representing this derivation appears on Stevin's tombstone as an epitaph, and concludes: "If you have such an epitaph on your tombstone, you're doing fine."

Regarding the pulley problem, the existence of a geometric solution based on statics provides reason for speculating that Archimedes could have solved this problem, and perhaps did solve similar ones. Indeed, although Archimedes did not have vectors as we know them for decomposing forces, his work on tackles certainly reflects sufficient insight, if not for tackling the general case, then certainly for cases of this particular kind where pulleys induce symmetry. The book *La Rivoluzione Dimenticata* by Lucio Russo, as presented in a recent review by Sandro Graffi **[1]**, certainly provides food for thought about how much scientific knowledge was available in antiquity.

References

1. Sandro Graffi, Review of *La Rivoluzione Dimenticata* by Lucia Russo, *Notices of the AMS* 45 (1998) #5 601–605.
2. A. J. Hahn, Two historical applications of calculus, this JOURNAL 29 (1998) #2 93–103.

The Derivative of Sin θ

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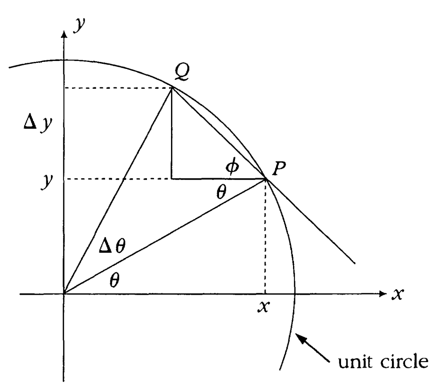
An unusual argument supporting the derivative formula for $\sin\theta$ can be offered using Figure 1.

Since the distance from P to Q is $2\sin(\Delta\theta/2)$ (when Δy and $\Delta\theta$ are positive quantities as in the figure), we have $\sin\phi = \Delta y/2\sin(\Delta\theta/2)$. Also, we see that as $\Delta\theta$ approaches 0, the line PQ approaches the tangent line to the circle at P so ϕ



approaches $\pi/2 - \theta$. Thus

$$\frac{d(\sin\theta)}{d\theta} = \frac{dy}{d\theta} = \lim_{\Delta\theta\to 0^+} \frac{\Delta y}{\Delta\theta} = \lim_{\Delta\theta\to 0^+} \frac{\Delta y}{2\sin\frac{\Delta\theta}{2}} \frac{2\sin\frac{\Delta\theta}{2}}{\Delta\theta}$$

$$= \lim_{\Delta\theta\to 0^+} \sin\phi = \cos\theta.$$

The companion formula for the derivative of $\cos\theta$ can be obtained in the same way.

Measuring the Curl of Paper

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The following problem came to us from an employee of a paper manufacturer. It can be posed to an introductory trigonometry class and provides a good illustration of how mathematics can be used in many fields.

In the paper manufacturing industry, one indicator of the quality of paper produced is its *curl*. Paper with too much curl for its intended use is considered defective and is discarded. To measure the curl, a paper sample is held up to a chart that shows circular arcs of different curvature, each arc associated with a number d which is the *depth* of the arc (Fig. 1). The depth of the matching arc is recorded as the curl of the sample. For example, an $8\frac{1}{2}$ by 11 inch sheet of paper would be held up at the midpoint of a long edge and the opposite edge would be compared with the arcs on the chart and matched to one of them to determine the curl of the paper. The allowable curl for the 11 inch sample would differ from the allowable curl of a

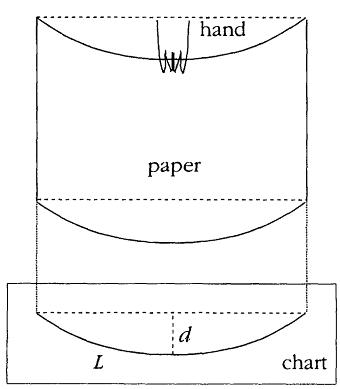

Figure 1. Illustration of the way in which curl is measured. The paper is held at the midpoint of one end and the dangling free end is compared to a chart lying flat on the table. The *depth* of the arc is d.