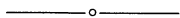


References

1. K. Kendig, *Elementary Algebraic Geometry*, Springer-Verlag, New York, 1977, Chapter II.
2. _____, Algebra, Geometry, and algebraic geometry: Some interconnections, *American Mathematical Monthly* 90 (1983) 161–173.
3. A. Seidenberg, *Elements of the Theory of Algebraic Curves*, Addison-Wesley, Reading, MA, 1968.



The Snowplow Problem Revisited

Xiao-peng Xu, University of Massachusetts, Amherst, 01003

A classic problem in elementary differential equations, commonly attributed to R. P. Agnew [*Differential Equations*, McGraw-Hill, 1942, pp. 30–32], is the following:

One day it started snowing at a heavy and steady rate. A snowplow started out at noon, going 2 miles the first hour and 1 mile the second hour. What time did it start snowing?

The problem is usually solved by setting $t = 0$ at noon, setting up the relevant differential equation, finding its general solution and then using the conditions of the problem to eliminate all the arbitrary constants. This procedure involves a fair amount of algebra which, if not done carefully, can be quite tedious. There is, however, a quick and easy way that avoids most of this algebra. It goes as follows:

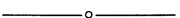
Let t denote time, measured in hours, and let $t = 0$ at 1:00 P.M. Let t_0 be the time it started snowing. Let $y(t)$ denote the distance traveled by the snowplow, measured in miles. Let $h(t)$ be the height of the snow at time t , so that $h(t_0) = 0$. Let s denote the rate of the snowfall, measured in any suitable units. Then, since it was snowing at a steady rate, $h(t) = s(t - t_0)$. Assume that the width of the snowplow is one unit and let k be the amount of snow that the plow can remove per unit time. Then we have

$$h(t) \frac{dy}{dt} = k \quad \text{or} \quad \frac{dy}{dt} = \frac{c}{t - t_0},$$

where $c = k/s$. Now (and this is the trick) instead of finding the general solution of this differential equation, we note that

$$2 = c \ln(t - t_0) \Big|_{-1}^0 \quad \text{and} \quad 1 = c \ln(t - t_0) \Big|_0^1$$

whence $2 \ln[(t_0 - 1)/t_0] = \ln[t_0/(t_0 + 1)]$, and a very little algebra yields $t_0^2 + t_0 - 1 = 0$, so that $t_0 = (-1 - \sqrt{5})/2$.



The Differentiability of Sin x

David A. Rose, East Central University, Ada, OK 74820

That $\sin x$ is differentiable with derivative $\cos x$ implies that $\sin x$ has derivative 1 at $x = 0$, i.e.,

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1. \tag{1}$$