

The condition that  $x^3 + px + q = 0$  have a rational root is essential, for  $x^3 - 6x - 6 = 0$  has  $\Delta = 1$  but has no rational roots and none of its roots are in  $\mathbb{Q}[\omega]$ .

*Final note.* All of these results may be proved by elementary means. For example, it is only moderately difficult to prove Theorem 1 by noting that if  $x_1$  is the rational root, then

$$\left( \frac{x_1}{2} - \frac{3x_1}{3q + 2px_1} \sqrt{\Delta} \right)^3 = -\frac{q}{2} + \sqrt{\Delta},$$

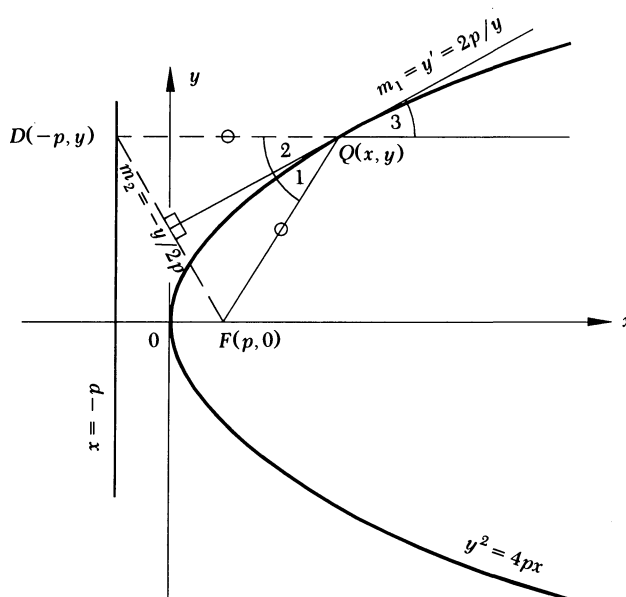
which gives an  $H \in \mathbb{Q}[\sqrt{\Delta}]$ . However, this method is unsystematic and makes the results seem to depend on fortuitous algebraic trivialities. The approach through field extensions gives a much better explanation of what is really going on.

## REFERENCES

1. Girolamo Cardano, *Ars Magna*, Nuremberg, 1543.
2. A. P. Hillman and G. L. Alexanderson, *A First Undergraduate Course in Abstract Algebra*, 4th edition, Wadsworth Publishing Co., Belmont, CA, 1988.

Proof without Words:

The reflection property of the parabola



$$QF = QD \quad \& \quad m_1 \cdot m_2 = -1 \quad \Rightarrow \quad \angle 1 = \angle 2 = \angle 3$$

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