

$$\begin{aligned}
2A &= 1 \\
3A + 2B &= 0 \\
2B + 2C &= 0 \\
C + 2D &= 0.
\end{aligned}$$

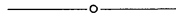
Since

$$A = 1/2, \quad B = -3/4, \quad C = 3/4, \quad D = -3/8,$$

we have

$$\int x^3 e^{2x} dx = \left(\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right)e^{2x} + K.$$

Other integrals, such as $\int e^{ax} \sin bx dx$, which require repeated integration by parts can also be evaluated more efficiently using this technique.



Relating Differentiability and Uniform Continuity

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For a continuous function $f: R \rightarrow R$ define function $F(x) = (f(x) - f(a))/(x - a)$ where a denotes some fixed real number. Clearly F is a continuous function defined on $I - a$, where I is any interval containing a . We wish to prove the following result.

The function $f(x)$ is differentiable at $x = a$ if and only if $F(x)$ is uniformly continuous on some punctured interval $I - a$.

If $f'(a)$ exists, then we may extend $F(x)$ to the continuous function

$$G(x) = \begin{cases} F(x), & x \neq a \\ f'(a), & x = a. \end{cases}$$

Since $G(x)$ is then uniformly continuous on any closed interval I containing a , it follows that $F(x)$ is uniformly continuous on $I - a$.

Suppose, conversely, that $F(x)$ is uniformly continuous on $I - a$, and let $\{x_n\} \in I - a$ be any sequence which converges to a . The sequence $\{x_n\}$ is then a Cauchy sequence and, since $F(x)$ is uniformly continuous, the sequence $\{F(x_n)\}$ is also a Cauchy sequence. By the completeness of the real numbers, there exists a number L such that $F(x_n) \rightarrow L$. Furthermore, L does not depend on the choice of $x_n \rightarrow a$. Indeed, suppose $\{y_n\} \in I - a$ is another sequence converging to a and let $F(y_n)$ converge to L' . Then the sequence $\{z_n\} \in I - a$ defined by

$$z_n = \begin{cases} x_{(n+1)/2}, & \text{if } n \text{ is odd} \\ y_{n/2}, & \text{if } n \text{ is even} \end{cases}$$

also converges to a , and $L = \lim_{n \rightarrow \infty} F(x_n) = \lim_{n \rightarrow \infty} F(z_n) = \lim_{n \rightarrow \infty} F(y_n) = L'$. This independence of L on the choice of $x_n \rightarrow a$ means that $L = \lim_{x \rightarrow a} F(x) = \lim_{x \rightarrow a} (f(x) - f(a))/(x - a) = f'(a)$.

