

Paul R. Halmos - Lester R. Ford Awards

David Lowry-Duda and Miles H. Wheeler

“Perturbing the Mean Value Theorem: Implicit Functions, the Morse Lemma, and Beyond,” *The American Mathematical Monthly*, 128:1, 50–61. doi.org/10.1080/00029890.2021.1840879

The classic mean value theorem from calculus states that, given a differentiable function f on an interval $[a, b]$, there exists at least one value c contained in $[a, b]$ such that the slope of the tangent line to f at $x = c$ is equal to the slope of the secant line through $(a, f(a))$ and $(b, f(b))$. How do the choices of c relate to varying the right endpoint of the interval and, in particular, can we write c as a continuous function of b in some interval? To answer this question the authors lead the reader through a lively mathematical exploration which includes visiting the implicit function theorem, a simplified version of the Morse lemma, and the theory of analytic functions.

Response from Both

We are humbled to have been selected to receive the Halmos-Ford Award for our article on perturbing the mean value theorem. We love the *Monthly*, both as a source of inspiration and as a paragon of clear writing. It is an honor to contribute to this legacy.

While the mean value theorem is a foundational result underpinning calculus, it often feels anticlimactic when first learned. This dichotomy drew our attention, and we were thrilled to find natural connections to other foundational results lurking in the corners.

This article grew from hallway conversations during graduate school, in which a pair of mathematicians in analytic number theory and the analysis of PDEs explored common interests. Initially we concentrated on examples, surprising ourselves with the different behaviors we could find. Once we had a general theory, we sought to craft examples that were both algebraically tractable and easy to visualize. As we attempted to distill our ideas to their simplest form, we learned a lot—both about fundamental analysis, and each other.

We would like to thank the long tradition of the Brown Graduate Student Seminar for providing an open, inviting atmosphere for fun projects like this. We also thank the MAA for promoting and stewarding high quality and approachable mathematical exposition. To everyone involved in these great institutions, may your hallways be covered in chalkboards and full of ideas!

Biographical Sketches

David Lowry-Duda received BS degrees in applied math, international affairs and modern languages from Georgia Tech in 2011, and his PhD from Brown University in 2017. As a senior research scientist at the Institute for Computational and Experimental Research in Mathematics, he researches analytic number theory, arithmetic geometry, and computation. He is an active developer of math software, including the L-Function and Modular Form Database (LMFDB) and SageMath. When not doing mathematics, you might find him biking or hiking.

Miles Wheeler received a BA in mathematics and physics from Cornell University in 2009, and a PhD from Brown University in 2014. He is a lecturer (assistant professor) in analysis in the Department of Mathematical Sciences at the University of Bath. His research applies techniques from mathematical analysis to partial differential equations coming from fluid mechanics, and he is one of the co-founders of the One World PDE Seminar.