in the theorem. This theorem is rather conservative and one can generally get much smaller bounds on the errors in approximation that hold for intuitively worst-case situations—thus, arguably everywhere.

There are other opportunities for undergraduate research that faculty members could craft. One interesting problem would be to work out the details of testing a null hypothesis that data is from a Poisson distribution by comparing the Poisson fit to the data with that of the best fitting almost-binomial distribution. If one suspected the data would be under-dispersed relative to the Poisson distribution (before collecting data), a one-sided test of this sort may be an interesting competitor to the Poisson dispersion test.

Other possible problems would include looking at some of the material in [**2**] that used the Poisson-binomial distribution, namely logistic regression and conditional Bernoulli models. The almost-binomial approximation to the Poisson-binomial may be of interest here.

References

1. A. D. Barbour, L. Holst, and S. Janson, *Poisson Approximation*, Oxford University Press, 1992.
2. S. Chen, and J. Liu, Statistical applications of the Poisson-binomial and conditional Bernoulli distributions, *Statistica Sinica* **7** (1997) 875–892.
3. A. Jalal, Error bounds involving almost-binomial approximations of hypergeometric probabilities, *Pi Mu Epsilon Journal*, **11** (2001) #4 187–193.
4. P. Thompson, Almost-binomial random variables, technical report, Wabash College, 1999.

The Roots of a Quadratic

Leonard Gillman (len@math.utexas.edu), The University of Texas, Austin, TX 78712

In a recent discussion among several math teachers, both college and precollege, someone remarked that we do students a disservice when we let them "solve" a quadratic equation by means of the formula without having them check their answers. The obvious question at this point is just how the students are expected to do the checking. Most of the group agreed that substituting into the quadratic is too hard, at least for beginning students, especially when complex numbers are involved.

Regarding this last assertion, observe that there is no need to use the i-notation in order to do the checking. Consider the equation $f(x) = 0$, where

$$\begin{aligned} f(x) &= ax^2 + bx + c \qquad (1) \\ &= 2x^2 - 5x + 7. \end{aligned}$$

I include enough detail to illustrate the mechanism. Introduce the symbol

$$u = \sqrt{b^2 - 4ac} = \sqrt{-31}, \text{ and note that } u^2 = -31.$$

The solutions given by the quadratic formula are $\frac{5 \pm u}{4}$. For the solution $s = \frac{5+u}{4}$, say, we get

$$f(s) = 2\left(\frac{5+u}{4}\right)^2 - 5\left(\frac{5+u}{4}\right) + 7$$

$$= 2\left(\frac{25 + 10u + u^2}{16}\right) - \frac{25}{4} - \frac{5u}{4} + 7$$

$$= \frac{1}{8}\,[(25 + 10u - 31) - 50 - 10u + 56]$$

$$= \frac{1}{8}\,[(10u - 6) - 10u + 6] = 0,$$

verifying that s is a root.

But we can check for errors without substituting into the equation. A well-known (and easily proved) theorem tells us that the sum of the roots of (1) is $-b/a$ and the product is c/a. So test the sum and product; if either fails the test then you know your solution-pair is wrong. In the example, call the two solutions s_1 and s_2:

$$s_1 = \frac{5+u}{4}, \quad s_2 = \frac{5-u}{4}. \tag{2}$$

Then

$$s_1 + s_2 = \frac{5+u}{4} + \frac{5-u}{4} = \frac{5}{2} = -\frac{b}{a},$$

and

$$s_1 s_2 = \left(\frac{5+u}{4}\right)\left(\frac{5-u}{4}\right) = \frac{25-u^2}{16} = \frac{25+31}{16} = \frac{7}{2} = \frac{c}{a},$$

so we have failed to prove we made a mistake.

Moreover, these test results show that s_1 and s_2 are in fact the actual roots—that is, the converse of the theorem quoted above is a theorem. To see this, let f be as in (1) and let r_1 and r_2 be numbers satisfying

$$r_1 + r_2 = -\frac{b}{a}, \quad r_1 r_2 = \frac{c}{a}. \tag{3}$$

Then

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$
$$= a\left[x^2 - (r_1 + r_2)x + r_1 r_2\right] = a(x - r_1)(x - r_2),$$

whose roots are precisely r_1 and r_2.

I remark that the same idea works for polynomials of arbitrary degree, though admittedly the practical usefulness of this fact appears doubtful. For instance, if r_1, r_2, r_3 are the roots of

$$g(x) = ax^3 + bx^2 + cx + d \quad (a \neq 0), \tag{4}$$

then the general theorem on sums and products of roots tells us that

$$r_1 + r_2 + r_3 = -\frac{b}{a}, \quad r_1r_2 + r_2r_3 + r_3r_1 = \frac{c}{a}, \quad r_1r_2r_3 = -\frac{d}{a}, \tag{5}$$

and an argument analogous to the one given for the quadratic case shows that any numbers r_1, r_2, r_3 satisfying (5) are necessarily the three roots of the cubic polynomial (4).