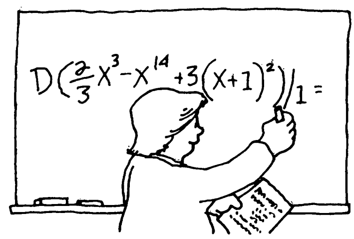
# CLASSROOM CAPSULES






A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Frank Flanigan.

## Timing Is Everything

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As every comedian knows, *timing is everything*. The same is true of an elementary differential equations course when establishing the method of variation of parameters to solve the second order linear ODE

$$y'' + p(x)y' + q(x)y = f(x). \tag{1}$$

Many textbooks (for example, [W. E. Boyce & R. C. diPrima, *Elementary Differential Equations*, 4th ed., Wiley, New York, 1986], [M. M. Guterman & Z. H. Nitecki, *Differential Equations: A First Course*, Saunders, Philadelphia, 1984]) establish the method of variation of parameters to solve (1) in somewhat the following manner.

Suppose that the general solution of the associated homogeneous ODE is

$$y_h(x) = c_1 y_1(x) + c_2 y_2(x).$$

For a particular solution of (1), we guess the function

$$y_p(x) = c_1(x) y_1(x) + c_2(x) y_2(x).$$

Next, to determine the functions $c_1(x)$ and $c_2(x)$, we substitute $y_p(x)$ into (1). To begin, we evaluate $y_p'(x)$:

$$y_p'(x) = c_1'(x) y_1(x) + c_1(x) y_1'(x) + c_2'(x) y_2(x) + c_2(x) y_2'(x).$$

We now make the simplifying assumption

$$c_1'(x) y_1(x) + c_2'(x) y_2(x) = 0,$$

etc., etc.

I claim that the simplifying assumption here is made *too early* in the procedure, for to most students it is *completely unmotivated* at this point. In fact, often, the only reason given for the simplifying assumption is that it reduces the amount of work required in determining $y_p''(x)$; unfortunately, this leaves many students wondering why none of the other terms was chosen to be zero instead, for that would just as well make evaluating $y_p''(x)$ easier.

I propose that, rather than making the simplifying assumption at this stage, it is far better to labor through evaluating $y_p''(x)$ in all its goriness. For doing so gives (after some rearrangement)

$$\begin{aligned}
f &= y_p'' + py_p' + qy_p \\
&= c_1[y_1'' + py_1' + qy_1] + c_2[y_2'' + py_2' + qy_2] \\
&\quad + [c_1''y_1 + c_1'y_1' + c_2''y_2 + c_2'y_2' + c_1'y_1' + c_2'y_2'] + p[c_1'y_1 + c_2'y_2] \\
&= [c_1''y_1 + c_1'y_1'] + [c_2''y_2 + c_2'y_2'] + p[c_1'y_1 + c_2'y_2] \\
&\quad + [c_1'y_1' + c_2'y_2'] \\
&= \frac{d}{dx}(c_1'y_1) + \frac{d}{dx}(c_1'y_2) + p[c_1'y_1 + c_2'y_2] + [c_1'y_1' + c_2'y_2'] \\
&= \frac{d}{dx}[c_1'y_1 + c_2'y_2] + p[c_1'y_1 + c_2'y_2] + [c_1'y_1' + c_2'y_2'].
\end{aligned}$$

Now, at this point, making the assumption that $c_1'y_1 + c_2'y_2 = 0$ is well motivated, for it eliminates almost all the terms in the last line *and* yields the system

$$\begin{cases} c_1'y_1 + c_2'y_2 = 0 \\ c_1'y_1' + c_2'y_2' = f \end{cases}$$

in one fell swoop!

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## Teaching the Laplace Transform Using Diagrams

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In this capsule, we present an approach to evaluating the Laplace transform and its inverse using commutative diagrams. The value of this technique is twofold. First, it presents a visual approach to a symbolic process. Second, it introduces the concept of commutative diagrams, which embody the important idea of distinct processes producing identical results.

The *Laplace transform* of a function $f(t)$ $(t > 0)$ is defined by

$$F(s) = \mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

for all values $s$ for which the integral is defined. We write $f(t) = \mathscr{L}^{-1}\{F(s)\}$ and call $f(t)$ the *inverse Laplace transform* of $F(s)$. If $a$ is a real constant and $n$ is a