absolute values as

$$\ln|x+2|,$$

which, however, does not indicate that, because of the initial condition, $x+2$ must have the same sign as -3. In general, without absolute values,

$$\int_a^b \frac{1}{x}\,dx = \ln\frac{b}{a}, \quad \textbf{provided that } ab > 0.$$

References

1. Ralph Palmer Agnew, *Differential Equations*, McGraw-Hill, New York and London, 1942.
2. R. Courant, *Differential and Integral Calculus*, Vol. 1, 2nd ed., translated by E. J. McShane, Blackie and Son, London and Glasgow, 1937.

A Shortcut in Partial Fractions

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The method of partial fractions is the basic technique for preparing rational functions for integration. It is also a useful tool for finding inverse Laplace transforms. This method enables us to write a rational function as a sum of simpler quotients that can be integrated directly or transformed easily by the inverse Laplace operator.

The basic technique to find partial fractions for a rational function is based on the method of undetermined coefficients. However, the computation involved in this method is often tedious. The following is a simple shortcut to expanding certain rational functions in partial fractions. We believe it is worthwhile to include this method in the texts.

Shortcut. Let $p(x)$ be a function and a, b distinct scalars. Then

$$\frac{1}{(p(x)+a)(p(x)+b)} = \left(\frac{1}{p(x)+a} - \frac{1}{p(x)+b}\right)\frac{1}{b-a}.$$

This is a special case of a general algebraic identity, and it is really useful. Let us look at some applications.

Example 1.

$$\frac{x}{(x^2+1)(x^2+4)} = \frac{x}{4-1}\left(\frac{1}{x^2+1} - \frac{1}{x^2+4}\right) = \frac{1}{3}\left(\frac{x}{x^2+1}\right) - \frac{1}{3}\left(\frac{x}{x^2+4}\right).$$

Example 2.

$$\int \frac{5x-3}{(x+1)(x-3)}\,dx = \int \frac{5(x+1)-8}{(x+1)(x-3)}\,dx$$

$$= \int \left(\frac{5}{x-3} - \frac{8}{(x+1)(x-3)} \right) dx$$

$$= \int \left(\frac{5}{x-3} - \frac{8}{4}\left(\frac{1}{x-3} - \frac{1}{x+1} \right) \right) dx$$

$$= \int \left(\frac{3}{x-3} + \frac{2}{x+1} \right) dx$$

$$= 3\ln|x-3| + 2\ln|x+1| + C.$$

Example 3.

$$\int \frac{1}{\left(x^2+x+\frac{5}{4}\right)\left(x^2+x+\frac{9}{4}\right)}\,dx = \int \left(\frac{1}{x^2+x+\frac{5}{4}} - \frac{1}{x^2+x+\frac{9}{4}} \right) dx$$

$$= \int \left(\frac{1}{\left(x+\frac{1}{2}\right)^2+1} - \frac{1}{\left(x+\frac{1}{2}\right)^2+2} \right) dx$$

$$= \tan^{-1}\left(x+\frac{1}{2}\right) - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\sqrt{2}}\right) + C.$$

Example 4.

$$\frac{1}{s^2(s-a)} = \frac{s+a}{s^2(s^2-a^2)}$$

$$= \frac{s+a}{a^2}\left(\frac{1}{s^2-a^2} - \frac{1}{s^2} \right)$$

$$= \frac{1}{a^2}\left(\frac{1}{s-a} \right) - \frac{1}{a^2}\left(\frac{s+a}{s^2} \right)$$

$$= \frac{1}{a^2}\left(\frac{1}{s-a} \right) - \frac{1}{a^2}\cdot\frac{1}{s} - \frac{1}{a}\cdot\frac{1}{s^2}.$$

Example 5. Find the inverse Laplace transform for $1/s(s^2+1)(s^2+2)$.

Solution. Since

$$\begin{aligned}\frac{1}{s(s^2+1)(s^2+2)} &= \frac{1}{s}\left(\frac{1}{s^2+1} - \frac{1}{s^2+2}\right)\\ &= \frac{s}{s^2(s^2+1)} - \frac{s}{s^2(s^2+2)}\\ &= \frac{s}{s^2} - \frac{s}{s^2+1} - \frac{1}{2}\left(\frac{s}{s^2} - \frac{s}{s^2+2}\right)\\ &= \frac{1}{s} - \frac{s}{s^2+1} - \frac{1}{2}\cdot\frac{1}{s} + \frac{1}{2}\left(\frac{s}{s^2+2}\right)\\ &= \frac{1}{2}\cdot\frac{1}{s} - \frac{s}{s^2+1} + \frac{1}{2}\left(\frac{s}{s^2+2}\right),\end{aligned}$$

we have

$$\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+1)(s^2+2)}\right\} = \frac{1}{2} - \cos t + \frac{1}{2}\cos\sqrt{2}\,t.$$

Exercise. Obtain the analogous identity for

$$\frac{1}{(p(x)+a)(p(x)+b)(p(x)+c)}.$$

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Differentiation via Partial Fractions: A Case against CAS

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Manipulations versus thinking: two views. On an autumnal day in 1989 I was introduced for the first time to CAS. Wade Ellis, Jr. presented a workshop on True Basic Calculus at the Second Technology Conference [**1**]. After seeing how the program worked I typed in the function $G(x) = \dfrac{-x}{x^2+x-1}$ and pressed the key that yielded derivatives. The answer appeared on the screen in about 15 seconds. Pressing several more keys yielded, almost immediately, the second, third, and fourth derivatives. The entire process took about one minute. This was impressive. My colleagues, when sampled, took between three and ten minutes to calculate these derivatives, and some students whom I asked took between five and thirty minutes!

The argument that CAS software performs manipulations, leaving the instructor more time to teach concepts, now seemed cogent, and without a reasonable counterargument.

The day before, however, in a keynote lecture, former MAA president Lynn Steen asked if computer packages for calculus won't lead to much meaningless