What is algebra and why do students find it so hard?

Algebraic thinking is not just arithmetic with letters standing for numbers. It is a different kind of thinking.

Many people find arithmetic hard to learn, but most succeed, to varying degrees, though only after a ***lot*** of practice. What makes it possible is that the basic building blocks of arithmetic, numbers, arise naturally in the world around us, when we count things, measure things, buy things, make things, use the telephone, go to the bank, check the baseball scores, etc. Numbers may be abstract — you never saw, felt, heard, or smelled the number 3 — but they are tied closely to all the concrete things in the world we live in.

Algebra is thinking ***logically*** about numbers rather than computing with numbers. In algebra you are a second step of abstraction removed from the everyday world: those x's and y's usually denote numbers ***in general***, not particular numbers. In algebra you use analytic, ***qualitative*** reasoning **about** numbers, whereas in arithmetic you use numerical, ***quantitative*** reasoning ***with*** numbers.

For example, you need to use algebraic thinking if you want to write a macro to calculate the cells in a spreadsheet like Microsoft *Excel*. It doesn't matter whether the spreadsheet is for calculating scores in a sporting competition, keeping track of your finances, running a business, or figuring out the best way to equip your character in *World of Warcraft*, you need to think algebraically to set it up to do what you want — that means thinking about or across numbers, rather than in terms of numbers.

When students start to learn algebra, they inevitably try to solve problems by arithmetical thinking. That's a natural thing to do, given all the effort they have put into mastering arithmetic, and at first, when the algebra problems they meet are particularly simple (that's the teacher's classification), this approach works. In fact, the stronger a student is at arithmetic, the further they can progress in algebra using arithmetical thinking. (Many students can solve the quadratic equation $x^2 = 2x + 15$ using basic arithmetic, using no algebra at all.) Paradoxically, or so it may seem, however, those better students may find it harder to learn algebra. Because to do algebra, for all but the most basic examples, you have to ***stop*** thinking arithmetically and learn to think algebraically.