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Teaching Tip: Actuarial Science and Gompertz's Law of Mortality

Jesse Byrne (jbyrne@uco.edu)

Actuaries use a unique combination of mathematical, statistical, and business skills to solve a variety of financial and social problems, such as the computation of premiums for insurance policies. A highly stratified profession, actuarial science dates back to the 17th century and welcomes mathematics majors. This short primer gives students and their teachers some idea of what actuaries do as well as describing Gompertz's Law of Mortality, a major contribution to actuarial science. (For more about Gompertz, see Charlotte Simmons' article on p. 33.)

Pricing life insurance If an insurance company knew exactly when the person buying insurance would die, this would be simple. Unfortunately (from the actuary's point of view), this date is unknown and the company must use a simple but lengthy process based on expected values and the Law of Large Numbers.

Specifically, actuaries create a table giving the number of years the insured might live from the date the insurance policy was written, the present value of the benefit being purchased for each of these corresponding years of life, and the probability that the insured will die in the corresponding year. Table 1 illustrates with a benefit of \$1,000. The expected value of the policy is computed by multiplying the values from columns two and three and then summing the results. For example, using a \$1,000 benefit and a 6% interest rate, the premium turns out to be \$817.65, the sum of the entries in the fourth column.

Table 1.

Number of Years	Present Value of $1000 at 6% interest	Probability of Mortality	Expected Value Computation
1	$1{,}000(1.06)^{-1} = 943.40$	.10	$94.34
2	$1{,}000(1.06)^{-2} = 890.00$	.15	$133.50
3	$1{,}000(1.06)^{-3} = 839.62$	.20	$167.92
4	$1{,}000(1.06)^{-4} = 792.09$	.25	$198.02
5	$1{,}000(1.06)^{-5} = 747.26$	.30	$224.18
Total			$817.65

After receiving the premium, the insurance company invests the $817.65 at 6% interest in order to accumulate the $1,000 benefit it must pay when the insured dies. Note that the insurance company loses money if the insured actually dies within one year of buying the policy; it should have charged a premium of $943.40 instead of $817.65. On the other hand, the insurance company makes a profit if the insured lives until year five; then a premium of $747.26 would have sufficed. It is essential the company sell a large number of policies in order to break even.

Gompertz's Law of Mortality A natural question is how the values in the third column of Table 1 are computed. For this, actuaries use a life table, a list of values giving the proportion of x-year olds that survive to the following year. Specifically, let $d(x, k)$ be the probability that an x-year old dies in year k (at age $x + k$). Let $p(x)$ be the probability that an x-year old survives one year (to age $x + 1$). Then $q(x) = 1 - p(x)$ is the probability that an x-year old dies within a year and,

$$d(x, k) = p(x)p(x + 1) \cdots p(x + k - 1)q(x + k) \tag{1}$$

Prior to Gompertz's Law of Mortality, actuaries attempted to determine such values by gathering census information from the population under consideration, quite a difficult task in the mid-1800's. Gompertz discovered a simple analytic function that accurately approximates the probability distribution $p(x)$ for the future lifetimes of a given population. Let $S(x)$ be the probability that a newborn survives to the age of x. Gompertz modeled S by

$$S(x) = \exp(-m(c^x - 1)), \tag{2}$$

where m and c are constants depending on the specific population under consideration (e.g., humans, elephants, insects, etc.). Using S, we obtain $p(x) = S(x + 1)/S(x)$, and from $p(x)$, using (1), we obtain $d(x, k)$, which is the third column of Table 1.

Gompertz's key observation is that the rate of mortality of a given population increases in geometric proportion. That is, if $L(x)$ is the total population of x-

year olds, then

$$\frac{L'(x)}{L(x)} = -Bc^x.$$

This differential equation is separable and the solution has the form

$$\frac{L(x)}{L(0)} = \exp\left(\frac{-B}{\ln(c)}(c^x - 1)\right).$$

Therefore, since $S(x)$ is the percentage of newborns that survive to age x, i.e., $L(x)/L(0)$, it follows that S has the form (2). This is Gompertz's Law of Mortality. This law greatly simplified the computation of life insurance premiums since life tables were now much easier to determine. Rather than finding accurate census information for each age group in the table, actuaries now only needed to find the data for two age groups, because the survival function (2) only has two parameters.

Conclusion Gompertz's law had great impact on the insurance industry but it has applications outside human populations. It has been broadly studied and used to predict maximum life expectancies for many animals and the growth of tumors as well.

101. Mr Jeavons said that I liked maths because it was safe. He said I liked maths because it meant solving problems and these problems were difficult and interesting but there was always a straightforward answer at the end. And what he meant was that maths wasn't like life because in life there are no straightforward answers at the end. I know he meant this because this is what he said.

This is because Mr. Jeavons doesn't understand numbers.

Here is a famous story called **The Monty Hall Problem** which I have included in this book because it illustrates what I mean.

There used to be a column called **Ask Marilyn** . . .

. . .   .

And this shows that intuition can sometimes get things wrong. And intuition is what people use in life to make decisions. But logic can help you work out the right answer.

It also shows that Mr. Jeavons was wrong and numbers are sometimes very complicated and not very straightforward at all. And that is why I like **The Monty Hall Problem**.

—from *The Curious Incident of the Dog in the Night-time*
by Mark Haddon [pp. 61–65],
For more about the Monty Hall problem see page 71.

