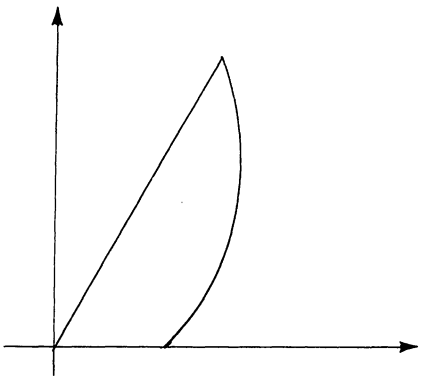

Figure 3

the shape of the curved blade. But further simplification yields $\mathrm{Arctan}(y/x) - \ln\sqrt{x^2+y^2} = C$, which suggests... polar coordinates! The polar form of the general solution is the logarithmic spiral $\theta - \ln r = C$ or $r = ke^{\theta}$.

Since the polar point $(5, \pi/3)$ is on the curve, $k \cong 1.8$ and the solution to our design problem is

$$r = 1.8e^{\theta} \quad \text{for } 0 \le \theta \le \pi/3.$$

Figure 3 shows a plot of the actual blade shape, which could be scaled to build a template for the blade manufacturing process.

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A Visual Proof of Eddy and Fritsch's Minimal Area Property

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In a recent Classroom Capsule, R. H. Eddy and R. Fritsch [*CMJ* 25 (1994) 227–228] established a remarkable fact: For any convex curve Γ on the interval AB, the point at which the tangent minimizes the shaded area in Figure 1 is the midpoint C of AB.

This fact is all the more interesting because it has a very simple geometric proof that does not use calculus.

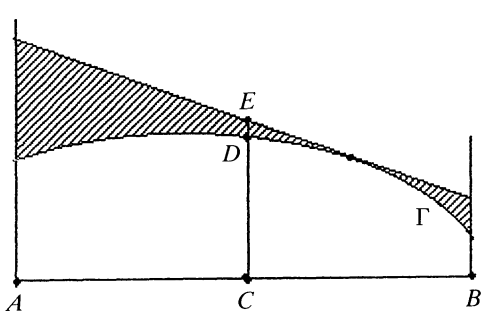

Figure 1

Proof. As the area under the curve Γ is constant, the shaded area will be minimized when the area of the trapezoid is minimized. The area of the trapezoid is the base times its height at the midpoint, so it suffices to minimize that height, CE. This will occur when $E = D$, that is, the tangent at D is taken. ■

This proof suggests the following generalization. Let Σ be a convex surface over some convex region R in the (x, y) plane. Assume Σ has a tangent at each point in R. Then one can seek the point in R at which the tangent minimizes the volume between the tangent plane and the surface (see Figure 2). As the volume under the surface is constant, again it will suffice to minimize the volume under the tangent plane. This volume is the area of R times the height above the centroid C of R (a nice exercise with the definition of the centroid using double integrals). Thus we see that the minimum occurs precisely at the centroid.

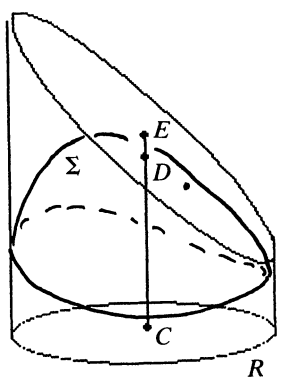

Figure 2

This fact can also be shown by a computation in the style of Eddy and Fritsch. It turns out to be an interesting exercise, which uses at the very end the fact that the Hessian is nonsingular—that is $f_{xx}f_{yy} - f_{xy}f_{yx}$ is nonzero, a consequence of the convexity of f. Our earlier argument shows, however, that the region R need not be convex, provided the centroid lies inside R. It is sufficient that the surface have a tangent at each point in R and that the surface always lie below (or always above) these tangents.

Acknowledgment. I thank Pat Stewart and Tony Thompson for helpful remarks.

Editors' note. The idea for the three-dimensional generalization above appeared in an exercise with solution in R. C. Buck, *Advanced Calculus*, 3rd ed., McGraw-Hill, New York, 1978, p. 541.