Proof Without Words: Pythagorean Triples and Factorizations of Even Squares

There is a one to one correspondence between Pythagorean triples and factorizations of even squares of the form \( p^2 = 2nm \).

\[
\begin{align*}
\text{If } & a^2 + b^2 = c^2, \quad \text{then } \quad (a + b - c)^2 = 2(c - a)(c - b). \\
\text{If } & p^2 = 2nm, \quad \text{then } \quad (p + n)^2 + (p + m)^2 = (p + n + m)^2.
\end{align*}
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EDITOR’S NOTE: Author Darryl McCullough, whose article Height and Excess of Pythagorean Triples appears in this issue, points out that another way to see the correspondence between factorizations of even integers and Pythagorean triples is through \((e, h)\)-coordinates: In the notation of his article, each factorization \( p^2 = 2nm \) corresponds to the two pairs \((p, m)\) and \((p, n)\), which represent a Pythagorean triangle and its mirror image.