

Paul R. Halmos - Lester R. Ford Awards

The Paul R. Halmos-Lester R. Ford Awards recognize authors of articles of expository excellence published in *The American Mathematical Monthly*. The awards were established in 1964 as the Ford awards, named for Lester R. Ford, Sr., a distinguished mathematician, editor of *The American Mathematical Monthly*, 1942–1946, and President of the Mathematical Association of America, 1947–1948. In 2012, the Board of Governors designated these awards as the Paul R. Halmos–Lester R. Ford Awards to recognize the support for the awards provided by the Halmos family and to recognize Paul R. Halmos, a distinguished mathematician and editor of the *Monthly*, 1982–1986.

William Dunham

“Euler and the Cubic Basel Problem,” *The American Mathematical Monthly*, 128:4, 291–301. doi.org/10.1080/00029890.2021.1865014

In 1689, Jakob Bernoulli showed that the sum of the reciprocals of the squares of positive integers converges, although he did not find the exact value of the sum. Due to Bernoulli’s location at the time, this became known as the “Basel problem.” In 1734, Leonhard Euler summed it up! This article explains Euler’s solution to the Basel problem, as well as how Euler revisited the problem many times, including an initial attack on the “cubic Basel problem” in 1741. First, Euler proves a formula relating the sum of the reciprocals of the n th powers of the integers to the sum of the reciprocals of the n th powers of the odd integers. Then, he cleverly relates a definite integral to the sum of the reciprocals of the squares of the odd integers, obtaining his 1734 result. Euler was hoping to apply the same idea to the sum of the reciprocals of the cubes of positive integers to obtain a result involving a rational multiple of the cube of π . He used “closed forms” of some definite integrals and yet he could not succeed. The article mentions that, in spite of others - including Ramanujan - tackling the cubic Basel problem across two and a half centuries, the problem has not yielded a closed form sum. In 1978 Apéry astonishingly showed the sum is irrational, an accomplishment alluded to on his gravestone.

Response

I am most grateful to the MAA and to the Halmos-Ford committee for recognizing my article about Euler and the cubic Basel problem. What a thrill!

Let me especially thank Penny Dunham, who has always been my best, and favorite, editor.

I also send a shout-out to the Collier Science Library at Bryn Mawr College, where this paper took shape over the course of many months. On its shelves, I have found Euler’s mathematical works, some fascinating 19th century journals, and volumes inscribed by faculty superstars like Charlotte Angas Scott and Emmy Noether. It is a treat to graze through the remarkable collections at Bryn Mawr.

Finally (albeit a few centuries late) I must thank Leonhard Euler. The breadth and depth of his achievements are legendary, and he has kept me busy—and enthralled—across four decades. In my view, the long history of mathematics features no one as compelling as the “Master of Us All.”

Biographical Sketch

William Dunham (PhD, Ohio State, 1974) is a historian of mathematics who has authored four books on the subject: *Journey Through Genius*, *The Mathematical Universe*, *Euler: The Master of Us All*, and *The Calculus Gallery*. In 2015 – 2016 he was the MAA’s George Pólya Lecturer, and he is featured in the Teaching Company’s DVD course, “Great Thinkers, Great Theorems.”

After retiring from Muhlenberg College (emeritus, 2014), he has held visiting positions at Harvard, Princeton, Penn, Cornell, and Bryn Mawr, where he is currently a research associate in mathematics.