$$= \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & mk_1 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & mk_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & mk_n \end{vmatrix} = m \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & k_1 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & k_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & k_n \end{vmatrix},$$

which is a multiple of m.

Euler's Constant

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The purpose of this note is to show that the existence of the limit

$$\lim_{n\to\infty}\left(1 + \frac{1}{2} + \cdots + \frac{1}{n-1} - \ln n\right)$$

(Euler's constant $\gamma = 0.57721566\ldots$) is an immediate consequence of

$$\left(1 + \frac{1}{n}\right)^n < e < \left(1 + \frac{1}{n}\right)^{n+1}$$

and the fact that $\ln x$ is an increasing function of x.

Since $\left(1 + \frac{1}{n}\right)^n < e$ is equivalent to $1 < \frac{e^{1/n}}{(n+1)/n}$, and $\left(1 + \frac{1}{n}\right)^{n+1} > e$ implies that $e^{1/n(n+1)} > \frac{e^{1/n}}{(n+1)/n}$, we readily obtain

$$\begin{aligned} 0 < \gamma_n &= 1 + \frac{1}{2} + \cdots + \frac{1}{n-1} - \ln n = \ln\left[\frac{e^1}{2/1}\cdot\frac{e^{1/2}}{3/2}\cdot\cdots\cdot\frac{e^{1/(n-1)}}{n/(n-1)}\right] \\ &< \ln\left[\frac{e^1}{2/1}\cdot\frac{e^{1/2}}{3/2}\cdot\cdots\cdot\frac{e^{1/(n-1)}}{n/(n-1)}\cdot\frac{e^{1/n}}{(n+1)/n}\right] \\ &= 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln(n+1) \\ &= \gamma_{n+1} \\ &< \ln\left[e^{1/1\cdot 2}\cdot e^{1/2\cdot 3}\cdot\cdots\cdot e^{1/n(n+1)}\right] \\ &= \frac{n}{n+1} \\ &< 1. \end{aligned}$$
