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Einstein's Principle

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In his remarkable Spencer Lecture, delivered at Oxford in 1933, Albert Einstein advanced an astonishing epistemological view:

> If then it is the case that the axiomatic basis of theoretical physics cannot be an inference from experience, but must be free invention, have we any right to hope that we shall find the correct way?... To this I answer with complete assurance, that in my opinion there is *the* correct path and, moreover, that it is in our power to find it. Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity. It is my conviction that pure mathematical construction enables us to discover the concepts and the laws connecting them which give us the key to the understanding of the phenomena of Nature... In a certain sense, therefore, I hold it to be true that pure thought is competent to comprehend the real, as the ancients dreamed.
>
> To justify this confidence of mine, I must necessarily avail myself of mathematical concepts. The physical world is represented as a four-dimensional continuum. If in this I adopt a Riemannian metric, and look for the simplest laws which such a metric can satisfy, I arrive at the relativistic gravitation-theory of empty space. If I adopt in this space a vector-field, ..., and if I look for the simplest laws which such a field can satisfy, I arrive at the Maxwell equations for free space. [**1**, p. 167]

The elevation of mathematical simplicity from helpful adjunct to indispensable guiding principle for the discovery of natural law seems radical even today. That so many of his last years were spent in vain on a quest for the simple field which would comprise, as special cases, gravity's Riemannian metric and electromagnetism's vector-field, may have unduly obscured the basic soundness of the principle, which he believed to be founded on the rock of general relativity.

This note gives an elementary illustration of the principle applied to pre-relativity physics. If a vector-field is thought of as first rank, and a metric as second rank, then the next step down is a scalar-field, of rank zero. For it the following, itself based on a hint of Einstein's (in [**2**, p. 29]), holds:

And if in the ordinary three-dimensional continuum, time aside, one adopts a scalar-field, and looks for the simplest laws which such a field can satisfy, namely, spherical symmetry and Laplace's equation, then one arrives at Newton's inverse square law.

The argument is simple indeed. Let the scalar-field be $\phi(x,y,z)$. If it is spherically symmetric, then $\phi=\phi(r)$, where $r=\sqrt{x^2+y^2+z^2}$. The analogue of relativity's vanishing curvature condition is Lapace's equation: $\phi_{xx}+\phi_{yy}+\phi_{zz}=0$. Computing gives

$$\phi_{xx}=\frac{\phi'}{r}+\frac{x^2\phi''}{r^2}-\frac{x^2\phi'}{r^3},$$

and similarly for ϕ_{yy} and ϕ_{zz}.

Thus Laplace's equation becomes $\phi''+2\phi'/r=0$, which, upon integration, yields $\phi=-1/r$. This potential has force-field,

$$-(\phi_x,\phi_y,\phi_z)=-\left(\frac{x}{r^3},\frac{y}{r^3},\frac{z}{r^3}\right),$$

whose magnitude is $1/r^2$.

Therefore, Newtonian physics also may be regarded as satisfying Einstein's principle, that in Nature is actualized the ideal of mathematical simplicity.

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Another Hysterical Gram

There was a statistics prof
Who went too oft to the trough
He acquired a mode
At his navel node
Which was more than he could doff.

A standard snore with a standard score.

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