0 or 1 (modulo 2) times, then this point may be designated as the root and the algorithm will produce a walk which visits every point the specified number of times.

An *endline* of a tree T is a line incident with an endpoint of T, i.e., a point of degree one. The tree in Figure 2 has eight endpoints.

Theorem 2. *Given any endline of a tree T and $i \in \{0,1,2\}$, there exists a walk which originates at one of the points of the endline and visits every point i(modulo 3) times.*

If T has only two points, the result is immediate. Thus, assume T has at least three points. Let uv be an endline, where u is the endpoint. Embed T in the plane with v as root, u as the leftmost son of v and point w as the rightmost son. Now apply the algorithm of our Corollary. If all points, including v, are visited i(modulo 3) times, we are done. Otherwise, we may assume that the resulting walk W ends in w and visits all points i(modulo 3) times except for v. If v is visited $(i-1)$(modulo 3), then the number of times the walk Wv visits v is i(modulo 3) and we are done. Thus, it remains only to consider the case where the number of times W visits v is $(i+1)$(modulo 3). In this case, the walk $uvuWvu$ constructed from W visits all the points a correct number of times.

A slight modification of the preceding proof shows that of any two adjacent points in a tree, at least one can be used as the root of a tree for which the algorithm of our Corollary will produce a walk which visits each point of the tree i(modulo 3) times.

Habitués of video arcades may recognize the applicability of the preceding results to the game of "Q*Bert."

A Note on Integration by Parts

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The point of a textbook exercise such as evaluating $\int x^n e^{ax}\,dx$ is to illustrate repeated integration by parts. Since this technique can be tedious for $n > 1$, students who have learned integration by parts may appreciate the following approach.

To evaluate $\int x^n e^{ax}\,dx$, assume that the answer is of the form $e^{ax}p(x)$, where $p(x)$ is a polynomial of degree n. Then obtain the coefficients of $p(x)$ by setting $D_x\{e^{ax}p(x)\}$ equal to $x^n e^{ax}$. This approach is not only simpler, it introduces students to a technique (the method of undetermined coefficients) they will encounter again in differential equations courses. We illustrate this approach as follows:

Example. To evaluate $\int x^3 e^{2x}\,dx$, we assume that an antiderivative of x^3e^{2x} is of the form $e^{2x}(Ax^3 + Bx^2 + Cx + D)$. Then

$$D_x\left\{e^{2x}(Ax^3 + Bx^2 + Cx + D)\right\} = x^3e^{2x}$$

yields

$$e^{2x}\left\{2Ax^3 + (3A + 2B)x^2 + (2B + 2C)x + (C + 2D)\right\} = x^3e^{2x}.$$

This identity yields:

$$\begin{aligned} 2A \quad\quad\quad\quad &= 1 \\ 3A + 2B \quad\quad &= 0 \\ 2B + 2C \quad &= 0 \\ C + 2D &= 0. \end{aligned}$$

Since

$$A = 1/2, \qquad B = -3/4, \qquad C = 3/4, \qquad D = -3/8,$$

we have

$$\int x^3 e^{2x}\, dx = \left(\tfrac{1}{2}x^3 - \tfrac{3}{4}x^2 + \tfrac{3}{4}x - \tfrac{3}{8}\right)e^{2x} + K.$$

Other integrals, such as $\int e^{ax} \sin bx\, dx$, which require repeated integration by parts can also be evaluated more efficiently using this technique.

Relating Differentiability and Uniform Continuity

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For a continuous function $f\colon R \to R$ define function $F(x) = (f(x) - f(a))/(x - a)$ where a denotes some fixed real number. Clearly F is a continuous function defined on $I - a$, where I is any interval containing a. We wish to prove the following result.

> *The function $f(x)$ is differentiable at $x = a$ if and only if $F(x)$ is uniformly continuous on some punctured interval $I - a$.*

If $f'(a)$ exists, then we may extend $F(x)$ to the continuous function

$$G(x) = \begin{cases} F(x), & x \neq a \\ f'(a), & x = a. \end{cases}$$

Since $G(x)$ is then uniformly continuous on any closed interval I containing a, it follows that $F(x)$ is uniformly continuous on $I - a$.

Suppose, conversely, that $F(x)$ is uniformly continuous on $I - a$, and let $\{x_n\} \in I - a$ be any sequence which converges to a. The sequence $\{x_n\}$ is then a Cauchy sequence and, since $F(x)$ is uniformly continuous, the sequence $\{F(x_n)\}$ is also a Cauchy sequence. By the completeness of the real numbers, there exists a number L such that $F(x_n) \to L$. Furthermore, L does not depend on the choice of $x_n \to a$. Indeed, suppose $\{y_n\} \in I - a$ is another sequence converging to a and let $F(y_n)$ converge to L'. Then the sequence $\{z_n\} \in I - a$ defined by

$$z_n = \begin{cases} x_{(n+1)/2}, & \text{if } n \text{ is odd} \\ y_{n/2}, & \text{if } n \text{ is even} \end{cases}$$

also converges to a, and $L = \lim_{n\to\infty} F(x_n) = \lim_{n\to\infty} F(z_n) = \lim_{n\to\infty} F(y_n) = L'$. This independence of L on the choice of $x_n \to a$ means that $L = \lim_{x\to a} F(x) = \lim_{x\to a}(f(x) - f(a))/(x - a) = f'(a)$.
