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Euler's Other Proof

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$$\frac{\pi^2}{6} = \frac{4}{3}\frac{(\arcsin 1)^2}{2} = \frac{4}{3}\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$$

$$= \frac{4}{3}\int_0^1 \frac{x + \sum_{n=1}^{\infty} \frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots 2n}\frac{x^{2n+1}}{2n+1}}{\sqrt{1-x^2}}\,dx$$

$$= \frac{4}{3}\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx + \frac{4}{3}\sum_{n=1}^{\infty}\frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots 2n(2n+1)}\int_0^1 x^{2n}\frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \frac{4}{3} + \frac{4}{3}\sum_{n=1}^{\infty}\frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots 2n(2n+1)}\left[\frac{2n(2n-2)\cdots 2}{(2n+1)(2n-1)\cdots 3}\right]$$

$$= \frac{4}{3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = \frac{4}{3}\left(\sum_{n=1}^{\infty}\frac{1}{n^2} - \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2}\right) = \sum_{n=1}^{\infty}\frac{1}{n^2}.$$

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