Calculus with Analytic Geometry. By George F. Simmons. McGraw-Hill Book Company. New York, 1985. xiii + 950 pp. $46.95.

UNDERWOOD DUDLEY
Department of Mathematics and Computer Science, DePauw University, Greencastle, Indiana 46135

This is about calculus books, and there are seven important conclusions.

Let us go back to the beginning and look at the first calculus book, *Analyse des Infiniment Petits, pour L'intelligence des Lignes Courbes*, published in Paris in 1696 and written by Guillaume François Antoine L'Hospital, Marquis de Sainte-Mesme, Comte d'Entrement, Seigneur d'Orques, etc. To almost everyone, L'Hospital is just a name attached to L'Hospital's Rule and almost no one knows anything about him. His memory deserves better. He displayed mathematical talent early, solving a problem about cycloids at fifteen, and he was a lifelong lover and supporter of mathematics who, unfortunately, died young, at the age of 43. Also, L'Hospital was no mean mathematician. He published several papers in the journals of the day, solving various nontrivial problems. I know that I could not have found, as he did, the shape of a curve such that a body sliding down it exerts a normal force on it always equal to the weight of the body. Further, as Abraham Robinson has written,

> According to the testimony of his contemporaries, L'Hospital possessed a very attractive personality, being, among other things, modest and generous, two qualities which were not widespread among the mathematicians of his time.

His book was a huge success. There was a second edition in 1715, and there were commentaries written on it. I have the 1781 edition, with additions made by another author. Not many textbooks last almost 100 years. Birkhoff and MacLane is not yet 50 years old. L'Hospital's book is about differentials and their applications to curves and the style is exclusively geometrical. There are not many equations, but there are an awful lot of letters and pictures, just as I remember in my tenth grade geometry text. Mathematics was geometry then, and mathematicians were geometers.

There are many things not in the book. There are no sines or cosines, no exponentials or logarithms, only algebraic functions and algebraic curves. There are also no derivatives, only differentials. Here is L'Hospital's proof of L'Hospital's Rule, using differentials. In Figure 1, points on the graph of $f(x)/g(x)$ are found by dividing lengths of abscissas, except at a. But if you dx past a, the point on the graph will be df/dg. Since dx is infinitesimal, that ratio gives you the point at a. Is that not nice?

It took some time for calculus to become generally taught in colleges. Eventually it made it, and calculus textbooks began to appear in the nineteenth century. I have a copy of one, *Elements of the Differential and Integral Calculus*, by Elias Loomis, Ll. D., Professor of Natural Philosophy at Yale College. His calculus was first published in 1851, and my copy of it is the 1878 edition. It sold in excess of 25,000 copies, so it must reflect accurately the style and content of calculus teaching of the time. Just as with L'Hospital, the differential was the important idea. Loomis derived the formula for the differential of x^n with no use of the binomial expansion, ($d(xy)/xy = dx/x + dy/y$, $d(x^n)/x^n = dx/x + \cdots + dx/x$, add and simplify) and his proof of L'Hospital's Rule was short, simple, and clear, and also one which

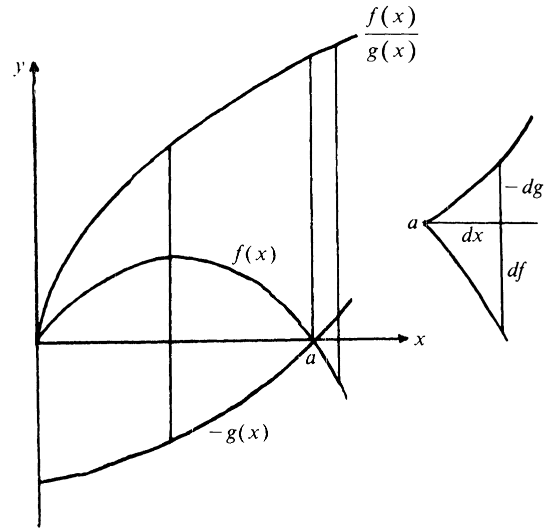

FIG. 1.

does not appear in modern texts because it fails for certain pathological examples. Also, Loomis put *all* of his formulas in words, italicized words. After deriving the formula for the differential of a power of a function, he wrote

> The differential of a function affected with any exponent whatever is the continued product of the exponent, the function itself with its exponent diminished by unity, and the differential of the function.

It was a good idea. It is also probably a good idea to do as L'Hospital and Loomis did and talk about differentials instead of derivatives whenever possible. Little bits of things are easier to understand than rates of change. It is a still better idea to strive for clarity and let students see what is really going on, which is what Loomis did, rather than putting rigor first. But nowadays authors cannot do that. They must protect themselves against some colleague snootily writing to the publisher, "Evidently Professor Blank is unaware that his so-called proof of L'Hospital's Rule is faulty, as the following well-known example shows. I could not possibly adopt a text with such a serious error." It is a shame, and probably inevitable that calculus books are written for calculus teachers, but I have nevertheless concluded

> CONCLUSION #1: CALCULUS BOOKS SHOULD BE WRITTEN FOR STUDENTS.

It would be worth a try. *Calculus Made Easy* by Silvanus P. Thompson was quite successful in its time, which ran for quite a while. The second edition appeared in 1914, and my copy was printed in 1935. It is still in print. The book has a motto:

What one fool can do, another can.

and a prologue:

> Considering how many fools can calculate, it is surprising that it should be thought either a difficult or a tedious task for any other fool to learn how to master the same tricks. ...

> Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

Chapter 1, whose title is "To Deliver You From The Preliminary Terrors" forthrightly says that dx means "a little bit of x." Thompson did not include L'Hospital's Rule.

Both Loomis and Thompson are like L'Hospital when it comes to giving applications and examples: they are all almost entirely geometrical. Loomis's applications of maxima and minima are all about inscribing and circumscribing things, and so are all of Thompson's except one. In fact, all three books are full of geometry. Thompson concluded with arc length and curvature, Loomis had involutes and evolutes, cusps and multiple points, and lots of curve sketching. Did you know that the asymptote to $y^3 = x^3 + x^2$ is $y = x + 1/3$? I didn't until I read Loomis. It is nice to know. There must have been some reason why calculus books for more than 200 years taught so much geometry. Mathematics may no longer be synonymous with geometry, but we have discarded, wrongly I think, the wisdom of the ages, and I have concluded

CONCLUSION #2: CALCULUS BOOKS NEED MORE GEOMETRY.

Before writing this essay, I examined 85 separate and distinct calculus books. I looked at all of their prefaces, all of their applications of maxima and minima, and all of their treatments of L'Hospital's Rule. By the way, I found five different spellings of L'Hospital. There were the two you would expect, and Lhospital, as L'Hospital sometimes spelled his name. In addition, one author, not wanting to take chances, had it L'Hôspital, and one thought it was Le Hospital. Why are there so many calculus books, and why do they keep appearing? One could be cynical and say that the authors are all motivated by greed. But I do not think so. I think that authors write new calculus books because they have observed that students do not learn much from the old calculus books. Therefore, prospective authors think, "if I write a text and do things properly, students will be able to learn." They are wrong, all of them. The reason for that is

CONCLUSION #3: CALCULUS IS HARD.

Too hard, I think, to teach to college freshmen in the United States in the 1980s, but that is another topic.

If you plot the books' numbers of pages against their year of publication, you have a chart in which an ominous increasing trend is clear. The 1000-page barrier, first pierced in 1960, has been broken more and more often as time goes on. New highs on the calculus-page index are made almost yearly. Where will it all end? We can get an indication. The magic of modern statistics packages produces the least-squares line: Pages = 2.94 (Year) − 5180, showing that in the middle of the next millennium, the average calculus book will have 2,270 pages and the longest one, just published, will have 3,783 pages exclusive of index.

Why do we need 1000 pages to do what L'Hospital did in 234, Loomis in 309, Thompson in 301, and the text I learned calculus from, used exclusively for four whole semesters, 14 semester-hours in all, in 416? There are several reasons. One, of course, is the large number of reviewers of prospective texts. No more can an editor make up his mind about the merits of a text, it has to go out to fifteen different people for opinions. And if one of them writes that the author has left out the $\tan(x/2)$ substitution in the section on techniques of integration, how can he or she do that, we won't be able to integrate $3/(4 + 5\sin 6x)$, how can anyone claim to know calculus who can't do that; isn't the easiest response to include the $\tan(x/2)$ substitution? Of course it is, in it goes, and in goes everything else that is in every other 1000-page text. It is impossible to escape

CONCLUSION #4: CALCULUS BOOKS ARE TOO LONG.

Another reason for the length is the current mania for Applications. If you go to *Books in Print* and look in the subject index under "Calculus" what you see is

> The Usefulness of Calculus for the Behavioral, Life, and Managerial Sciences
> Essentials of Calculus for Business, Economics, Life Sciences, and Social Sciences

and many, many similar titles. Now authors have to explain, with examples, what marginal revenue is, and consumer surplus, and what tracheae are whereas in the old days, all their readers knew what a cone was. A third reason is the supposed need to be rigorous. Now we see statements of L'Hospital's Rule that take up half a page and proofs of it that go on for three pages. My 416-page calculus book never even mentioned L'Hospital's Rule, and I never felt the lack. Its author never proved that the derivative of x^e was ex^{e-1}, but I was willing to believe it. Trying to include everything and trying to prove everything makes for long books. Everything gets longer. Prefaces used to be short, a page or less. Now they are five and six pages, hard sells for the incredible virtues of the text that follows, full of thanks to reviewers, to five or six editors, to wives, to students, even to cats.

Let me return to "applications." There aren't many, you know. In the 85 calculus books I examined, almost all of them had the Norman window problem—the rectangle surmounted by a semicircle, fixed perimeter, maximize the area. The semicircle always "surmounts." This is the sole surviving use of "surmounted" in the English language, except for the silo, a cylinder surmounted by a hemisphere. Only one author had the courage to say that the window was a semicircle on top of a rectangle. All the books had the box made by cutting the corners out of a flat sheet, all have the ladder sliding down the wall, all had the conical tank with changing height of water, all had the tin can with fixed surface area and maximum volume, all had the V-shaped trough, all had the field to fence, with or without a river flowing (in a dead straight line) along one side, all had the wire—usually wire, but sometimes string—cut into two pieces to be formed into a circle and a square, though some daring authors made circles and equilateral triangles. There are only finitely many calculus problems, and their number is *very* finite.

"Applications" are so phony. Ladders do not slide down walls with the base moving away from the wall at a constant rate. Authors know the applications are phony. One book has the base of the ladder sliding away from the wall at a rate of 2 feet per *minute*. At that rate, you could finish up your painting with time to spare and easily step off the ladder when it was a foot from the ground. Another author has the old run-and-swim problem—you know, minimize the time to get somewhere on the other side of the river—with the person able to run 25 feet per second and swim 20 feet per second. That's not bad for running (it's a 3:31.2 mile), but it is super swimming, 100 yards in 15 seconds, a new world's record by far. There are no conical reservoirs outside of calculus books. Real reservoirs are cylindrical, or perhaps rectangular. The reason for this is found in the texts: in the problems, the conical reservoirs usually have a leak at the bottom. Tin cans are not made to minimize surface area. I could give any number of examples of absurd applications in which businessmen "observe" the price of their product decreasing at the rate of \$1 per month, or where the S. D. S. (remember them?) "find" that staging x demonstrations costs \$250x^3. Why will authors not be honest and say that these artificial problems provide valuable practice in translating from English into mathematics and that is all they are for? Surely they cannot disagree with

CONCLUSION #5: FIRST-SEMESTER CALCULUS HAS *NO* APPLICATIONS.

Before getting to my next conclusion, here is my favorite "application."

> A cow has 90 feet of fence to make a rectangular pasture. She has the use of a cliff for one side. She decides to leave a 10 foot gap in the fence in case the grass should get greener on the other side. Find ... .

Hardly any authors dare to do that. Calculus books are Serious. The text from which that problem came was titled *Calculus Without Analytic Geometry* and it is no surprise that it did not catch on.

The existence of all those calculus books with "Applications" in their titles implies a market for them. There must be students out there who are being forced to undergo a semester of calculus before they can complete their major in botany and take over the family flower shop. I cannot believe that any more than a tiny fraction of them will ever see a derivative again, or need one. Calculus is a splendid screen for screening out dummies, but it also screens out perfectly intelligent people who find it difficult to deal with quantities. I don't know about you, but I long ago concluded

CONCLUSION #6: NOT EVERYONE NEEDS TO LEARN CALCULUS.

The book by Simmons is a fine one. It was written with care and intelligence. It has good problems, and the historical material is almost a course in the history of mathematics. It is nicely printed, well bound, and expensive. Future historians of mathematics will look back on it and say, "Yes, that is an excellent example of a late twentieth-century calculus book." This leads to my last conclusion

CONCLUSION #7: THAT'S *ENOUGH* ABOUT CALCULUS BOOKS.