CLASSROOM CAPSULES

EDITOR

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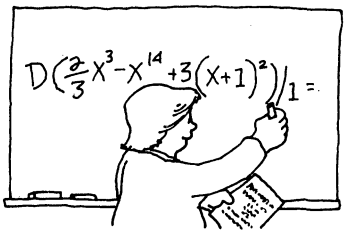

A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Nazanin Azarnia.

Calculus in the Brewery

Susan Jane Colley, Oberlin College, Oberlin, OH 44074

When I was a thirsty graduate student, I went to my corner market to buy some beer. Many cans of many brands were neatly stacked in the refrigerated case. One set of cans stood out, for they appeared to give much better value for the money (i.e., more beer); see Figure 1. To my surprise, however, all the beers were packaged in the standard 12 fl. oz. amounts. What could explain this well-marketed optical illusion?

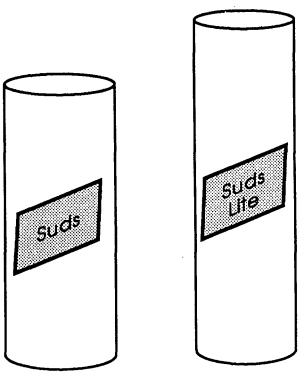

Figure 1
Which would you buy?

Multivariable calculus provides the answer. A standard beer can is, essentially, a cylinder of radius $r \approx 1$ in. and height $h \approx 5$ in. The volume V of such a cylinder is

$$V(r,h) = \pi r^2 h$$

and therefore the total differential of the volume is given by

$$\begin{aligned} dV &= \frac{\partial V}{\partial r}\,\Delta r + \frac{\partial V}{\partial h}\,\Delta h \\ &= 2\pi r h\,\Delta r + \pi r^2\,\Delta h. \end{aligned}$$

Moreover, the quantity dV approximates ΔV (the exact change in volume) when Δr and Δh are small.

Suppose that the cylinder is a standard beer can with dimensions as given above. Then we have

$$\Delta V \approx dV = \pi(10\Delta r + \Delta h).$$

This result expresses the fact that, at these dimensions, the volume is approximately 10 times more sensitive to changes in radius than to changes in height. Hence one can make a beer can appear to be larger than the standard one by decreasing the radius slightly (so little as to be hardly noticeable) and increasing the height so no change in volume results. The sensitivity analysis above shows that even a tiny decrease in radius forces an appreciable compensating increase in height.

Smart people, those marketing and sales types.

An Optimization Oddity

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Suppose f is a differentiable function ($f(x) \geq 0$) on the interval $[0, t]$ ($t > 0$) such that d^2f/dx^2 exists, is different from zero, and does not change sign on the same interval. In addition, without loss of generality, we assume $d^2f/dx^2 < 0$ for all $x \in [0, t]$. Consider a tangent m to f at the point $(a, f(a))$, $a \in [0, t]$. The problem is to find the place $x = a$ such that the tangent m minimizes the area $\Phi(a)$ of the region determined by $m, f, x = 0$, $x = t$. (See Figure 1.) Surprisingly, the result is independent of f!

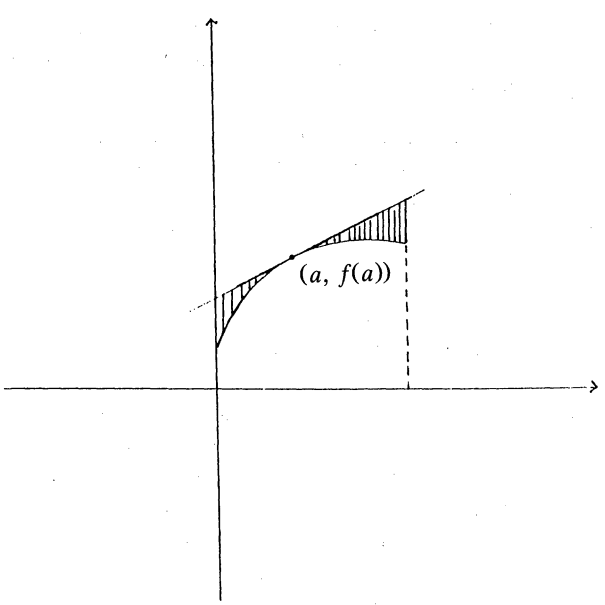

Figure 1