An advertising opportunity. In Spring 1997 Macy's department store featured just such a lamp in an advertisement, but the number of levels of illumination was not mentioned. Needing a floor lamp, I could not resist purchasing this tangible instance of my oft-used example. Its accompanying printed materials were silent on the count, so I wrote to the manufacturer (Stiffel), who sent me copies of several pages from their catalog. Among the units with evocative names such as "Pharmacy Lamp" and "Club Floor Lamp," my purchase was listed unsentimentally as the "6-Way Floor Lamp." The advertising copy not only missed a chance to endow the lamp with a fetching name such as "Fireworks" or "Candlepower Cornucopia," its six ways fell below the global minimum of classroom responses over the decades. The students in my course on discrete mathematics were amused by this firsthand evidence of real-world innumeracy, and perhaps they were happy to see that their instincts were closer to the mark than those of a commercial outfit. This example aroused more interest than any in years.

References

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A Geometric View of a Vector Identity

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The vector identity

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$$

is usually proved by algebraic methods. However, by using a basis for 3-space that is adapted to the problem, we can give a simple geometric proof. This proof makes it clear why the vector A appears on the right in the form of inner products, and it explains the signs of the two terms.

Let's write the decomposition $B = \mathrm{proj}_C B + (B - \mathrm{proj}_C B)$ of B into its components parallel to C and orthogonal to C as simply $B = B_{\|} + B_{\perp}$. Similarly, let $C = C_{\|} + C_{\perp}$ denote the decomposition of C into its components parallel to B and orthogonal to B. Let θ be the angle between B and C. If $\theta \geq \pi/2$, then the angle between B and $B_{\perp}$, and also the angle between C and $C_{\perp}$, is $\theta - \pi/2$. On the other hand, if $\theta < \pi/2$, this angle is $\pi/2 - \theta$. In either case, we have

$$\frac{|B_{\perp}|}{|B|} = \frac{|C_{\perp}|}{|C|} = \cos\left(\theta - \frac{\pi}{2}\right). \tag{1}$$

Since both sides of the identity $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$ are clearly 0 if C is a multiple of B, we may assume that B and C are linearly independent. Then the vectors $B_{\perp}$, $C_{\perp}$, and $B \times C$ are linearly independent, so we may write $A = \alpha B_{\perp} + \beta C_{\perp} + \gamma(B \times C)$ for suitable scalars α, β, γ. Then

$$A \times (B \times C) = (\alpha B_{\perp} + \beta C_{\perp}) \times (B \times C). \tag{2}$$

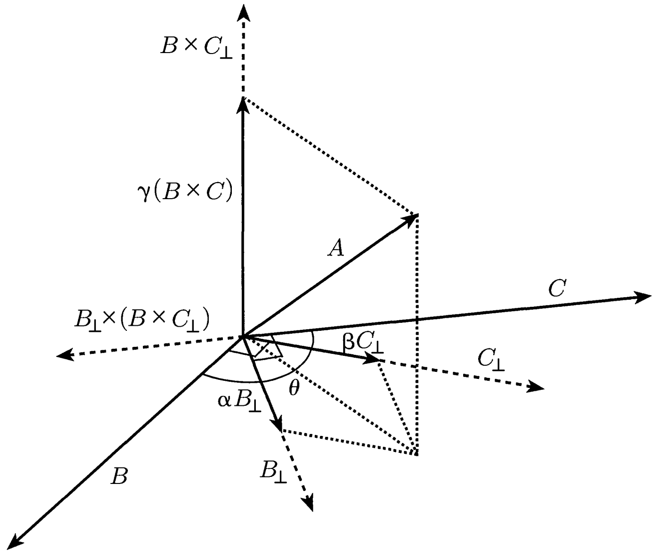

Figure 1

Notice that, as Figure 1 indicates, $B \times C = B_\perp \times C = B \times C_\perp$; also, $|B_\perp \times C| = |B_\perp|\,|C|$ and $|B \times C_\perp| = |B|\,|C_\perp|$. Moreover, $A \cdot B = \alpha B_\perp \cdot B$ and $A \cdot C = \beta C_\perp \cdot C$. By the right-hand rule, we find

$$\alpha B_\perp \times (B \times C) = \alpha B_\perp \times (B \times C_\perp) = \alpha\,|B_\perp|\,(|B|\,|C_\perp|)\left(\frac{-C}{|C|}\right) \tag{3}$$

and

$$\beta C_\perp \times (B \times C) = \beta C_\perp \times (B_\perp \times C) = \beta\,|C_\perp|\,(|B_\perp|\,|C|)\left(\frac{B}{|B|}\right), \tag{4}$$

where $B/|B|$ and $C/|C|$ are unit vectors with the directions of B and C. The key observation is that $B_\perp \times (B \times C_\perp)$ has direction opposite that of C, but $C_\perp \times (B_\perp \times C)$ has the same direction as B.

Recalling equation (1), we can write the right sides of (3) and (4) as

$$-\alpha\,|B_\perp|\,|B| \cos\left(\theta - \frac{\pi}{2}\right) C = -\left(\alpha B_\perp \cdot B\right) C = -(A \cdot B)C \tag{5}$$

and

$$\beta\,|C_\perp|\,|C| \cos\left(\theta - \frac{\pi}{2}\right) B = \left(\beta C_\perp \cdot C\right) B = (A \cdot C)B. \tag{6}$$

The identity $A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$ then follows from equation (2), using (3) and (4) and then (5) and (6).

This proof is a nice example of the maxim that a result in linear algebra often becomes apparent when one uses a basis that is properly adapted to the situation.