MONICA NEAGOY'S EDITED VERSION

What is algebra and why some students find it so hard?

Algebra is not just arithmetic with letters standing for numbers. It's a different kind of thinking. While the path from arithmetic to algebra may be steep, anyone with enough effort and good instruction can climb it.

Some people find arithmetic hard to learn, but most succeed, though only after a ***lot*** of practice. What makes mastery possible is that the basic building blocks of arithmetic—numbers—abound in our world, when we count, measure, buy, make things, call someone, go to the bank, check baseball scores, etc. Numbers may be abstract — you never saw, felt, heard, or smelled the number 3 — but they are tied to concrete things in the world we live in.

The basic building blocks of algebra on the other hand are variables. If algebra is introduced as generalized arithmetic, then variables are generalized numbers. For example 2 + 3 = 3 + 2 generalizes to x + y = y + x. There is nothing to solve here. If algebra is presented from the problem-solving perspective, variables are viewed as unknowns to solve for. We all remember those senseless word problems! If algebra is the study of functions (relationships among changing quantities), then variables represent these quantities that change. The many uses of the symbol x can be confusing to students. But an astute teacher with a profound understanding of algebra can help students overcome this confusion.

When students start to learn algebra, they inevitably try solving algebraic problems arithmetically. But not all properties of arithmetic transfer seamlessly to the objects of algebra. Therein lies a major hurdle. In algebra you use analytic, ***qualitative*** reasoning (thinking logically) ***about*** numbers, whereas in arithmetic you use numerical, ***quantitative*** reasoning (computing explicitly) ***with*** numbers.

For example, you need to use algebraic thinking if you want to write a macro to calculate the cells in a spreadsheet like Microsoft *Excel*. It doesn't matter whether the spreadsheet is for calculating scores in a sporting competition, keeping track of your finances, running a business, or figuring out the best way to equip your character in *World of Warcraft*, you need to think algebraically to set it up to do what you want — that means thinking about or across numbers, rather than in terms of numbers.

To do algebra, you have to ***stop*** thinking arithmetically and ***start*** thinking algebraically.

[Note from MN edits: I kept Keith Devlin's main idea about the distinction between arithmetic and algebra, between arithmetic thinking and algebraic thinking because I was not to address content here. I felt however that the title of

the essay posed two questions but the body of the text essentially addressed the first. So I inserted an example of a source of difficulty for beginning algebra students.—variables—in the attempt to answer part of “why some students find it so hard?”

I agree with JB, the editor of *Big City Times* opinion pages that you shouldn’t underestimate the power of a mathematician’s words. I suggest changing “Why do students find it so hard?” to “Why do *some* students find it so hard?” as not all students find algebra hard. And to those who don’t, we wouldn’t want to suggest that they should…]