indicates that $R_n(0) = 0$ and $R_n^{(k)}(0) = 0$ for each $1 \le k \le n$, thus (2) implies the claim.

So far we have sought to obtain the Taylor polynomials for a rational function f at the origin. For a point a other than the origin, we simply make the change of variables $z = x - a$ and find the Taylor polynomials of this new function at the origin.

These ideas make a good introduction to Taylor polynomials for general functions. I begin by asking my students to use the division algorithm to calculate the Taylor polynomials for a simple function like $f(x) = 1/(1-x)$. Next the students calculate Taylor polynomials about various base points and of various degrees for other rational functions. Students can then graph the original functions and corresponding Taylor polynomials to discover that these polynomials are good approximations of the functions. They might also graph the remainders R_n and their derivatives to discover that all of these functions vanish at the base point of the Taylor polynomial. The form of the remainder, (3), lets students rigorously prove their conjecture. Finally, using the idea that the value of the function and its first n derivatives must agree with the polynomial and its first n derivatives at the base point, students can define and calculate Taylor polynomials for more general functions.

Inquisitive students will want to know if the remainders for Taylor polynomials of general functions behave in the same fashion as the remainders for Taylor polynomials of rational functions; this leads directly to a discussion of Taylor's theorem.

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Pursuit and Regular *N*-gons

Michael J. Seery (seerymj.pl@usafa.af.mil), United States Air Force Academy Preparatory School, Colorado Springs, CO 80840

Many readers will recognize the following situation as a generalization of a problem that has appeared in many mathematical textbooks and collections of mathematical puzzles.

> One person is positioned at each vertex of a regular n-gon. Simultaneously, and at the same speed, each person walks directly toward the person who is located k vertices away in the counterclockwise direction. Find an equation that describes the shape of the path taken by each person.

(The case where $n = 4$ and $k = 1$ is the familiar situation.) To approach the generalized problem, begin by positioning a regular "unit" n-gon (i.e., the distance from the center to each vertex is 1) so that the center lies at the origin and a vertex V_0 lies at $(1, 0)$. Therefore, the polar coordinates of the vertex located k vertices counterclockwise from V_0 are $V_k(1, 2k\pi/n)$. If you were to chase someone located *more* than halfway around the n-gon, it would be wiser to move in the direction of $-\theta$; so we impose the restriction that $1 \le k < n/2$. If n is even, the case where $k = n/2$ makes sense and will be examined separately.

Let P_0 and P_k represent the people positioned initially at V_0 and V_k respectively. Due to symmetry, everyone will travel through the same angle during a given time interval, and at any point they will be the same distance from the center. Therefore, when the polar coordinates of P_0 are (r, θ), those of P_k are $(r, \theta + 2k\pi/n)$. The corresponding rectangular coordinates are $P_0(r\cos\theta, r\sin\theta)$ and $P_k\big(r\cos(\theta + 2k\pi/n), r\sin(\theta + 2k\pi/n)\big)$.

Since P_0 moves directly toward P_k, the line connecting their positions is also tangent to the path of P_0. Thus, the slope of the line connecting their positions is $dy/dx = (dy/d\theta)/(dx/d\theta)$; that is,

$$\frac{r\sin\left(\theta + \dfrac{2k\pi}{n}\right) - r\sin\theta}{r\cos\left(\theta + \dfrac{2k\pi}{n}\right) - r\cos\theta} = \frac{\left(\dfrac{dr}{d\theta}\right)\sin\theta + r\cos\theta}{\left(\dfrac{dr}{d\theta}\right)\cos\theta - r\sin\theta}.$$

Solving for $dr/d\theta$ and simplifying involves trigonometric identities and a bit of algebra or use of a computer algebra system such as *Mathematica*. Using each approach, I obtained $dr/d\theta = -r\tan(k\pi/n)$. This differential equation is easily integrated, using the initial condition $r(0) = 1$, to yield $r(\theta) = e^{-\theta\tan(k\pi/n)}$. Hence each path is a spiral that approaches the origin. The graphs in Figure 1 demonstrate the spiral paths for $n = 6$ and various values of k.

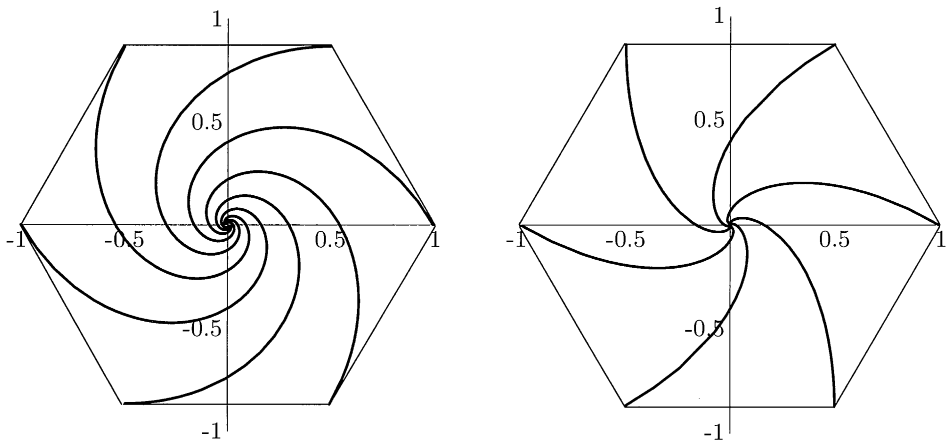

Figure 1. Pursuit paths in a hexagon. *Left*: $k = 1$; *right*: $k = 2$.

The following exercises offer some insight into the behavior of the spirals.

Exercise 1. Show that for any fixed values of k and θ, $\lim_{n\to\infty} r(\theta) = 1$. Thus as n increases, the spirals remain close to the unit circle through a larger angle, before converging to the origin.

Exercise 2. For a given value of n, show that, as k increases toward its maximum value of $n/2$, the spiral paths become flatter. What happens if n is even and $k = n/2$?

Exercise 3. Show that for any angle α, the arc length of the spiral path for $0 \le \theta \le \alpha$ is given by $s(\alpha) = \csc(k\pi/n)(1 - e^{-\alpha\tan(k\pi/n)})$. By evaluating $\lim_{\alpha\to\infty} s(\alpha)$, show that the distance traveled by each person is $\csc(k\pi/n)$.

Acknowledgment. I am grateful to my colleagues Bill Kiele, John Simeoni, and Suzanne Wheeler for their comments and to the referees for helpful suggestions.

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