

George Pólya Awards

The George Pólya Awards, established in 1976, are made to authors of articles of expository excellence published in the *College Mathematics Journal*. The awards are named for George Pólya, who was a distinguished mathematician, well-known author, and professor at Stanford University.

Joseph Previte and Michelle Previte

“The Beautiful Chaotic Dynamics of i^z ,” *The College Mathematics Journal*, 52:5, 364–372.
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In “The Beautiful Chaotic Dynamics of i^z ,” Joseph and Michelle Previte guide their readers on an engaging exploration of the principal branch of the complex map $f(z) = i^z$. While Brouwer’s fixed-point theorem guarantees that this function has at least one fixed point, the authors establish that there are, in fact, an infinite number of fixed points—all but one of which are unstable. Of course, with this initial groundwork in place, exotic and ever-enchanting fractal images cannot be far behind! By iterating i^z numerically for a large collection of initial points, the Prevites create a graph to identify those points in the plane which lie in the basin of attraction of the stable fixed point and those initial points which escape to infinity. Earlier in the paper, technology was used to explore the locations of the sought after fixed points and to follow up with careful mathematical analysis to verify the information alluded to in the resulting graphs. This helpful side of technology is counter-balanced as the Prevites use mathematical analysis to carefully point out the limits of technology by identifying points within the basin of attraction that the computer-generated plot clearly mis-identified.

While some of the points the computer identified as being outside the basin of attraction actually approach the stable fixed point, the authors provide a compactness argument to show that there are indeed points in the plane with orbits whose moduli tend to infinity.

The paper continues by examining the composite maps $f^2(z)$ and $f^3(z)$ to identify period two and period three points of $f(z)$. Thus, one concludes that $f(z)$ is a chaotic map having periods of all orders. The authors conclude by giving readers six open problems to investigate on their own.

The Prevites’ clear exposition makes it easy for a reader to interact with this paper at a variety of levels. There are five exercises sprinkled throughout the paper that allows one to, at first, skip some of the technical details and more quickly get to the “good stuff”—that is, the beautiful chaotic dynamics of i^z . However, these exercises contain some very nice analysis for students to grapple with and help to reiterate the usefulness of one-sided limits, monotonicity, and notions of convergence that students have likely seen in their mathematics courses.

Students who already have some familiarity with complex numbers could use this paper as a nice introduction to the ideas of fractals and chaos. Taking time to fill in some of the details, to reproduce some of the lovely plots, and to explore the open problems would make for a truly engaging and worthwhile project for students and instructors alike.

Response

Part of the enterprise of mathematics is to convey ideas and concepts to others in a way that entertains and inspires. The *College Mathematics Journal* is at the forefront of this endeavor. So to learn that our article was recognized as making an excellent contribution to the *CMJ* was a surprising honor. However, upon further reflection, the structures that we discovered were so intrinsically striking and beautiful, our job was both a joy and relatively easy. We would like to thank the MAA, Stanley Tuznik, who was a student in the complex analysis class where this problem was first considered, Dr. Russell Howell for his help and insight, and our Lord and Savior Jesus Christ without whom we could accomplish nothing (John 15:5).

Biographical Sketches

Joseph Previte is an associate professor of mathematics at Penn State Erie, The Behrend College. He did his graduate work at the University of Maryland under the direction of Michael Brin and received his doctorate in 1997. His research interests include mathematical biology, topology, geometry, and dynamical systems. He and his wife have five children. Besides mathematics, he is active in advising the Christian group on campus.

Michelle Previte is an associate professor of mathematics at Penn State Erie, The Behrend College. She earned a BS from Westmont College and a PhD from the University of Maryland, College Park. She enjoys running, homeschooling her five children, cheering loudly at their sporting events, and spending time with friends, especially her best friend, her husband, Joe.