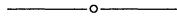


$$= \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & mk_1 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & mk_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & mk_n \end{vmatrix} = m \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n-1} & k_1 \\ a_{21} & a_{22} & \cdots & a_{2n-1} & k_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & k_n \end{vmatrix},$$

which is a multiple of  $m$ .



### Euler's Constant

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The purpose of this note is to show that the existence of the limit

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{2} + \cdots + \frac{1}{n-1} - \ln n \right)$$

(Euler's constant  $\gamma = 0.57721566 \dots$ ) is an immediate consequence of

$$\left( 1 + \frac{1}{n} \right)^n < e < \left( 1 + \frac{1}{n} \right)^{n+1}$$

and the fact that  $\ln x$  is an increasing function of  $x$ .

Since  $\left( 1 + \frac{1}{n} \right)^n < e$  is equivalent to  $1 < \frac{e^{1/n}}{(n+1)/n}$ , and  $\left( 1 + \frac{1}{n} \right)^{n+1} > e$  implies that  $e^{1/n(n+1)} > \frac{e^{1/n}}{(n+1)/n}$ , we readily obtain

$$\begin{aligned} 0 < \gamma_n &= 1 + \frac{1}{2} + \cdots + \frac{1}{n-1} - \ln n = \ln \left[ \frac{e^1}{2/1} \cdot \frac{e^{1/2}}{3/2} \cdot \cdots \cdot \frac{e^{1/(n-1)}}{n/(n-1)} \right] \\ &< \ln \left[ \frac{e^1}{2/1} \cdot \frac{e^{1/2}}{3/2} \cdot \cdots \cdot \frac{e^{1/(n-1)}}{n/(n-1)} \cdot \frac{e^{1/n}}{(n+1)/n} \right] \\ &= 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln(n+1) \\ &= \gamma_{n+1} \\ &< \ln \left[ e^{1/1 \cdot 2} \cdot e^{1/2 \cdot 3} \cdot \cdots \cdot e^{1/n(n+1)} \right] \\ &= \frac{n}{n+1} \\ &< 1. \end{aligned}$$

