Shanille Practices More

HEATHER ANDERTON
Houghton College
Houghton, NY 14744
heather.anderton@houghton.edu

RICHARD JACOBSON
Houghton College
Houghton, NY 14744
jake.jacobson@houghton.edu

The following problem appeared in the 63rd annual Putnam Exam on December 7, 2002.

> Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of the shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots? [**1**]

A natural generalization of this problem would be to increase the number of shots in determining her skill level. Suppose her skill level is determined by the fact that she made a and missed b shots. Then what is the probability, based on the criteria above, that she will make k out of the next n shots?

Richey and Zorn recently developed a solution to this problem involving an approach in a statistical setting; see this MAGAZINE [**2**]. Their argument was an interesting application of Bayesian analysis. It is the purpose of this note to solve this problem using a direct tree diagram approach.

At this point we introduce some combinatorial notation that is convenient, but not used universally. For natural numbers a and k we define the falling factorial to be $a_{[k]} = a(a-1)(a-2)\cdots(a-k+1)$. This is another name for the number of permutations $P(a,k)$. Similarly, we define the rising factorial $a^{[k]} = a(a+1)(a+2)\cdots(a+k-1)$. This notation will be useful later in the paper.

Let $[a:b]$ denote Shanille's skill level at the beginning of her shooting the n additional shots. The probability of making k of these shots is the sum of the probabilities of the distinct ways in which she can make those k additional shots. The number of these different ways can be counted by specifying which of the n shots were made. It follows that there are $C(n,k)$ distinct paths.

The probability associated with any particular path will be the product of the n probabilities of making or missing a basket on any specific attempt. The probability of making (missing) a basket is the number of previous shots made (missed) divided by the number of previous attempts. The denominator of this product will be $(a+b)(a+b+1)\cdots(a+b+n-1) = (a+b)^{[n]}$. The factors in the numerator will be $a, a+1, a+2, \ldots, a+k-1$ for the events when the shot is made and $b, b+1, b+2, \ldots, b+n-k-1$ for the $n-k$ events when the shot is missed. The product of these factors can be rearranged so the numerator is $a^{[k]}b^{[n-k]}$ for each of the $C(n,k)$ paths.

This leads to the theorem:

Let $[a:b]$ denote a skill level of making a shots and missing b shots in $a+b$ attempts. Let $P([a:b];k,n)$ be the probability of making exactly k of the next n shots.

Then

$$P([a:b];k,n) = \frac{C(n,k)a^{[k]}b^{[n-k]}}{(a+b)^{[n]}}.$$

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REFERENCES

1. 63rd Annual William Putnam Mathematical Competition, this MAGAZINE **76** (2003) 76–80.
2. Matthew Richey and Paul Zorn, this MAGAZINE **78** (2005) 354–367.

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Caleb J. Emmons
Pacific University
Emmons@pacificu.edu