CUPM Curriculum Guide
Engineering Program Area Study Group Report

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Engineering can be described as the "science" of design, and mathematics is the language of science. Thus, mathematics and engineering are inextricably linked in both theory and application, as "real-world" problems investigated in undergraduate mathematics courses attest. Mathematics instructors who focus class activities on application as well as theory can stimulate students' curiosity about the world of engineering in which analysis, design, and creation are key activities. Mathematics program leaders who seek out opportunities to collaborate with those in client disciplines such as engineering can maintain or increase the relevance, both actual and perceived, of their mathematics courses to faculty and students in engineering.

Student Audience

This report focuses on two distinct types of students: mathematics majors interested in a minor in engineering, and engineering majors interested in taking additional mathematics courses beyond the required core. These students are similar in that they are likely to intend graduate study, a career, or both in mathematics or engineering. They also gain similar benefits from taking upper-level mathematics courses designed to equip them both for employment in engineering upon completion of a bachelor's degree and for advanced study in engineering.

Curricular Issues for Engineering

Mathematical content. Building on the foundation of calculus and differential equations, the following topics are of most value for the aforementioned mathematics and engineering students: discrete mathematics, linear and matrix algebra, applied mathematics, engineering mathematics, partial differential equations, statistics and probability, operations research, and modeling (to include validation, verification, visualization, and building algorithms). Students can build these topics into their curriculum through a combination of independent study, special projects, and formal course work. Of particular benefit to these students are courses that provide opportunities for internships in industry.

Students might choose a "breadth-first" approach and take a set of varied but related courses: partial differential equations, linear algebra, statistics and probability, discrete mathematics, and operations research. Alternatively, students might choose a "depth-first" sequence of courses that focus on a specific area, such as statistics and probability. Mathematics departments can package courses focused on any subset of the topics listed above as a track, option, concentration, or minor.

Engineering skill set. Prospective engineers must acquire proficiency with an ever-increasing set of skills. Reports such as the National Academy of Engineering's *The Engineer of 2020* [1] and *Educating the Engineer of 2020* [2] describe this expanding list of skills, which includes problem solving, oral and written communication, computation (both "by hand" and with technology), and experience with programming. Engineering skills, like any other, improve only with practice, so students need multiple opportunities to hone them. Situating these opportunities within settings that also require and build reading comprehension provides training most relevant to the work required of an engineer. Well-structured activities in mathematics courses can provide such settings and thereby play a key role in the development of these critical skills.

Faculty collaboration across disciplines. Mathematics faculty who seek to understand how mathematics actually interfaces with engineering learning and practice are best able to engage students interested in advanced study or a career in engineering. These faculty members can interweave illustrations and activities with content, beginning early in their curricula, in ways that capture the imagination of students already drawn to engineering. Practicing engineers continually seek to improve approaches to modeling physical reality and better techniques for framing, posing, and solving problems in design and development work. Mathematics faculty can engage students' creativity in ways that will effectively prepare them for further study or a career in engineering. Focusing solely on mathematical algorithms and theory is less likely to excite students aiming for or interested in engineering.

Conceptual approaches. Many and varied strategies can be used to teach mathematics. These include algebraic, numerical, and graphical representations, continuous and discrete approaches, and stochastic and deterministic viewpoints. Useful perspectives on content may be quantitative, qualitative, or structural. Students benefit from exposure to as many of these variations as possible, and from practice in identifying and applying appropriate mathematical tools—fundamental activities of working engineers. Examples of such tools include linear programming for constrained optimization; the Fast Fourier Transform to reveal patterns in data; stochastic processes to model uncertainty; and differential equations and finite element methods to analyze large and small structures.

Pedagogy. There is a variety of promising, evidence-based teaching practices that work especially well in engineering education, where students draw on concepts from multiple disciplines to design practical solutions to real-world problems (see [2], [3], [4], [5], [6]). Project-based approaches, inquiry-based learning, flipped classrooms, and other "active learning" techniques fit well with what these students will encounter in disciplinary engineering courses and labs. Combined with a focus on applications and modeling and supplemented with appropriate technology, active learning environments can spur these students to greater engagement in the learning process. Innovative approaches can also help manage acknowledged pedagogical tensions: breadth vs. depth of coverage, theory vs. applications, and lecture vs. time spent improving skills.

Because technology and knowledge are advancing so quickly, mathematics departments are preparing students for careers that don't even exist today. And because it is impossible to replicate even the current workplace demands (e.g., actual engineering problems are much more complex than undergraduates are capable of solving), it is crucial that undergraduates learn how to learn. The best career preparation faculty can offer students is a spirit of inquiry, curiosity, and tenacity. Faculty can encourage students to engage with peers, instructors, text books, research journals, seminars, webinars, online interest groups, conferences, other web resources, and to forge professional and social connections. Faculty can also model lifelong learning by availing themselves of professional development opportunities about student cognition and evidence-based pedagogies.

***Technology*.** The incorporation of computational technologies into mathematics courses is particularly important for students interested in engineering, but the required repurposing of class time can dissuade faculty. Using such technologies introduces a new learning curve for students and raises often time- and resource-consuming issues of technical support. In addition, mathematics faculty may not initially best suit engineering applications. Nevertheless, the benefits of such technology can far outweigh its disadvantages. Using technology allows students to complete calculations more quickly, and thus explore more examples, than would be possible by hand. Using technology also allows students to explore and practice its appropriate uses in their work.

Even though technology is evolving and advancing quickly, and students will probably use different software packages in careers than those encountered during formal education, many of the skills students gain during formal education will transfer to workplace uses. This transfer will likely be stronger, moreover, if students are exposed to the most current tools available while in school. Applying technology to problem solving allows students to engage more complex problems than they could otherwise, and it provides context for the mathematics students see in engineering courses—and powerful motivation to learn the fundamental concepts and techniques taught in mathematics courses.

One approach to mathematics education technology is to have students use software packages (e.g., MATLAB) that are standard in engineering education and practice, which are different from those (e.g., *Maple* and *Mathematica*) used in mathematics. Access to powerful computational tools also affords students the opportunity to engage with more complex, authentic tasks and to produce meaningful results. Employers of engineering graduates often consider such experience an important part of students' training. Time can be spent focused on interpreting and communicating the results rather than on performing tedious calculations by hand.

Open communication with engineering faculty can provide mathematics faculty with important information regarding the computational tools used in engineering courses as well as existing resources for technical support. Using the same tools across all related courses allows faculty to save instructional time by capitalizing on the software expertise students have or will be expected to acquire in their engineering courses. This approach also serves to reinforce the notion of relevance of the mathematical concepts to the field of engineering.

Alternatively, mathematics departments might already use a particular software package. In this case, it may not be practical to abandon the current investment in favor of another, particularly one intended for a narrow audience. Mathematics departments serve many client disciplines, not only engineering, and the current software package is likely to be serving students from all of those disciplines well. In either case, we recommendation that appropriate technology be integrated throughout the mathematics curriculum, not isolated in a separate "computers for math" course.

Programming in the context of high performance computing, modeling, and data. In light of the current needs of employers and the forward-looking recommendations of professional organizations and national advisory groups (e.g., [1], [7], [8]), mathematics departments should consider how and where to include introductions to programming and high performance computing, including code writing, parallelization of codes, Message Passing Interface (MPI), and code/algorithm optimization for targeted computing environments. A strong foundation in mathematical modeling is also key for engineering-interested students. Students need experience visualizing and constructing simple models, refining models to account for the physics, biology, chemistry laws that govern the system under investigation, using optimization in parameter identification to construct the "best" model, and run simulations to test and validate the model. Students also need experience dealing with large data sets; engineers will increasingly be expected to possess adequate knowledge of and proficiency with data analysis.

Using computational technology to facilitate modeling and other applications may lie outside mathematics faculty members' comfort zones. Administrators and other academic leaders can serve many students efficiently by supporting these faculty with training, hardware, software, released time, and by supporting changes to curricular components and requirements. Careers in engineering will require increasing technological and modeling expertise; students in other STEM disciplines will also benefit.

Learning Goals

Following is a sample of potential mathematics program goals for students interested in advanced study or careers in engineering. While many of these goals apply to all mathematics students, some have particular meaning for student engineers.

Cognitive goals.

- *Critical thinking.* Students will be able to think creatively and analytically.
- *Problem solving*. Students will be able to apply the necessary mathematical tools and appropriate computational technology to solve complex problems.
- *Modeling*. Students will be able to construct valid mathematical models for engineering applications.
- *Visualization*. Students will be able to use visualization skills to assist with problem-solving and modeling.
- *Communication.* Students will be able to clearly articulate ideas orally and in writing.

Additional learning goals.

- *Breadth of knowledge.* Students will develop a broad view of the mathematical sciences, encompassing algebraic, numeric, graphic, continuous, discrete, and stochastic themes, as well as explicit connections among mathematics and engineering theory, applications, and models.
- *Technology.* Students will be able to use computational technology appropriately and apply programming skills as necessary.

Further Considerations for Engineering Programs

Accreditation. Engineering programs are accredited by ABET [9]. ABET accreditation requires a specific number of hours of college-level mathematics and science, but it does not specify the level or content of the required mathematics courses. Input from engineering faculty and administrators is important as mathematics faculty consider curriculum for students interested in engineering. Of critical concern is that the alignment of curricula across programs facilitates successful student transition from one course to the next, regardless of the department in which the course is housed. Additional information about the mathematical requirements for engineers can be found in the American Society for Engineering Education's publications; the most current information is also regularly disseminated at their conferences.

Modes of delivery. Online, hybrid, and other modes of delivery, along with new ways to assess and verify learning, hold great potential to change the teaching and learning of mathematics. Technology-assisted approaches have already made content more accessible to the masses, including those who have had no access previously. Mathematics faculty might best serve students by remaining open to the possibility of improvements in teaching and learning via adaptation of the modes that prove to be most promising.

Extracurricular engagement. Participation in extracurricular competitions such as the Mathematical Contest in Modeling, the Interdisciplinary Contest in Modeling (see [10]), and the National Academy of Engineering Grand Challenge Scholars Program (see [11]) is beneficial for students interested in engineering as well. Entrepreneurship courses and competitions also offer added value for these students.

Growth of community colleges**.** Recent data suggest that approximately 40% of engineering majors either start their education at a community college or attend one at some time ([12], [13]). As financial pressures and demand to educate students from broader demographics continue to increase, it will be increasingly important for institutions to implement effective articulation agreements and plans to ensure that students who transfer between institutions experience a seamless transition.

Recommendations for Faculty and Administrators

Although this report emphasizes what *students* should be able to do, we should also consider how faculty and administrators can support this mission. They should

1. encourage faculty collaboration across disciplines, particularly among mathematics and client disciplines;
2. expose students to a broad range of mathematical concepts and approaches;
3. employ and promote evidence-based pedagogical practices (e.g., project-based and inquiry-based learning);
4. integrate use of technology throughout the mathematical curriculum, for discovering, exploring, understanding, and using concepts;
5. incorporate modeling in many mathematics courses, both to motivate students and to enrich the mathematics under study;
6. ensure that the transition process for transfer students is transparent and effective.

References and Resources

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