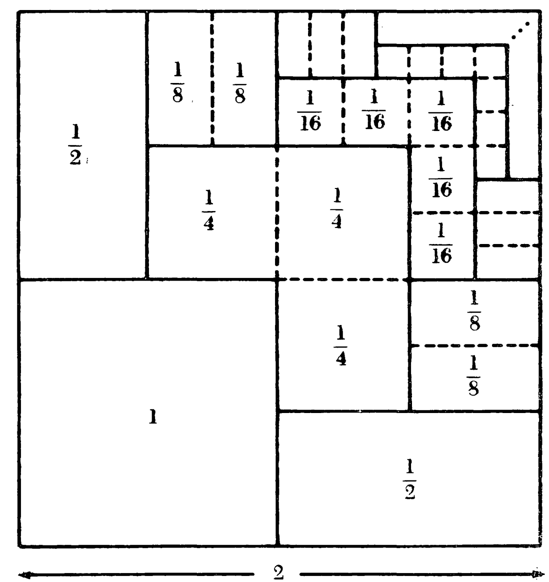
Proof without Words: Differentiated geometric series

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+4\left(\frac{1}{8}\right)+\cdots=4$$



$$1=4-3(1)$$
$$1+2\left(\frac{1}{2}\right)=4-4\left(\frac{1}{2}\right)$$
$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)=4-5\left(\frac{1}{4}\right)$$
$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+4\left(\frac{1}{8}\right)=4-6\left(\frac{1}{8}\right)$$
$$\vdots$$
$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+\cdots+n\left(\frac{1}{2}\right)^{n-1}=4-(n+2)\left(\frac{1}{2}\right)^{n-1}$$

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right)+\cdots=4$$

$$1+2r+3r^2+4r^3+\cdots=\left(\frac{1}{1-r}\right)^2, \qquad 0\le r<1$$

$$1=\left(\frac{1}{1-r}\right)^2-r(2-r)\left(\frac{1}{1-r}\right)^2$$

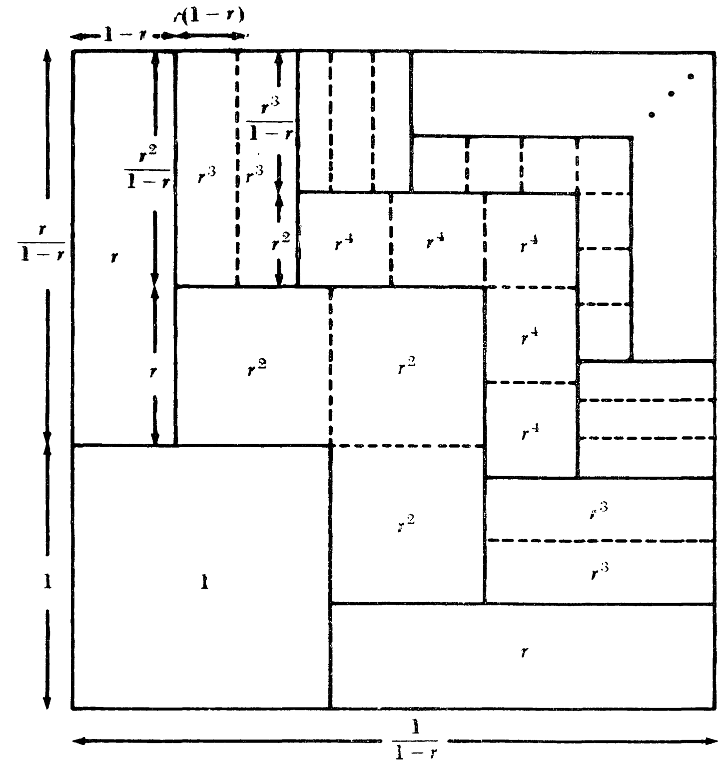
$$1+2r=\left(\frac{1}{1-r}\right)^2-r^2(3-2r)\left(\frac{1}{1-r}\right)^2$$

$$1+2r+3r^2=\left(\frac{1}{1-r}\right)^2-r^3(4-3r)\left(\frac{1}{1-r}\right)^2$$

$$\vdots$$

$$1+2r+3r^2+\cdots+nr^{n-1}=\left(\frac{1}{1-r}\right)^2-r^n(n+1-nr)\left(\frac{1}{1-r}\right)^2$$

$$1+2r+3r^2+\cdots=\left(\frac{1}{1-r}\right)^2$$



—Roger B. Nelsen
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