CLASSROOM CAPSULES

Classroom Capsules are short (1–3 page) notes that contain new mathematical insights on a topic from undergraduate mathematics, preferably something that can be directly introduced into a college classroom as an effective teaching strategy or tool. Classroom Capsules should be prepared according to the guidelines on the inside front cover and sent to any of the above editors.

Winning at Rock-Paper-Scissors

Derek Eyler (eylerd@gmail.com) 169 Lismore Avenue, Glenside PA 19038, Zachary Shalla (zms2285@gmail.com) 2415 Western Avenue, Seattle WA 98121, Andrew Doumaux (andrew.doumaux@gmail.com), 730 Toftrees Avenue, State College PA 16803, and Tim McDevitt (McDevittT@etown.edu) Elizabethtown College, Elizabethtown PA 17022

Rock-paper-scissors (RPS) is a popular way to settle simple disputes because it is can be played quickly, can accommodate multiple players, and is presumably fair. (We assume that the reader is familiar with the rules of RPS. If not, consult [**7**].) Game theory confirms that RPS between two players *is* fair, provided that at least one of the two players follows the optimal strategy of choosing rock, paper, and scissors uniformly at random. This note discusses experimental results that show that people do *not* follow the optimal strategy in practice, and suggests two strategies for defeating human opponents in RPS.

A first strategy. In recent decades, experimental psychologists have cast doubt on the ability of people to detect randomness. Perhaps the most famous example is the controversy over the “hot hand” phenomenon in basketball, which has generated numerous articles in the mathematics, statistics, and psychology literatures. In 1985, Gilovich and Tversky [**2**] claimed to show that, when basketball players make several consecutive baskets and are deemed to have a “hot hand”, their success is readily explained as a sequence of independent Bernoulli trials. Subsequent authors have contested this claim and the issue does not appear to be resolved. (Korb and Stillwell [**3**] is a recent overview of the literature on the subject.)

In addition to being poor judges of random behavior, there is also experimental evidence that people are poor generators of random sequences. For example, Kubovy and Psotka [**4**] demonstrate that people have a tendency to choose 7 far too often when asked to choose a number between 0 and 9. Budescu [**1**] constructed a successful Markov model for the way in which people tend to generate random outcomes of Bernoulli trials and he found that most subjects tended to switch between “heads” and “tails” too frequently, although some subjects switched too infrequently.

RPS is particularly easy to study, and we simplify matters even more by concentrating on RPS competitions between only two players, whom we call Bart and Lisa. To deduce an optimal strategy, assume that Bart plays rock, paper, and scissors randomly with respective probabilities r, p, and s. If each game pays 1 to the winner and -1 to the loser, then Lisa has the following expected values for each choice of symbol.

Lisa's Symbol	Rock	Paper	Scissors
Lisa's Expected Value	$s-p$	$r-s$	$p-r$

$$r + p + s = 1. \qquad (1)$$

If Bart chooses $r = p = s = 1/3$, then the game is fair regardless of Lisa's strategy. However, if Bart does not choose symbols uniformly at random, then Lisa can find a strategy with a positive expected value. Suppose, for example, that $r > p, s$. In this case, Lisa should not play scissors since $p - r < 0$. If $s > 1/3$ then $s - p > r - s$ so she should choose rock, if $s < 1/3$ then she should choose paper, and if $s = 1/3$ then she can choose either rock or paper. Lisa's play in this case conveniently generalizes to the following strategy:

- Never choose the symbol that loses to the most likely symbol.
- Choose the most likely symbol if the symbol that it beats has probability greater than 1/3.
- Otherwise, choose the symbol that beats the most likely symbol.

We collected data from 119 people who each played 50 games of RPS against a computer playing this optimal RPS strategy. Rather like Kubovy and Psotka's [**4**] results, our subjects had a nonuniform preference—for rock. Of the 119 participants, 66 (55.5%) started with rock, 39 (32.8%) with paper, and 14 (11.8%) with scissors. Symbol choices beyond the first seem to depend on previous choices, with empirical transition probabilities shown in Table 1. These probabilities are far from uniform. Notably, players seem to have a distinct preference for repeating plays. For example, a player who chose paper on one trial has a 0.421 probability of repeating this play on the next trial, well above 1/3. So, if Bart plays symbols with the probabilities in Table 1, then Lisa has the extremely simple, but effective, strategy of always playing the symbol that defeats the symbol that Bart previously played. For example, if Bart plays rock in the nth game, then Lisa should play paper in the $(n + 1)$st game. It is noteworthy that this strategy is not listed in *The Official Rock Paper Scissors Strategy Guide* [**6**].

Table 1. Empirical transition probabilities for RPS.

Previous Choice	Next Choice		
	Rock	Paper	Scissors
Rock	0.445	0.354	0.201
Paper	0.288	0.421	0.292
Scissors	0.176	0.308	0.516

Lisa's strategy, however, can only apply in the short term. If Bart always plays with the probabilities in Table 1, then, viewed as a Markov process, his choices approach a long-term state in which he has probabilities $r \approx 0.297$, $p \approx 0.362$, $s \approx 0.341$ of choosing rock, paper, and scissors, respectively. The expected values in (1) imply that Lisa *always* chooses scissors in the long run, and even Bart would notice and change his strategy.

A Second Strategy. A more sophisticated approach analyzes an opponent's previous *two* choices. The results for the same 119 subjects are shown in Table 2. For example, if Bart chooses rock-paper, then he has a 0.449 probability of choosing scissors as his third choice. Table 2 shows that people have a tendency to repeat symbols (e.g., rock-rock-rock) or to cycle through the symbols (e.g., rock-paper-scissors).

Table 2. Empirical transition probabilities for RPS based on subjects' previous two choices.

Previous Two Choices	Next Choice		
	Rock	Paper	Scissors
Rock-Rock	0.684	0.203	0.115
Rock-Paper	0.226	0.325	0.449
Rock-Scissors	0.185	0.618	0.197
Paper-Rock	0.238	0.381	0.381
Paper-Paper	0.212	0.591	0.198
Paper-Scissors	0.357	0.330	0.313
Scissors-Rock	0.241	0.648	0.111
Scissors-Paper	0.455	0.293	0.253
Scissors-Scissors	0.075	0.183	0.742

We developed a second program in the form of a website [**5**] that bases its choice of symbol adaptively on its human opponent's previous two choices. Specifically, it starts with the matrix in Table 2 for each opponent and then updates the probabilities every time the human opponent chooses a symbol. Each competition consisted of 100 games of RPS and there were 241 participants. On average, the computer won 42.1%, lost 27.7%, and tied 30.2% of individual games. This represents a 26.4% increase in wins and a 9.51% decrease in losses *per game* as compared to expected outcomes of 100 fair games. Figure 1 shows the cumulative results in all 100 trials for all 241 participants. Implementing our adaptive scheme, the computer won more games out of 100 than its human opponent 79.7% of the time.

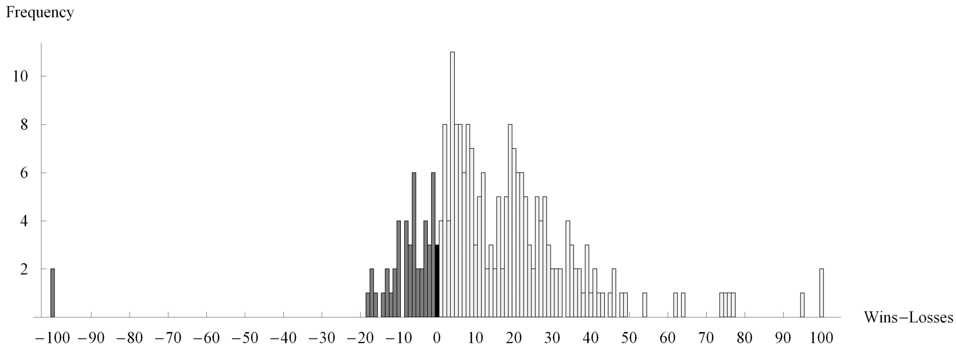

Figure 1. Wins minus losses for the computer versus 241 human opponents.

In closing, we note that deviating from the optimal strategy is always dangerous because a superior strategy always exists. Figure 1 shows that two players defeated the web program 100 times out of 100. Conversations with those two players revealed that one of them stopped and started the match repeatedly until he found the perfect strategy through trial and error, and the second accessed and analyzed the program to determine the computer's strategy.

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References

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Proving that Three Lines Are Concurrent

Daniel Maxin (daniel.maxin@valpo.edu), Valparaiso University, Valparaiso IN 46383

The role of elementary geometry in learning proofs is well established. Among the more challenging problems that a student may encounter, those asking to prove that three lines are concurrent occupy a special place. The common approach in solving these problems is finding a suitable triangle where the three lines are known cevians such as medians or perpendicular bisectors. Yet many other problems are even more general and involve arbitrary concurrent cevians. Ceva's Theorem is a standard approach in this case:

Theorem (Ceva's Theorem). *Given a triangle ABC, and points A', B', and C' that lie on lines BC, CA, and AB respectively, the lines AA', BB', and CC' are concurrent if and only if*

$$\frac{|C'A|}{|C'B|} \cdot \frac{|A'B|}{|A'C|} \cdot \frac{|B'C|}{|B'A|} = 1.$$

Finding a suitable triangle and expressing the ratios in the equation above is not always straightforward. We present an elementary solution to an interesting problem emphasizing two important steps in applying Ceva's Theorem:

Problem. Let ABC be a triangle. Construct rectangles $ACDE$, $AFGB$, and $BHIC$, one on each side of ABC. Prove that the perpendicular bisectors to the segments EF, GH, and ID are concurrent.

