traditional SMET careers. The first is a "math for all citizens" example. One of us (RMR) teaches a freshman-level general education science course for nonscience majors. Reading graphs as pictures or graphs as active conveyors of quantitative information is a desired learning outcome for the course. For the past five years we have used, in classes with as many as 325 students, the so-called Keeling CO_2 data set. This data set consists of a nearly continuous 50-year record of monthly atmospheric CO_2 concentration levels at Mauna Loa, Hawaii (Keller 2000; Keeling and Whorf 2001). Working in groups during a single lecture period, students randomly select 15 data points from about a six-year (100-point) portion of the data set. They plot this subset on an overhead transparency and estimate the slope of the data. Using their slopes, they estimate the number of years it will take for CO_2 concentration to double, an important component of all climate models of global warming. Because the small sample size of the data is insufficient to accurately reflect both an annual cycle and a long-term increase, student estimates of slope and doubling times vary by at least a factor of two. When all transparencies are superimposed using an overhead projector, the annual cycle is clearly visible. This exercise, although simple and completed in a single class period, includes basic mathematical operations (slopes, rates, doubling times) and issues of data quality and completeness, as well as a contextual setting that is arguably one of the most important for everyday life in the twenty-first century. (The Mauna Loa CO_2 data set is very rich for quantitative literacy instruction. For example, geologist Len Vacher at the University of South Florida uses the same data set to show that errors in estimating slopes from graphs are very common unless the axes of such graphs are understood.)

Another example is a business mathematics course recently developed at the University of Arizona by a collaboration between the Department of Mathematics and the College of Business and Public Administration. In this course, students use mathematical and technological tools to make business decisions based on realistic (in some cases, real) data sets. In one project, for example, students decide whether to foreclose on a business loan or work out a new payment schedule. They have available some information about the value of the business, the amount of the loan, and the likely future value if the business is allowed to continue but still fails. They also have some demographic information about the person running the business. Using historical records about the success and failure of previous arrangements to work out a payment schedule, they make successively more sophisticated calculations of expected value to arrive at their decision. Students are expected to understand both the mathematics and the business context, and to make professional oral presentations of their conclusions in which they are expected to express themselves mathematically, with clarity, completeness, and accuracy.

A noteworthy feature of this course is the level of involvement of the business college. The impetus to create the course came from the college, as do the basic ideas for projects. The visible involvement of the college makes their students take seriously the requirement to understand the mathematics. Thus, our second criterion:

> *A curriculum for quantitative literacy must be engaged with a context, be it everyday life, humanities, business, science, engineering, or technology.*

Mathematics and Democracy (Steen 2001) lists elements that might compose quantitative literacy: confidence with mathematics, cultural appreciation, interpreting data, logical thinking, making decisions, mathematics in context, number sense, practical skills, prerequisite knowledge, symbol sense. Many of these elements arise naturally in applying the criteria we have given here, and are, in varying proportions, necessary ingredients of a curriculum for quantitative literacy. The precise proportions depend on the educational level and background of the students. Symbol sense, for example, is a rich vein in the everyday life of students in the physical and social sciences and engineering, but perhaps not as important to students in art or literature. On the other hand, because all citizens are bombarded daily with statistical data and inferences from it, reasoning logically and confidently with data is a crucial component of any curriculum for quantitative literacy.

The odd man out in this list is "cultural appreciation." In recent years there has been a proliferation of general education courses, taught by mathematics departments, that study such topics as voting schemes, symmetry, and periodic tilings of the plane. Although these "mathematics appreciation" courses often provide the serious engagement with quantitative information necessary for quantitative literacy, they focus primarily on fostering a general understanding of the uses of mathematics. Although there is certainly a place in the curriculum for mathematics appreciation courses, we believe an important difference exists between such courses and quantitative literacy—just as there is a difference between the ability to appreciate a great work of art and the ability to make some sketches of one's own, however rudimentary. For quantitative literacy, the element of engagement is crucial.

Who Is Responsible for Teaching Quantitative Literacy?

This question has both a vertical dimension (in which grades should quantitative literacy be taught?) and a horizontal dimension (in which departments?). Along the vertical dimension, we are faced with the question of whether quantitative literacy is properly a college subject at all. It can be argued that a good K–12 education should be sufficient to lay the foundations of quantitative literacy, and that the proper role of colleges and universities, typically in general education courses, is not to teach it but to use it.

Whatever the ideal, the reality is that there are well-documented problems with the mathematical performance of students in grades K–12; see, for example, the Third International Mathematics and Science Study (TIMSS). As a local confirmation of a national issue, each year over 35 percent of freshman students entering the University of Arizona, which has a four-year high school mathematics entrance requirement, place *below* college algebra.

Many people, including mathematics faculty, are working to improve the situation. Mathematics faculty have focused on improving the teaching of mathematics in high schools and reforming courses in mathematics departments in the first two years of college. The National Council of Teachers of Mathematics (NCTM) issued standards for K–12 mathematics in 1989 (NCTM 1989) and revised them in 2000 (NCTM 2000). Many states and localities have endeavored to improve mathematics education by implementing standards, frameworks, or high-stakes tests. At the college level, the National Science Foundation (NSF) has funded projects to reconsider curricula in pre-calculus, calculus, differential equations, and linear algebra. More recently, the Curriculum Foundations Project of the Mathematical Association of America, cited above, has initiated an ambitious undertaking aimed at formulating recommendations for the first two years of undergraduate mathematics.

Would improvements in mathematics education be sufficient to remedy current deficiencies in quantitative literacy? Attempts to change the mathematics curriculum—to make room earlier for statistics and probability (as recommended by NCTM), to teach mathematics in context, to pay attention to conceptual understanding, and to improve the mathematics education of K–12 teachers—would certainly move partway toward the changes needed to improve quantitative literacy. Such improvements are necessary. However, although the way mathematics is taught has a lot to do with quantitative literacy, so do other things.

Quantitative literacy cannot be taught by mathematics teachers alone, not because of deficiencies in teaching but because quantitative material must be pervasive in all areas of a student's education. Quantitative literacy is not simply a matter of knowing how to do the mathematics but also requires the ability to wed mathematics to context. This ability is learned from seeing and using mathematics regularly in contexts outside the mathematics classroom: in daily life, in chemistry class, in the business world. Thus, quantitative literacy cannot be regarded as the sole responsibility of high school mathematics teachers or of college teachers in mathematics departments. It has long been recognized, for example, that instruction in writing literacy, isolated in English composition courses, cannot succeed. Students quickly recognize that a requirement satisfied by a course or two in a single department is a local "hoop" to be jumped through, not a global requirement central to their education. Students often behave as if mathematical ideas are applicable only in mathematics courses, so that once they enter the world of their chosen major they can safely forget whatever they learned in those courses.

It must therefore be the common responsibility of both mathematicians and those in other disciplines to provide students with basic skills, to develop conceptual understanding, and to model the systematic use of mathematics as a way of looking at the world. The pervasiveness of quantitative information in the world outside the classroom also must be reflected throughout academe. A beautiful example of this pervasiveness is the recent foray into art history by optical scientist Charles Falco and contemporary artist David Hockney. These two have recently challenged traditional art historian interpretations of fifteenth-century art. Using simple optics, they have argued persuasively that a number of important painters of the fifteenth century, from van Eyck to Bellini, used lenses or mirrors to produce some of their paintings nearly 200 years earlier than had been believed possible. They argue that the use of such optical instruments accounts for the sudden surge in the reality of portraits in the fifteenth century (Hockney and Falco 2000). We can easily envision a wonderful application of quantitative literacy in fine arts education if their arguments stand the test of further scrutiny.

A persuasive argument can be made that the skills component of quantitative literacy is essentially precollege in nature. What, this argument goes, beyond the topics of precollege education (graphs, algebra, geometry, logic, probability, and statistics) is foundational to quantitative literacy for everyday life? Looking at the curriculum as a list of topics, however, misses an important point: quantitative literacy is not something that a person either knows or does not know. It is hard to argue that precollege education in writing fails to cover the basics of grammar, composition, and voice, for example. Yet it is widely accepted that writing is a skill that improves with practice in a wide variety of settings at the college level. We argue here that quantitative literacy at the college level also requires an across-the-curriculum approach, providing a wide variety of opportunities for practice.

The challenges to incorporating quantitative literacy across the curriculum are many, including math anxiety on the part of both faculty and students, lack of administrative understanding and support, and competing pressures for various other literacy requirements. We discuss below a variety of approaches that have demonstrated success at the college level in moving quantitative literacy across the curriculum. A more comprehensive discussion would address how these approaches should be coordinated with efforts to improve K–12 education, an issue we do not feel qualified to address. It is worth pointing out, however, that improving quantitative literacy at the college level would have an important effect on K–12 education for the simple reason that it would influence the mathematics education of K–12 teachers.

Mathematics Across the Curriculum

The term "mathematics across the curriculum" refers to attempts to incorporate mathematical thinking in courses throughout the university. The following excerpt from the vision statement of the Mathematics Across the Curriculum committee at the University of Arizona expresses the goals (2000):

> The purpose of Mathematics Across the Curriculum at the University of Arizona is to help students recognize the utility of mathematics across disciplines and majors and to improve their skills in mathematics. Just as all students should be able to write an essay in any class they take, all students should be able to look at a problem or situation in any class and be able to formulate appropriate mathematical approaches to finding solutions. They should also have the mathematical skills to know how to seek solutions. Particular attention must be paid to such fundamental processes as graphic representation of quantitative data; estimation; basic numeracy (i.e., ability to perform "basic" mathematical operations); and logic, among other mathematical concepts and topics.

Various approaches to implementing mathematics across the curriculum have been tried. We consider six approaches here: collaboration between mathematics and other faculty, gateway testing, intensive instructional support, workshops for nonmathematics faculty, quantitative reasoning requirements, and individual initiative by nonmathematics faculty. This is not intended to be an exhaustive list of all possible approaches or all approaches that have been tried, but rather an illustration of the range of possibilities.

"Friendly Conspiracies" Between Mathematicians and Others

Collaboration between mathematics faculty and faculty from other departments is one powerful approach. This could involve a sort of pact between mathematicians and others: mathematicians will add more context to their courses, others will add more mathematical concepts to theirs. Deborah Hughes Hallett writes of the need for friendly conspiracies between mathematicians and other departments to make sure this happens (Hughes Hallett 2001). The two-course business mathematics sequence developed at the University of Arizona is an example of such a conspiracy: students know that the problems they are studying in their mathematics course will come up again in their business courses, because they know that the course was developed with significant input from the business college. Team-teaching arrangements between mathematics and other departments are another example of this collaborative approach.

One cautionary note on such collaborations is illustrated by a survey conducted by the Mathematics Across the Curriculum group at the University of Arizona. This survey was sent to faculty in the College of Social and Behavioral Sciences who teach some of the largest general education courses on campus. The responses, completed by almost 33 percent of the group, included some telling results. More than half responded positively to the question, "Does any course you teach include any mathematical or quantitative elements?" The most common elements included statistics, slopes and rates, analysis of experimental outcomes, graphs, formal reasoning, and decision theory. Faculty were also asked, "Would you be willing to integrate some mathematical elements into your courses?" Again, more than half responded positively, although the response was cautious. For example, even among the positive responses, faculty said, "I don't see this as central to the usefulness of the course. Emphasis on the mathematics might actually distract students from the more important (in this course!) learning," and, "I would be reluctant to assign stats-heavy reading, as most students do not seem to pay close attention to such materials." The faculty responding negatively were split about evenly between "The course doesn't seem compatible with the addition of mathematical content" and "My background in mathematics is insufficient." These responses, typical of faculty everywhere, highlight some of the challenges inherent in establishing these friendly conspiracies.

Gateway Testing in Courses Outside Mathematics

The University of Nevada, Reno, approaches quantitative literacy by offering a set of mathematics competency tests in courses across the university, based on a model described in an article by Steven F. Bauman and William O. Martin (1995). The mathematics covered by the tests is required for success in the courses, and is no more than the students reasonably can be expected to have learned already. The main purpose of the tests is to inform instructors and students. Students may retake a test until they pass (a passing grade is 80 percent). The initial test is held in class during the first week of classes, to make clear that the test is an integral part of the course; after that, a separate office, the Math Center, handles grading and retesting, to make it feasible for instructors to use the system. The Math Center also provides tutors who go over a failed examination with each student and help him or her correct mistakes. Courses that have been involved with this program include agricultural economics, anthropology, art, biology, chemistry, economics, English, environmental studies, geography, geology, mathematics core courses, nutrition, philosophy, physics, political science, psychology, recreation, physical education, dance, sociology, and Western traditions.

Mathematics Instructional Support

The Center for Mathematics and Quantitative Education at Dartmouth College (2001) functions as a laboratory support office for the mathematics department, analogous to similar services available in other science departments. It houses equipment for use in mathematics classrooms, books, videos, and prepared laboratory activities. Some of these materials come from Mathematics Across The Curriculum (MATC) courses at Dartmouth. The center also provides consulting, classroom visitation, and videotaping services, and runs a departmental Teaching Seminar during the summer. The center works in collaboration with similar offices in other departments and supports courses in other departments that feature mathematics as a key component. It reviews materials that come out of the university's MATC courses and makes those that are suitable for K–12 available to teachers. It also facilitates links between K–12 teachers and college professors for conversation and collaboration across levels and disciplines.

Workshops for Faculty Outside Mathematics

Many disciplines, most commonly the SMET disciplines, have come to recognize the importance of quantitative literacy and some have organized regional or national workshops on the topic. One group that has facilitated such workshops is Project Kaleidoscope (PKAL 2002), an informal national alliance working to build strong learning environments for undergraduate students in mathematics, engineering, and the various fields of science. One PKAL workshop, entitled "Building the Quantitative Skills of Non-Majors and Majors in Earth and Planetary Science Courses," was held in January 1999 at the College of William and Mary. The workshop brought together over 30 earth and planetary science faculty from research-intensive, and four-year and two-year institutions to work together on such questions as:

- Which quantitative skills are important in our curriculum, and at what levels?
- How do we include appropriate quantitative expectations in our courses for nonmajors without sending some students running for less quantitative offerings elsewhere on campus?
- How can a department work to build the quantitative skills of its majors?
- Many students, nonmajors and majors, bring tremendous fear, or "math anxiety," to our courses. What support is necessary to help students understand, use, and enjoy mathematics in our courses?

Such workshops have had a significant impact on how faculty outside of mathematics view quantitative literacy, and have provided concrete strategies and "best practices" to help them transform their courses. One example is the development of "Q-Courses" (e.g., Marine Environmental Geology and Introduction to Environmental Geology and Hydrology) at Bowdoin College by a geology team that grew out of the 1999 PKAL workshop.

Quantitative Reasoning Requirements

Recognizing that quantitative literacy often is not ensured by their entrance requirements, many colleges have instituted quantitative reasoning requirements that must be satisfied by all graduates. At some institutions, such as Harvard University and the University of Michigan, there is an approved list of courses that satisfy the requirement. An example of a more formal quantitative reasoning requirement is the one at Wellesley College (2002). This consists of a basic skills component, which is satisfied either by passing a quantitative reasoning test or by taking a specific course, and an overlay course component. The topics covered by the test are arithmetic, algebra, graphing, geometry, data analysis, and linearity. Overlay courses are taught within departments and engage students in using these skills in reasoning about and interpreting data in specific contexts. Guidelines specify the minimum necessary exposure to data analysis for a course to qualify as an overlay course. For example, such a course must address issues of collecting, representing, and summarizing data and must require a working knowledge of probability, distributions, and sampling. The goals of the overlay requirement are worth quoting for their resonance with the issues of quantitative literacy:

Literacy. The number of topics, and depth of coverage, should be sufficient to ensure that students have the basic knowledge they need in order to function in real-life situations involving quantitative data.

Authenticity. Students should have experience in using authentic numerical data. The experience should arise naturally in the context of the course and actually advance the work of the course. Only with such experience is the literacy goal likely to be realized.

Applicability. The examples used in an overlay class should be adequate to convince the average student that the methods used in the analysis of data are of general applicability and usefulness.

Understanding. A student's experience with data analysis should not be limited to rote application of some involved statistical procedure. Rather, students should understand enough of what they are doing so that their experience of data analysis is likely to stay with them, at least as a residue of judgment and willingness to enter into similar data analyses in the future.

Practicality. The breadth of topics covered, and the depth of coverage, should be consistent with what an average Wellesley student can realistically absorb in a course that devotes only a part of its time to data analysis.

Good-Citizen Model of Concerned Nonmathematics Faculty

All the previous approaches involve either mathematics faculty or specialized administrative units, but we should never underestimate the power of nonmathematics faculty or departments acting on their own initiative to advance quantitative literacy. There are many such examples of individual faculty revising courses and curricula simply because it is the right thing to do. Examples include Len Vacher at the University of South Florida, Bill Prothero at the University of California at Santa Barbara, Kim Kastens at Columbia University, Larry Braille and Jon Harbour at Purdue University, and Alexandra Moore at Cornell University. These faculty may take advantage of some of the approaches listed above, but they often are essentially lone crusaders for quantitative literacy working in the trenches. Although they may attend workshops or seek NSF funding, for example, just as often they proceed with little administrative support or interaction with mathematics faculty. In fact, some are hampered by administrations that depend on student credit hours as the coin of the realm, or student evaluations that can tend to favor less quantitatively challenging courses.

Given strong evidence of the success of these independent initiatives, we cannot but wonder at how much more effective such efforts could be with the full involvement and cooperation of mathematics faculty and college or university administrations. We argue that one critical component of quantitative literacy across the curriculum must be the support and nurturing of such initiatives. As one example of administrative support, we cite the reform of the promotion and tenure system in the College of Science at the University of Arizona for faculty whose primary scholarly contribution is in the area of mathematics and science education. This reform was recognized by NSF with one of just 10 Recognition Awards for the Integration of Research and Education (University of Arizona 1998).

Conclusions and Challenges

We have argued that conceptual understanding and everyday life are two aspects of quantitative literacy deserving special attention. The ability to adapt mathematical ideas to new contexts that is part of conceptual understanding is a key component of quantitative literacy. The everyday-life component of quantitative literacy argues forcefully for engagement of faculty across the curriculum. Quantitative literacy thus must be the responsibility of teachers in all disciplines and cannot be isolated in mathematics departments.

We have illustrated curricular approaches to quantitative literacy at the college or university level that range from friendly conspiracies between mathematics and other faculty to administrative structures and requirements to initiatives by individual nonmathematics faculty. All offer success stories as well as war stories, both of which serve as models for how we can work to improve quantitative literacy.

We end with two challenges. The approaches we have illustrated must be only the start of continued and sustained efforts on the parts of faculty and institutions. Significant institutional change must occur to achieve the sort of pervasive use of mathematical ideas that we think essential in teaching quantitative literacy. Neither administratively imposed solutions nor grassroots movements will succeed alone; initiatives solely from within mathematics departments or solely from without are bound to fail. The first challenge, therefore, is to cross the boundaries that separate disciplines and levels of administration. Administrators of university-wide requirements must talk with the faculty who do the teaching on the ground; pioneers in the classroom must talk to each other and to administrators; departments of mathematics must collaborate with other departments.

Second, we must not lose sight of the fact that our goal is student learning. It is far too easy, in the heat of battle over establishing quantitative literacy requirements, setting up support centers, or revising our individual courses, to forget that the student must be the focus of our efforts. The question of "what works best" must be answered in terms of student learning. To do this, we must establish clearly defined student learning outcomes in quantitative literacy. We must be able to develop measures for these outcomes as part of an ongoing assessment program. Key to the success of such an assessment program is feedback on the way we are teaching quantitative literacy. Without such formative assessment, debates on how to improve quantitative literacy will be driven by anecdotal experience and the force of individual personality. Students deserve better.

We welcome the national focus on quantitative literacy and are hopeful that the kinds of approaches described here may serve as models for others.

Acknowledgments

The authors wish to thank H. Len Vacher and Stephen Maurer for very constructive reviews of a preliminary version of this manuscript.

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