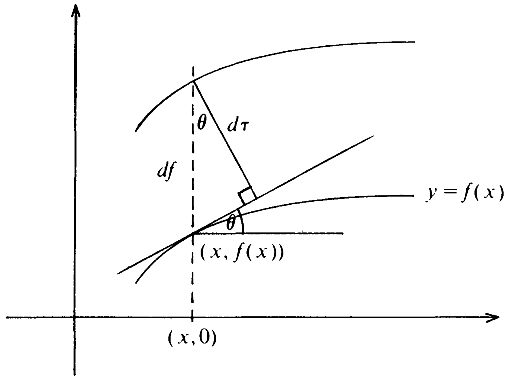

Figure 3

Beginning with the formula for the volume of the corresponding solid of revolution

$$V=\int_a^b \pi\,[f(x,\tau)]^2\,dx, \tag{12}$$

taking the derivative with respect to τ and evaluating at $\tau=0$ yields

$$\begin{aligned}\left.\frac{dV}{d\tau}\right|_{\tau=0} &= \int_a^b 2\pi f(x,\tau)\left.\frac{\partial f(x,\tau)}{\partial\tau}\right|_{\tau=0} dx\\ &= \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx.\end{aligned} \tag{13}$$

Thus, (13) and (4) yield the well-known formula

$$\text{Lateral Surface Area} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

for a surface of revolution.

$\mathbf{Sin^2}\,x$: A Sheep in Wolf's Clothing

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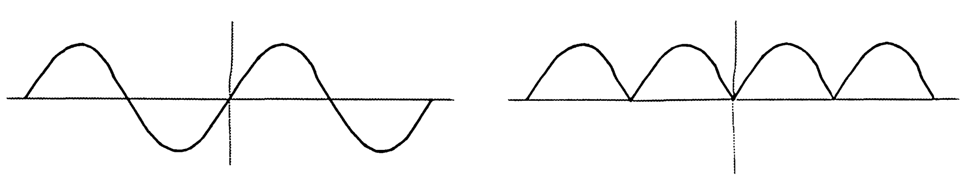
Students occasionally must construct the graph of $y=\sin^2 x$. Noting that both squaring and taking absolute values produce positive numbers as results, they frequently construct the graph of $y=|\sin x|$ instead, by taking the elementary sine curve and "flipping it up" to get positive values only:



Since this procedure gives an accurate representation of the periodicity of the function as well as its sign, many students accept it as a good picture of the graph of the function $y = \sin^2 x$.

But this picture of the function misses one of its important properties: it is infinitely differentaible, and hence its derivatives are continuous. How, then, does it acquire those cusps at each intersection with the x-axis? Such cusps are usually signs that something is wrong with the function's derivative.

The answer is, of course, that there are no such cusps in the graph of the function. Students generally react to this criticism of their graph in the standard way by rounding the cusps slightly, making them points of tangency of the x-axis to the curve. But how extensive is this "rounding"? How far from the alleged cusps does it influence the curve?

Most students (and some faculty!) are surprised by an identity they knew all along:

$$\cos 2x = 1 - 2\sin^2 x,$$

or

$$\sin^2 x = (1/2)(1 - \cos 2x),$$

which shows that the graph $y = \sin^2 x$ is a simple variant of the graph $y = \cos 2x$. See [David Cohen, *Precalculus*, 2nd ed., West, St. Paul, p. 399 #30]. It's just an ordinary sinusoidal curve, wtih period π and amplitude 1/2, which has been translated up half a unit.

My thanks to Eric Lander and Gregory Greene, who, in two comments over twelve years, helped me to see the obvious.

To See Ourselves as Others See Us

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If Charles J. Sykes headed the employment office of a big research university, his advertisements to fill faculty openings would read something like that.

Larry Gordon, A Muckraker's View of the Privileges of Academic Life, *The Los Angeles Times/Part V*, March 7, 1989, which is a review of Charles J. Sykes, *Profscam: Professors and the Demise of Higher Education*, Regnery Gateway