Applied Mathematics and Mathematical Modeling

Joseph Malkevitch, York College (CUNY), Chair
Ricardo Cortez, Tulane University
Michael A. Jones, Mathematical Reviews
Michael O'Neill, Claremont McKenna College

Introduction

In the last ten years the pattern of the prior sixty years—rapid growth in both the theory and range of uses for applied and applicable mathematics—has continued, and accelerated. Examples of situations where applied mathematics has had important influence and where new applications are emerging rapidly include:

- cell phones and global positioning systems
- medical imaging
- weather and climate modeling
- financial mathematics
- mechanism design, market design, and auctions
- web searching, routing, and sensor networks
- extracting information, such as genome data, from large data sets

These areas have drawn on mathematical tools from both classical applied mathematics and discrete applied methodologies. The tremendous increase in storage and speed of computers has revitalized many areas of applied mathematics because, with computer power now available, problems can be pushed towards solutions in ways that were formerly impossible.

Serving students

In recent years roughly 18,000 undergraduate degrees in mathematics and allied areas have been awarded; the analogous figures for master's degrees and PhD's are around 3000 and 1400, respectively. Thus the overwhelming majority of students who receive mathematics degrees enter the world of work, whether or not their jobs use their mathematical training. In addition, many mathematics majors who continue to graduate work do so in areas traditionally allied with mathematics: physics, engineering, computer science, biology, economics, finance, etc. These data argue that *all* students should acquire skills in mathematical modeling, knowledge of theory accepted as useful in applied mathematics, and a broad sense of where mathematics is being applied. Such students will be more successful either in chosen careers or as they pursue higher degrees beyond pure mathematics.

Calculus and Linear Algebra courses, for example, have plenty of theory to fill up any course time available. But such courses should, at minimum, present examples of where the material being discussed is applicable. In Linear Algebra, instructors should call attention to the broad range of uses of their subject in mathematical programming, search engine design, numerical analysis, and cryptography. Calculus, Abstract Algebra, and Discrete Mathematics courses should be structured so that students learn not only methods and ideas but also ways that these subjects are used. Abstract Algebra courses, for instance, may treat finite fields; if so, students can explore uses of finite fields in coding theory, which in turn finds applications in communications technology.

A challenge to applied mathematics

Applied mathematics tends to thrive best in relatively large colleges and universities, especially ones with associated undergraduate and graduate programs in engineering and science. Smaller colleges often have trouble attracting applied mathematicians, especially in geographic areas remote from companies that employ applied mathematicians. (Modern communications technologies have somewhat alleviated this problem.)

Three types of applied mathematics

Very roughly speaking we can classify applied mathematics under three broad headings, the last one cutting across the boundaries of the first two.

- classical applied mathematics
- discrete applied mathematics
- statistics and probability

Classical applied mathematics. This area draws on or includes calculus, differential equations, PDEs, real and complex analysis, integral equations, Fourier series and transforms, optimization, fluid dynamics, engineering mathematics, mathematical physics, tensor and vector analysis, signal processing, image processing, etc. Many schools offer formal programs in classical applied mathematics, but not all can do so. All schools, however, should aim to equip students for the possibility of pursuing higher degrees with an applied flavor.

Discrete applied mathematics. This area draws on or includes mathematical programming (in linear, integer, and binary forms), graph and network theory, scheduling, error correction systems, data compression systems, etc. Because discrete applied mathematics has emerged relatively recently, courses related to that field may have a greater variety of titles than we find in classical applied mathematics. These include Cryptography, Operations Research (which may also includes tools from continuous mathematics), Discrete Optimization, Linear Programming, Network Science, Graph Theory, Combinatorics, Game Theory, Social Choice, etc.

Statistics and probability. Some, typically larger, institutions offer undergraduate degrees in statistics and in related areas such as bioinformatics. Most mathematics departments, regardless of size, offer a range of courses in probability and/or statistics. These courses, especially at lower levels, often attract large enrollments from students meeting general education requirements, and so require considerable teaching time from mathematics faculty.

The burgeoning of communications technology and computational biology generates floods of data that require collection and analysis. The term “data science” describes the amalgam of tools from mathematics, statistics, and computer science that are being developed for the analysis of data. Yet many mathematics majors know too little about the growing number of careers that use skills in statistics and probability, and mathematics department offerings have not always kept apace of current trends and emerging applications. More traditional applications of data science (e.g., actuarial science) remain important; new applications (e.g., analyzing data from weather satellites, cell phones, social networks, web searching) deserve attention as well.

(For much more information on related topics, see the separate Program Area Study Group report on Statistics.)

Mathematical modeling and applied mathematics

Mathematical modeling might be described as using mathematics "outside of mathematics," in "real-world situations." Mathematical modeling has become even more important since the newly adopted Common Core State Standards in Mathematics include a Standard for Mathematical Practice which stipulates that students be able to "model with mathematics." While there is a tradition of "story problems" in mathematics, many are silly (A collection of cows and chickens includes 28 legs. I see 9 heads. How many … .). Since many mathematics majors plan on K-12 teaching, it is important that courses in the major not only treat applied mathematics but in general but also present mathematical modeling in ways that help future teachers. Some mathematical modeling courses may serve both students who plan K-6 teaching and also students seeking general education requirements. Some institutions (especially two-year colleges) are substituting Mathematical Modeling courses for College Algebra, hoping that such courses will better serve students' needs—and perhaps encourage some to consider majoring in STEM disciplines.

Different views of mathematics

As a complex and multilayered subject, mathematics can be viewed from many vantage points. One view of mathematics stresses attempts to solve specific problems; a second emphasizes theoretical frameworks. Many people are attracted to mathematics through competitions and wide-ranging problems; some distinguished mathematicians who have reputations as problem solvers. Others are known for theoretical exploration, over long periods of time, of particular pieces of the mathematical landscape. Mathematical modeling can illuminate the connection between problem solving and theoretical mathematics. Teaching modeling helps students develop of skill in attacking applied questions; situations to be modeled play the same role in applied mathematics as theoretical problems play in theoretical mathematics.

Majors, minors, and tracks in applied mathematics

Departments will naturally strive to serve the needs of all majors in theoretical and applied mathematics. One possibility is to offer applied mathematics majors, minor, or tracks in specific areas (e.g., mathematical physics, economics, biology, ecology, actuarial science, network science, bioinformatics). Many departments design their majors with a relatively small set of "core" required courses; students then select one of several applied-flavor tracks, each with its own required and elective courses, to complete the major.

Because colleges vary greatly in size, presence or absence of graduate or engineering programs, access to local industry and business, and other dimensions, it is difficult to offer more than general guidance to departments seeking to establish or improve applied programs. To give some indication of the range of courses now required for in existing applied mathematics majors, minors, and concentrations, we present a table of information compiled for the SIAM Education Committee's upcoming report on undergraduate degree programs in applied mathematics.

	Major	**Minor/Concentration**
Required courses	Calculus sequence Differential equations Linear algebra Introduction to proofs Real analysis Numerical analysis Applied math or Modeling Probability Statistics Abstract algebra Introduction to programming Int. object-oriented languages	Calculus sequence Differential equations Linear algebra Introduction to programming
Recommended/elective courses	PDEs Math biology Operations research Complex analysis Discrete mathematics Geometry	Differential equations Linear algebra Discrete mathematics Numerical analysis Applied math or Modeling Operations research

Recommendations

The following recommendations for applied mathematics and mathematical modeling emerged from discussions among the committees acknowledged below. They appear in no particular order.

- Calculus courses should specifically and systematically include material that show how ideas and methods of calculus are foundational for classical applied mathematics. *Comment*: Since many undergraduate majors no longer require a Differential Equations course, it is especially important that a broad range of calculus applications be seen within the Calculus sequence.

- Mathematics majors should be required or encouraged to take a Discrete Mathematics course early in their major careers. This course should pay attention to algorithms and should give significant indication of discrete mathematical applications. *Comment*: Many mathematics departments teach discrete mathematics in part as a service course for computer science departments. In this event care significant attention should be paid to topics such as algorithms and tree structures.

- All mathematics majors who plan K-12 teaching and major in mathematics should be exposed to mathematical modeling, ideally in a course with that title.

- All mathematics course, no matter how theoretical, should pay attention to applicability of their topics.   *Comment:* One approach to accomplishing this infusion is by assign writing projects that require students to learn about applications of the theory under study.

- Students should be encouraged to take Discrete Mathematics early. Departments might consider offering parallel entry points, in Calculus and Discrete Mathematics, to the mathematics major. (Both would be required.)

- Majors should receive career information and graduate program information that builds on skills they have acquired in applied mathematics and mathematical modeling. Because students with undergraduate degrees in mathematics often take their mathematical skills to the business, industry, and government, attention should be paid to the BIG world relies on both elementary and advanced mathematics.

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