References

1. M. Marcus, Determinants of sums, *The College Mathematics Journal* 21 (1990) 130–135.
2. B. Wardlaw, *The College Mathematics Journal* 22 (1991) 70.

The Probability that $(a,b)=1$

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Note (by Henry L. Alder): To ask what the probability is for two positive integers a and b to have the greatest common divisor 1 is a natural question and was raised by students in my beginning number theory class in the Fall quarter of 1989. I answered it and gave a traditional, rather lengthy proof calling on considerable prior knowledge of number theory. The above named two students (the first a 16-year old freshman, the second a 17-year old high school student) came up with the following much shorter proof. I encouraged them to share it with the readers of the *College Mathematical Journal* who might be asked the same question in their classes.

Let g be the greatest common divisor of two integers a and b, that is $g=(a,b)$ and let p be the probability* that $g=1$. We will first show that the probability that $g=n$ for $n=1,2,\ldots$ is p/n^2.

Clearly the probability that n divides both a and b is $1/n^2$. The probability that no proper multiple of n divides both a and b is the same as the probability that $(a/n, b/n)=1$, which is p. Thus, the probability that $g=n$ is p/n^2.

The sum of the probabilities that $g=n$ for $n=1,2,\ldots$ must equal 1, so that

$$\sum_{n=1}^{\infty}\frac{p}{n^2}=1.$$

Solving for p, we obtain

$$p=\frac{1}{\sum_{n=1}^{\infty}\frac{1}{n^2}}=\frac{1}{\frac{\pi^2}{6}}=\frac{6}{\pi^2}.$$

*The probability refers, of course, to the

$$\lim_{N\to\infty}\frac{\#\{(a,b):(a,b)=1, 1\le a\le N, 1\le b\le N\}}{\#\{(a,b): 1\le a\le N, 1\le b\le N\}}.$$

That this limit exists is well known. (See, for example, A. M. Yaglom and I. M. Yaglom, *Challenging Mathematical Problems with Elementary Solutions*, Vol. I, Holden-Day, San Francisco, 1964, pp. 202–4).