A821. In the ring $\mathbf{Z}_p[x]$ of polynomials over the p element field $\mathbf{Z}_p$,

$$\sum_{k=0}^{p^n-1}\binom{p^n-1}{k}(-1)^k x^k=(1-x)^{p^n}=\frac{(1-x)^{p^n}}{1-x}=\frac{1-x^{p^n}}{1-x}=\sum_{k=0}^{p^n-1}x^k.$$

Equating coefficients of x^k gives the result.

A822. From the Maclaurin expansion for $\sin x$, we find that

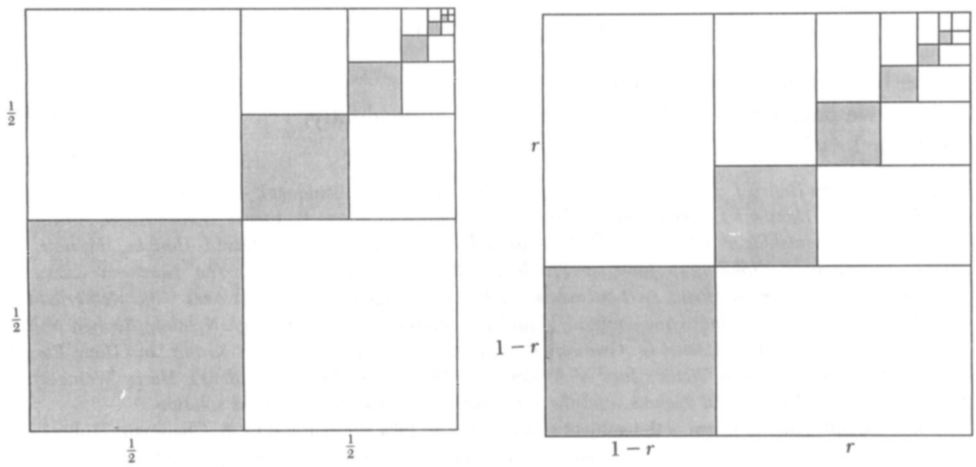
$$\sin x=\left(x+\frac{x^3}{6}\right)\left[1+O(x^4)\right].$$

Hence

$$\left(\frac{\sin x}{x}\right)^{1/x^2}=\left(1-\frac{x^2}{6}\right)^{1/x^2}\left(\left[1+O(x^4)\right]^{1/x^4}\right)^{x^2},$$

from which it follows that the desired limit is $e^{-1/6}$.

Proof without Words: Geometric Series



$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots=\frac{1}{3}$$

$$(1-r)^2+r^2(1-r)^2+r^4(1-r)^2+\cdots=\frac{(1-r)^2}{(1-r)^2+2r(1-r)}=\frac{1-r}{1+r}$$

$$1+r^2+r^4+\cdots=\frac{1}{1-r^2}$$

$$a+ar+ar^2+\cdots=\frac{a}{1-r}$$

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