Example 5. Find the inverse Laplace transform for $1/s(s^2+1)(s^2+2)$.

Solution. Since

$$\begin{aligned}\frac{1}{s(s^2+1)(s^2+2)} &= \frac{1}{s}\left(\frac{1}{s^2+1}-\frac{1}{s^2+2}\right)\\ &= \frac{s}{s^2(s^2+1)}-\frac{s}{s^2(s^2+2)}\\ &= \frac{s}{s^2}-\frac{s}{s^2+1}-\frac{1}{2}\left(\frac{s}{s^2}-\frac{s}{s^2+2}\right)\\ &= \frac{1}{s}-\frac{s}{s^2+1}-\frac{1}{2}\cdot\frac{1}{s}+\frac{1}{2}\left(\frac{s}{s^2+2}\right)\\ &= \frac{1}{2}\cdot\frac{1}{s}-\frac{s}{s^2+1}+\frac{1}{2}\left(\frac{s}{s^2+2}\right),\end{aligned}$$

we have

$$\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+1)(s^2+2)}\right\}=\frac{1}{2}-\cos t+\frac{1}{2}\cos\sqrt{2}\,t.$$

Exercise. Obtain the analogous identity for

$$\frac{1}{(p(x)+a)(p(x)+b)(p(x)+c)}.$$

Differentiation via Partial Fractions: A Case against CAS

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Manipulations versus thinking: two views. On an autumnal day in 1989 I was introduced for the first time to CAS. Wade Ellis, Jr. presented a workshop on True Basic Calculus at the Second Technology Conference [**1**]. After seeing how the program worked I typed in the function $G(x)=\dfrac{-x}{x^2+x-1}$ and pressed the key that yielded derivatives. The answer appeared on the screen in about 15 seconds. Pressing several more keys yielded, almost immediately, the second, third, and fourth derivatives. The entire process took about one minute. This was impressive. My colleagues, when sampled, took between three and ten minutes to calculate these derivatives, and some students whom I asked took between five and thirty minutes!

The argument that CAS software performs manipulations, leaving the instructor more time to teach concepts, now seemed cogent, and without a reasonable counterargument.

The day before, however, in a keynote lecture, former MAA president Lynn Steen asked if computer packages for calculus won't lead to much meaningless

calculation, in a way that many statistics packages are currently used. He suggested that we seek styles of instruction that minimize *inane use of mathematical software* [**1**,
p. 19].

I left the conference with these two opposing views in mind. The purpose of this note is to support Lynn Steen's warning and to suggest that the proper teaching of calculus may not need more software but, instead, more techniques and better theoretical applications.

Patterns versus manipulations. Returning to our function $G(x)$ consider the problem of finding a *pattern* in its successive derivatives. Equivalently, consider the problem of finding a closed formula for $G^{(n)}(x)$, the nth derivative of $G(x)$. A CAS can quickly give the first 10 derivatives of $G(x)$ (some of which will occupy a whole screen!), but the software cannot discern even an elementary pattern.

In this case, even with the first 10 derivatives computed, a human user cannot find a pattern either. The reader is invited to try for himself (herself)! We conclude in this case that the manipulative ability of CAS encourages the student to *waste* a great deal of time watching the computer perform long meaningless manipulations which do not suggest the ideas necessary to find a general pattern.

An elegant solution. The technical, yet elegant, details of finding a pattern in the derivatives of $G(x)$ are the following: A partial fraction expansion of $G(x)$ yields

$$G(x) = \frac{A}{x-\alpha} + \frac{B}{x-\beta} \quad \text{with } A = \frac{\alpha}{\beta-\alpha} \quad \text{and} \quad B = \frac{-\beta}{\beta-\alpha}$$

where α and β denote the two roots of the equation $x^2 + x - 1 = 0$. It immediately follows that

$$G^{(n)}(x) = \frac{(-1)^n n! A}{(x-\alpha)^{n+1}} + \frac{(-1)^n n! B}{(x-\beta)^{n+1}}.$$

This solves the problem of finding a closed formula for the nth derivative of $G(x)$. (G.H. Hardy's classic [**2**, p. 223] is a source for differentiation via partial fractions.)

The solution—more techniques? In summary, to find a general pattern for $G^{(n)}(x)$, a new technique must be introduced: *partial fraction expansions for differentiation*. Appreciation of specific techniques can be achieved by skillful choice of fresh examples that can be solved only by these techniques.

A calculus text appears as a collection of manipulations if its computational exercises repetitively request standard techniques that do not challenge the student. The current remedy is to fill calculus books with a variety of real world applications that challenge the student's modeling ability. We suggest also that a calculus text can be made interesting and challenging precisely by eliciting new applications of existing techniques.

References

1. F. Demana, B. Waits, and J. Harvey, eds., *Proceedings of the Second Annual Conference on Technology on Collegiate Mathematics*, Addison-Wesley, Reading, MA, 1991.
2. G. H. Hardy, *A Course of Pure Mathematics*, 10th ed., Cambridge University Press, London-New York-Melbourne, 1952.

Graphs and Derivatives of the Inverse Trig Functions

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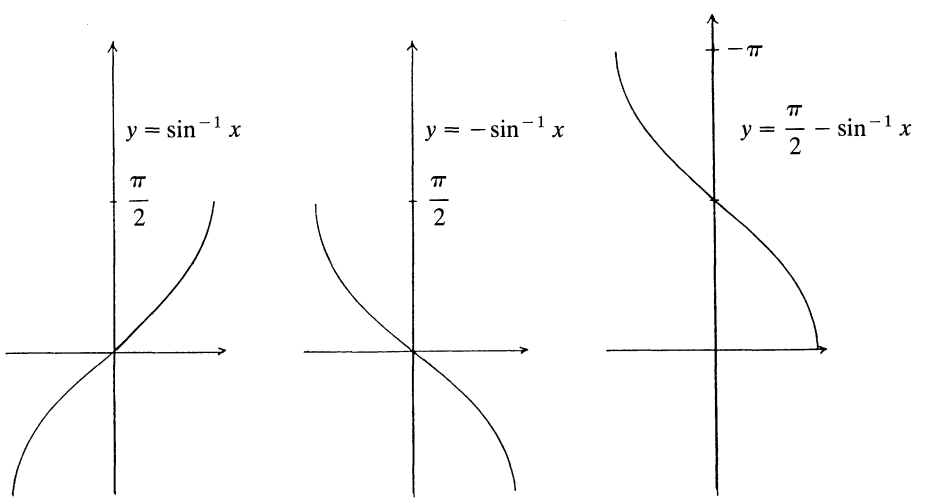
In a calculus course the differentiation formulas for the inverse trig functions are derived by implicit differentiation (at least for two or three of the functions). To avoid tedious repetition, the formulas for the others are merely stated, and their proofs omitted or left as an exercise.

The approach outlined below gives half of the differentiation formulas as immediate consequences of the others. After the inverse functions are defined, it is established that $f^{-1}(x)$ and $\mathrm{cof}^{-1}(x)$ are always complementary when f is sine, tangent or secant. Along the way, there is an opportunity to use graphics (computer-driven or otherwise) and strengthen the students' grasp of the elementary geometry of reflections and translations. And the whole process takes less classroom time than the conventional method!

The archetypical demonstration:



In the figure, the first graph is reflected in the horizontal axis to produce the second; the latter is then translated $\pi/2$ units upward to yield the third (which is evidently congruent to the graph of $y = \cos^{-1} x$). This establishes that $\sin^{-1} x + \cos^{-1} x = \pi/2$. We can now differentiate to discover that $D_x \cos^{-1} x = -D_x \sin^{-1} x$.

The demonstrations for $\tan^{-1}$ and $\sec^{-1}$ require practically no change from the above.