



## Pi Mu Epsilon

Pi Mu Epsilon is a national mathematics honor society with 382 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members. Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. Pi Mu Epsilon encourages scholarly activity in its chapters with grants to support mathematics contests and regional meetings established by the chapters and through its Lectureship program that funds Councillors to visit chapters. Since 1952, Pi Mu Epsilon has been holding its annual National Meeting with sessions for student papers in conjunction with the summer meetings of the Mathematical Association of America (MAA).



## MAA Student Chapters

The MAA Student Chapters program was launched in January 1989 to encourage students to continue study in the mathematical sciences, provide opportunities to meet with other students interested in mathematics at national meetings, and provide career information in the mathematical sciences. The primary criterion for membership in an MAA Student Chapter is “interest in the mathematical sciences.” Currently there are approximately 550 Student Chapters on college and university campuses nationwide.

## Schedule of Student Activities

Unless otherwise noted, all events are held at the Hilton Portland

Please note that there are no MAA Sessions #5 & 6 or #19-26.

### Wednesday, August 6

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
2:00 pm - 4:00 pm	CUSAC Meeting .....	3rd Floor, Executive Suite
4:30 pm - 5:30 pm	MAA/PME Student Reception .....	Plaza Level, Pavilion East
5:30 pm - 6:15 pm	Math Jeopardy .....	Plaza Level, Pavilion East

### Thursday, August 7

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:30 am - 11:30 am	PME Council Meeting .....	Executive Tower, Salon Ballroom I
8:30 am - 10:25 am	MAA Session #1 .....	3rd Floor, Forum Suite
	MAA Session #2 .....	3rd Floor, Council Suite
	MAA Session #3 .....	3rd Floor, Directors Suite
	MAA Session #4 .....	3rd Floor, Studio Suite
9:00 am - 5:00 pm	Student Hospitality Center .....	Exhibit Hall
1:00 pm - 1:50 pm	The Jean Bee Chan & Peter Stanek Lecture for Students .....	Ballroom Level, Grand Ballroom I
	MAA Session #7 .....	3rd Floor, Forum Suite
2:00 pm - 3:55 pm	MAA Session #8 .....	3rd Floor, Council Suite
	MAA Session #9 .....	3rd Floor, Directors Suite
	MAA Session #10 .....	3rd Floor, Studio Suite
2:00 pm - 3:55 pm	PME Session #1 .....	3rd Floor, Executive Suite
	PME Session #2 .....	3rd Floor, Senate Suite
4:00 pm - 6:15 pm	MAA Session #11 .....	3rd Floor, Forum Suite
	MAA Session #12 .....	3rd Floor, Council Suite
	MAA Session #13 .....	3rd Floor, Directors Suite
	MAA Session #14 .....	3rd Floor, Studio Suite
4:00 pm - 6:15 pm	PME Session #3 .....	3rd Floor, Executive Suite
	PME Session #4 .....	3rd Floor, Senate Suite
7:00 pm - 10:00 pm	Estimathon! .....	Plaza Level, Pavilion West

## Friday, August 8

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
7:30 am - 8:30 am	PME Advisor Breakfast Meeting .....	Ballroom Level, Galleria I
8:30 am - 11:45 am	MAA Session #15 .....	3rd Floor, Forum Suite
	MAA Session #16 .....	3rd Floor, Council Suite
	MAA Session #17 .....	3rd Floor, Directors Suite
	MAA Session #18 .....	3rd Floor, Studio Suite
9:00 am - 5:00 pm	Student Hospitality Center .....	Exhibit Hall
8:30 am - 11:45 am	PME Session #5 .....	3rd Floor, Executive Suite
	PME Session #6 .....	3rd Floor, Senate Suite
	PME Session #7 .....	23rd Floor, Skyline 2
1:00 pm - 1:50 pm	MAA Student Activity: Mathematical Research, It's Knot What You Think! .....	Plaza Level, Pavilion East
1:00 pm - 1:50 pm	MAA Student Activities Session: Using Puzzles to Illuminate Mathematics .....	Ballroom Level, Grand Ballroom II
2:00 pm - 3:55 pm	PME Session #8 .....	3rd Floor, Executive Suite
	PME Session #9 .....	3rd Floor, Senate Suite
	PME Session #10 .....	3rd Floor, Forum Suite
	PME Session #11 .....	3rd Floor, Council Suite
2:35 pm - 3:55 pm	Non-Academic Career Paths Panel .....	23rd Floor, Skyline 2
6:00 pm - 7:45 pm	Pi Mu Epsilon Centennial Celebration Banquet .....	Plaza Level, Broadway
8:00 pm - 8:50 pm	Pi Mu Epsilon J. Sutherland Frame Lecture .....	Ballroom Level, Grand Ballroom
9:00 pm - 10:00 pm	MAA Ice Cream Social and Undergraduate Awards Ceremony .....	Plaza Level, Broadway

## Saturday, August 9

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
9:00 am - 12:30 pm	Student Hospitality Center .....	Exhibit Hall
9:00 am - 10:30 am	MAA Modeling (MCM) Winners .....	Plaza Level, Broadway III & IV
1:00 pm - 2:15 pm	Student Problem Solving Competition ...	Ballroom Level, Galleria I

## **Jean B. Chan and Peter Stanek Lecture for Students**

### **THE FOUNDING OF PI MU EPSILON 100 YEARS AGO**

**Jack Graver**

Syracuse University

On December 8, 1913 the Mathematical Club of Syracuse University met on the 10th anniversary of its founding. “Miss Florence Richert presented the paper The Evaluation and Transcendence of Pi. Discussion followed the reading of the paper. There was a short business meeting before refreshments at which “Dr. Roe gave the report of committee appointed to consider changing the Club into a Mathematical Fraternity. The decision was made to proceed with this project and during the Spring semester of 1914 the details were worked out and PME was incorporated in New York State.

What did they want to accomplish by taking this action? What was background against which the decision to found PME was made? Using the Archives of Syracuse University and the detailed notes left by the founders, I hope to answer these questions and to develop an understanding of the historical context in which PME was born.

## MAA Undergraduate Student Activities Sessions

### MATHEMATICAL RESEARCH, IT'S KNOT WHAT YOU THINK!

**Lew Ludwig**

Denison University

*Hilton Portland, Plaza Level, Pavillion East*

Take a piece of string, tie a knot into it and glue the ends of the string together into a closed loop, and you have a mathematical knot. This simple process demonstrates the appeal of knot theory it is very hands-on and accessible to undergraduate research. In this student activity, we will work with two representations of knots: knot mosaics and cubic lattice knots. Using knot kits that participants can keep, we will explore the very new area of knot mosaics (2008, Lomonaco and Kauffman) and look at open problems that you can try to solve. We will also try to construct cubic lattice knots. Very little is known about these three-dimensional knots, so participants will be conducting on-the-spot mathematical research. This activity is open to everyone; no prior knowledge of knots is required.

### USING PUZZLES TO ILLUMINATE MATHEMATICS

**Jonathan D. Stadler**

Capital University

*Hilton Portland, Ballroom Level, Grand Ballroom II*

Solving a puzzle is a lot like solving a challenging problem in a math class. The “aha!” moment that is experienced in both circumstances is nearly identical, so it is natural that mathematicians enjoy puzzles and find engaging applications for them. Come explore a class of puzzles and their applications in courses such as discrete math. Participants will build their own version of an ancient Chinese linking puzzle to take with them.

## **J. Sutherland Frame Lecture**

### **FIBONACCI AND THE FIRST PERSONAL COMPUTING REVOLUTION**

**Keith Devlin**

Stanford University

The first personal computing revolution took place not in Silicon Valley in the 1980s but in Pisa in the 13th Century. The medieval counterpart to Steve Jobs was a young Italian called Leonardo, better known today by the nickname Fibonacci. Thanks to a recently discovered manuscript in a library in Florence, the story of how this genius, about whom we know so little, came to launch the modern commercial world can now be told.

Based on Devlin's book *The Man of Numbers: Fibonacci's Arithmetical Revolution* (Walker & Co, July 2011) and his co-published companion e-book *Leonardo and Steve: The Young Genius Who Beat Apple to Market by 800 Years*.

*The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer MathFests. He continually offered insight and inspiration to student mathematicians at these summer meetings.*

## Pi Mu Epsilon Speakers

<b>Name</b>	<b>School</b>	<b>MAA Session</b>
Camron Bagheri	Youngstown State University	7
Rachel Bass	Armstrong Atlantic State University	10
Robin Belton	Kenyon College	4
Meagan Benavides	Texas State University	10
Andrew Bishop	Willamette University	10
Sam Blakely	Saint Michael's College	11
Matthew Buhr	University of South Dakota	6
Andre Bunton	University of Alaska Southeast	9
Johnathan Bush	The University of Montana	4
Aaron Calderon	University of Nebraska-Lincoln	3
Daniel Catello	Youngstown State University	1
Sharat Chandra	University of California, Irvine	8
Justin Cook	Austin Peay State University	6
Heather Cook	Roanoke College	7
Noah Davis	Aquinas College	2
Martin Dean	Gonzaga University	9
Matthew DeVilbiss	University of Dayton	2
Shawn Doyle	Youngstown State University	1
Marcus Elia	SUNY Geneseo	5
Roger Estep	Marshall University	6
Andrew Evans	Hendrix College	7
Elizabeth Field	University of Illinois at Urbana-Champaign	9
James Ford	SUNY Geneseo	7
Melissa Gaddy	Wofford College	11
Maria Glenski	Saint Mary's University of Minnesota	11
Max Goering	Kansas State University	7
Elliot Golias	Kent State University	3
William Graft	Creighton University	4
Opal Graham	University of North Florida	3
Elizabeth Greco	Kenyon College	4
Heather Gronewald	Southwestern University	6
Elisha Hall	Austin Peay State University	6
Marissa Hartzheim	St. Norbert College	2
Kimberly Hildebrand	University of Mary Washington	7
Tung Hoang	Millsaps College	10
Robert Hoffman	University of California, Irvine	11
Edna Jones	Rose-Hulman Institute of Technology	5
Michael Kaufman	Kent State University	3

## Pi Mu Epsilon Speakers (Continued)

Name	School	MAA Session
Jake Kearley	Elmhurst College	4
Victoria Kelley	James Madison University	6
Krista Kernodle	University of California, Irvine	1
Dorothy Klein	Kent State University	2
Dodji Kuwonu	Austin Peay State University	9
Eric Lai	University of California - Irvine	3
Matthew Larson	Hendrix College	5
Greg Leclerc	Fitchburg State University	6
Robert Lehr	Southwestern University	5
William Linz	Texas A&M University	3
Alphonso Lucero	Iowa State University	3
Crystal Mackey	Youngstown State University	7
Dayna Mann	Pepperdine University	8
Jonathan Marino	Roanoke College	5
Ryan Matzke	Gettysburg College	10
Taylor McClanahan	University of Arkansas at Little Rock	1
Taylor Miller	St. Norbert College	2
Daniel Miller	Texas A&M University	8
Nakhila Mistry	Elon University	8
James Munyon	Youngstown State University	7
Lisa Naples	Fairfield University	4
Erika Ordog	Pepperdine University	10
Ashley Orr	Youngstown State University	1
Samantha Parsons	Roanoke College	6
Emily Paulson	University of Wisconsin Whitewater	6
Kimberly Phillips	Washington & Jefferson College	9
Adam Purcilly	Saint Peter's University	9
Shakil Rafi	Troy College	2
Adam Robinson	Portland State University	8
Fernando Roman	Kansas State University	4
Jenny Rustad	Luther College	6
Timothy Shaffer	Youngstown State University	7
Eric Shehadi	Youngstown State University	1
Joseph Sheppard	Kent State University	5
Garrett Shontrice	Jackson State University	8
Samantha Sprague	Marist College	5
Kelsey Swerdfeger	Concordia University Irvine	5
Sara Tandon	Pepperdine University	5
Shyam Vasanjee	University of North Florida	5
Erlan Wheeler	Carthage College	11
Jenna Wise	Youngstown State University	7
Barton Yadlowski	Kent State University	11



## MAA Student Speakers

Name	School	MAA Session
Bradley Abramson	Penn State Erie, The Behrend College	16
Michael Ackerman	Georgia Southern University	4
Daniel Adams	Utah Valley University	11
Pallavi Aggarwal	California Institute of Technology	1
Isaac Ahern	University of Alaska Anchorage	7
Michel Alexis	Northwestern University	3
Ayah Almousa	University of Wisconsin	15
Atl Arredondo	San Diego City College	9
Stanislav Atanasov	Yale University	17
Rob Rexler Baello	Montclair State University	13
Brenna Baker	Mills College	7
Taylor Ball	Indiana University Bloomington	3
Brealyn Beals	Grand Valley State University	3
Gaston Beaucage	British Columbia Institute of Technology	10
Baoyue Bi	Iowa State University	12
Wyatt Boyer	Williams College	14
Joseph Brosch	University of Delaware	2
Bryan Brown	Pomona College	7
Paula Burkhardt	Pomona College	17
Ian Calaway	Macalester College	3
Christopher Cericola	Seattle University	18
John Claxton	Loyola University Chicago	12
Ian Coleman	Tarleton State University	8
Donald Coleman	Essex County College	13
Nicholas Connolly	Kenyon College	13
Hannah Constantin	Yale University	12
Anna Cooke	McDaniel College	16
Michael Cork	Pomona College	4
Joann Coronado	Texas A&M University - Corpus Christi	15
Raameon Cowan	California State University, Northridge	16
Ram Dahal	Howard University	9
Julia Dandurand	California State University Northridge	7
Doug Decock	Western Oregon University	3
Benjamin Delaney	George Fox University	1
David DeSimone	Rutgers University	9
Andrew Deveau	Yale University	9
Thierno Diallo	New York City College of Technology	8
Hang Do	Linfield College	3

## MAA Student Speakers (Continued)

Name	School	MAA Session
Elizabeth Doebel	Iowa State University	12
Robert Dorward	Oberlin College	18
Mohamed EL Ghodhbane	U.S. Air Force Academy	17
Kathryn Flaharty	Canterbury School	18
Derek Francour	University of Wisconsin Madison	16
Robin French	Elon University	15
John Garwood	Austin Peay State Universtiy	14
Louis Gaudet	Yale University	9
Jonathan Gerhard	James Madison University	18
Mathew Gomez	University of North Texas at Dallas	14
Feixue Gong	Williams College	15
Harris Greenwood	St. Edward's University	17
Armando Grez	Florida Gulf Coast University	4
Anna Grim	University of St. THomas	2
Thomas Grubb	Michigan State University	18
Daniel Gulbrandsen	Utah Valley University	11
Jonathan Guzman	California State University, Long Beach	3
Madeleine Hanson-Colvin	Bryn Mawr College	3
Reed Haubenstock	Harvey Mudd College	14
Minda He	University of California - Irvine	11
Daniel Hirsbrunner	University of Illinois	16
Youkow Homma	Yale University	12
Hannah Horner	Whitman College	2
B. Houston-Edwards	Yale University	12
Taylor Huettenmueller	Emporia State University	17
Esther Hunt	George Fox University	1
Daniel Irvine	University of Notre Dame	7
Micah Jackson	Georgia Southern University	16
Stephanie Jones	Woodburn Wellness, Business, and Sports School	13
Samuel Justice	Kenyon College	17
Jenna Kainic	Yale University	9
Elliot Kaplan	Ohio University	10
Jessica Kawana	Willamette University	13
Mary Kemp	Occidental College.	9
Abdullah Khan	University of Texas at Dallas	18
Joshua Kiers	Taylor University	18
Danielle King	Southwestern University	8
Ethan Kowalenko	Metropolitan State University of Denver	12
Mitchell Krock	Bradley University	18
Eric Lai	University of California - Irvine	11
Justine Langman	Rutgers University	8
Dana Lapides	Lafayette College	13

## MAA Student Speakers (Continued)

Name	School	MAA Session
Tram Le	University of North Georgia	4
Alyssa Loving	University of Hawaii at Hilo	7
Benjamin Lovitz	Bates College	9
Mark Lugo	UNT Dallas	10
Susan Maslak	Ave Maria University	9
Marie Mauhar	Lenoir-Rhyne University	1
Jay Mayfield	Arkansas State University	4
Kelly McCurry	Augustana College	11
Autumn McMunn	University of North Alabama	18
Olivia Meyerson	Williams College	15
Jeremy Meza	Carnegie Mellon University	13
Alexander Middleton	Winthrop University	2
Peter Mielcarek	Dominican University	17
Joshua Mireles	University of Nebraska-Lincoln	9
Daniel Mitropolsky	Yale University	9
Javier Mondragon	University of North Texas - Dallas	10
Brent Moran	University of Colorado, Denver	3
Shyam Narayanan	Blue Valley West High School	16
Nwafor Nwoke	Benedictine University	2
Patricia O'Brien	The University of Texas	10
Timothy O'Connor	University of St. Thomas, Macalester College	1
Chloe Ondracek	Minot State University	17
Amrei Oswald	University of Puget Sound	16
Charles Payne	Southwestern University	8
Kelsey Pearson	University of St. Thomas	2
Kim Pham	University of California - Irvine	11
Rebecca Post	Augustana College	11
Carlin Purcell	Vassar College	18
Dylan Quintana	Carnegie Mellon University	9
John Rahmani	Virginia Tech	17
Jose Ramirez	San Diego City College	9
Lindsey Reppuhn	Kalamazoo College	18
Jasmine Rivers	Montclair State University	2
Austin Rodgers	University of Notre Dame	15
Tate Rogers	Western Connecticut State University	17
Delia Rojas	University of North Texas at Dallas	10
Jun Hwan Ryu	Yale University	12
Eric Samelson	Willamette University	3
Joanna Sasara	Dominican University	8

## MAA Student Speakers (Continued)

Name	School	MAA Session
Nikolas Schonsheck	Vassar College	3
Labeeby Servatius	Assumption College	1
Davis Shurbert	University of Puget Sound	3
Jeremy Siegert	Boise State University	15
Timothy Singer	Linfield College	3
Katie Sipes	James Madison University	8
Ryan Slechta	University of St. Thomas	1
Geoffrey Smith	Yale University	15
Nicole Soltz	Elon University	15
Hasan Sumdani	University of Texas at Arlington	8
Matthew Sutter	University of Michigan - Flint	17
Ryan Swartzentruber	Eastern Mennonite University	1
Sarah Tammen	University of Georgia	7
Preston Thompson	British Columbia Institute of Technology	4
Benjamin Tong	Yale University	12
Hanna Torrence	University of Chicago	18
Jose Torres	Montclair State University	2
George Trejo	Western Connecticut State University	17
Julie Vega	Burgundy Farm Country Day School	3
Adam Volk	University of Dayton	9
Yi Wang	Milburn High School, New Jersey	13
Zihao Wang	George Fox University	10
Abigail Ward	The University of Chicago	15
Nicholas Wawrykow	Yale University	9
Robert Weber	University of Illinois at Urbana-Champaign	13
Zach Weiner	Kenyon College	16
Theodore Weisman	Yale University	9
Rodney Wells	Penn State Erie: The Behrend College	16
Lauren White	California State University, Northridge	16
Sam Whitfield	McDaniel College	16
Bithiah Yuan	University of Hawaii at Hilo	16
Ksenia Zakirova	Harvey Mudd College	4
Tong Zhang	University of Illinois	16
Yukang Zhang	University of California - Irvine	11
Kelsey Zimmermann	Benedictine University	1

## MAA Session #1

3rd Floor, Forum Suite

8:30A.M. – 10:25A.M.

8:30–8:45

**No Talk**

Speaker Canceled

NA

8:50–9:05

**A Geometry of Sets**

Pallavi Aggarwal and Ryan Swartzentruber

*California Institute of Technology and Eastern Mennonite University*

The Hausdorff metric provides a way to measure distance between certain subsets of Euclidean space. This metric is important for its applications in fractal geometry, image matching, visual recognition by robots, and computer-aided surgery. We have used this metric to extend geometric notions from Euclidean geometry to create a geometry of sets. We will discuss interesting, and unexpected, aspects of this geometry, with particular emphasis on notions of orthogonality of sets and how orthogonality might lead to a trigonometry of sets. This research was conducted as part of the 2014 REU program at Grand Valley State University.

9:10–9:25

**Facet Description for Polytopes of Group-Based Models**

Marie Mauhar

*Lenoir-Rhyne University*

Given a group-based Markov model on a tree, one can compute the vertices of a polytope which describe the associated toric variety. In the case of  $\mathbb{Z}_2$  or  $\mathbb{Z}_2 \times \mathbb{Z}_2$ , these polytopes have applications in the field of phylogenetics. For other groups, these could describe different Markov processes. We provide a facet description for a class of these polytopes. Such a description is a key component in describing the geometry of these varieties.

9:30–9:45

**Fun with Cubes and Hexagonal Pyramids**

Esther Hunt and Benjamin Delaney

*George Fox University*

In *The College Mathematics Journal*, Volume 25 No. 2 in March 2014, authors Alsina, Nelsen, and Unal used a visual representation to wordlessly prove that the difference of consecutive integer cubes is congruent to 1 modulo 6. In our work, we created a visual model to generalize this statement: that for any integers  $n$  and  $a$ ,  $(n + a)^3 - n^3$  is congruent to  $a$  modulo 6.

9:50–10:05

**Parallel Parking with Ruler and Compass**

Labeeby Servatius

*Assumption College*

In this talk it will be explained how Euclidean geometry can be used to answer some questions about how to drive a car. Questions such as the following:

Which parts of the car are in danger of collision when turning?

At what point should one reverse the steering when parallel parking?

What kind of car is best for parallel parking?

How accurate are parallel parking simulators?

Why is it better to parallel park backing up, rather than driving forwards?

10:10–10:25

**Reassembling Humpty Dumpty: 3D Puzzles and Invariant Signature Curves**

Timothy O'Connor and Ryan Raymond Slechta

*The University of Minnesota and University of St. Thomas*

The end goal of this research is to virtually reconstruct a broken ostrich egg by devising an algorithm to automatically solve 3D jigsaw puzzles. In order to determine which egg pieces fit together, we look to use “snake” techniques to extract invariant signature curves and easily compare the egg piece boundaries. A method currently exists to assemble 2D puzzles, and we address the complications associated with extending it to 3D. In addition, we attempt to develop an efficient method to randomly generate 3D puzzles with no severe restrictions on shape or arrangement, enabling us to easily test the efficiency of the reassembly method.

## MAA Session #2

3rd Floor, Council Suite

8:30A.M. – 10:25A.M.

8:30–8:45

**A New Approach to Understanding Evolutionary Relationships  
Among Chagas Disease Vectors in Latin American Countries**

Jasmine Rivers and Jose Torres

*Montclair State University*

Our research applies a graph connectivity index to DNA sequence data of Chagas disease vectors from Latin American countries. It provides a new approach to understanding the evolutionary relationships among the insect vectors. We perform normalization and principle component analysis on the graph indices of the DNA data and then build phylogenetic trees in order to answer scientific questions about the disease.

This research is funded by MAA NREUP through NSF grants DMS-1156582 and DMS-1359016.

8:50–9:05

**A Nonlinear Model of Cancer Tumor Treatment with Cancer Stem Cells**

Alexander Middleton and Hannah Horner

*Winthrop University and Whitman College*

According to the American Cancer Society, cancer is one of the leading causes of death, second only to heart disease. We present a system of nonlinear, first-order, ordinary differential equations that describes tumor growth based on healthy cell, tumor cell, and cancer stem cell populations. We include terms within our model which reflect the differing effects of Chemotherapy and Anti-Angiogenic Agents to respective cell populations. We perform stability analysis on the equilibrium solutions to predict the long-term behavior of the cell populations. Special emphasis is placed on the biological implications of numerical simulations and the effects to the host (i.e. cure states or recurrence of the tumor).

9:10–9:25

**A Pulse Vaccination SVEIR Model with a Saturation Incidence Rate and Time Delay**

Kelsey Zimmermann

*Benedictine University*

Pulse vaccination is an effective strategy for controlling or eradicating an infectious disease within a population. We consider an SVEIR epidemic model with pulse vaccination, a nonlinear saturation incidence rate, and a time delay for individuals moving from the exposed class to the infectious class. This model is a variation of the SEIRS pulse vaccination model due to Gao, et al. 2007. We determine the conditions for which the disease-free periodic solution is globally attractive and for which the disease persists.

9:30–9:45

**A Stage Structured Integrated Pest Management with a Holling Type II Functional Response**

Nwafor Nwoke

*Benedictine University*

We present an impulsive integrated pest management model utilizing a combination of pesticide, a predator species, and a disease in the prey species. The model incorporates stage structure for both the predator and prey species and assumes that the prey species reproduces according to birth a pulse. We use a full saturation incidence rate for the spread of the disease within the prey population and a Holling Type II functional response for the predation. We determine the condition for which the pest (prey) free solution is globally asymptotically stable and for which the system is permanent. We also consider implications of varying of the timing of the impulsive events, which include the birth pulse, the spraying of pesticide, and the introduction of the predator and diseased prey.

9:50–10:05

**Applications of Mathematical Modeling in Diagnosing Breast Cancer**

Anna Grim and Kelsey Pearson

*University of St. Thomas*

Mammography persists as a challenge in radiology because the structural differences between benign and malignant tumors are indistinguishable to the human eye. However, the application of signature curves, fractal dimensions, and cumulative distance histograms can diagnose breast tumors by mathematically analyzing contours. Our signature curvature methodology quantifies a tumor contour  $C$  by the rigidly invariant curvature parametrization  $C = \{\kappa(t), \kappa'(t)\}$ , where  $\kappa(t)$  is curvature and  $\kappa'(t)$  is the derivative of curvature. The malignancy of a tumor is determined by the frequency and density of zero curvature points, where either  $\kappa(t) = 0$  or  $\kappa'(t) = 0$ .

Our methodology extends to using fractal dimensions, which are a numerical measure of the complexity and irregularity of an object. Using the box counting method to calculate the fractal dimension, tumors can be diagnosed as cancerous or benign. A higher fractal dimension corresponds to a more complex and irregular contour, indicating a cancerous tumor; in contrast, a lower fractal dimension indicates a benign tumor.

The final method we use is invariant distance histograms, which are formed by collecting a sample of distances between random points on the contour. Benign and malignant invariant distance histograms are compiled to create two respective cumulative distance histograms. The curve of best fit for these histograms can be compared for diagnosis because the curves of benign tumors differ from those of malignant tumors.

Our methodology has been implemented on over 150 tumors, demonstrating a correlation between curvature complexity and malignancy. Signature curves, fractal dimensions, and distance histograms are developing technologies in computer vision that have the potential to expedite diagnosis.



10:10–10:25

**Characterization of Blink Motion Through Curve Fitting**

Joseph Brosch

*University of Delaware*

The motion of the upper eyelid during blinking can be important in determining possible diseases and syndromes that affect the eye. In particular, combining machine learning with a low-dimensional model for the blink motion of the upper eyelid might yield an effective diagnostic tool for abnormalities in blinking and potentially for Dry Eye Syndrome (DES). Nonlinear least squares fitting was used to fit the position of the center of the upper lid during blinks of different subjects under four experimentally controlled conditions. The coefficients from the hypothesized lid motion functions are used for classification purposes. Preliminary use of clustering algorithms and support vector machines on the fit coefficients, suggests possible automatic identification of normal or abnormal blinks based solely on the nonlinear fits.

## MAA Session #3

3rd Floor, Directors Suite

8:30A.M. – 10:25A.M.

8:30–8:45

**Interval  $k$ -graphs**

Doug Decock

*Western Oregon University*

Interval  $k$ -graphs, introduced in 2002, are the interval graphs in which vertices correspond to intervals of  $k$  possible colors, and edges correspond to nonempty intersections of differently-colored intervals. Interval 2-graphs are traditionally called interval bigraphs and have been studied extensively. In this talk we will discuss recent work that has been done on interval  $k$ -graphs with  $k > 2$ .

8:50–9:05

**Competitive Graph Coloring (Part I)**

Michel Alexis and Davis Shurbert

*Northwestern University and University of Puget Sound*

The Graph Coloring Game has two players: Alice and Bob. The players alternate coloring the uncolored vertices (or edges) of a finite graph  $G$  from a fixed finite set of colors. At each step of the game, the players must choose to color an uncolored vertex (or edge) with a legal color. In the basic formation of the game, a color is legal for an uncolored vertex if the vertex has no neighbors already colored with that color. Alice wins the game if all vertices of the graph are colored; otherwise, Bob wins. The least  $r$  such that Alice has a winning strategy for this game on  $G$  is called the game chromatic number of  $G$ . If the definition of legal color is altered so that at each step in the game, each color class must have maximum degree at most  $d$ , for some fixed integer  $d > 0$ , then the least  $r$  such that Alice has a winning strategy is called the  $d$ -relaxed game chromatic number of  $G$ . The classes of graphs for which the most is known are trees and forests. While upper bounds on the game chromatic number (and  $d$ -relaxed game chromatic number) are known for these classes, and the bounds are known to be achievable, it is not known what structural properties the graphs must have in order to achieve the bounds. This project examines these properties, and considers other variations of the game in which the definition of legal color is altered.

9:10–9:25

**Competitive Graph Coloring (Part II)**

Eric Samelson and Julie Vega

*Willamette University and Burgundy Farm Country Day School*

The Graph Coloring Game has two players: Alice and Bob. The players alternate coloring the uncolored vertices (or edges) of a finite graph  $G$  from a fixed finite set of colors. At each step of the game, the players must choose to color an uncolored vertex (or edge) with a legal color. In the basic formation of the game, a color is legal for an uncolored vertex if the vertex has no neighbors already colored with that color. Alice wins the game if all vertices of the graph are colored; otherwise, Bob wins. The least  $r$  such that Alice has a winning strategy for this game on  $G$  is called the game chromatic number of  $G$ . If the definition of legal color is altered so that at each step in the game, each color class must have maximum degree at most  $d$ , for some fixed integer  $d > 0$ , then the least  $r$  such that Alice has a winning strategy is called the  $d$ -relaxed game chromatic number of  $G$ . The classes of graphs for which the most is known are trees and forests. While upper bounds on the game chromatic number (and  $d$ -relaxed game chromatic number) are known for these classes, and the bounds are known to be achievable, it is not known what structural properties the graphs must have in order to achieve the bounds. This project examines these properties, and considers other variations of the game in which the definition of legal color is altered.

9:30–9:45

**Competitive Graph Coloring (Part III)**

Hang Do, Brent Moran, and Timothy Singer,

*Linfield College, University of Colorado, Denver, and Linfield College*

The Graph Coloring Game has two players: Alice and Bob. The players alternate coloring the uncolored vertices (or edges) of a finite graph  $G$  from a fixed finite set of colors. At each step of the game, the players must choose to color an uncolored vertex (or edge) with a legal color. In the basic formation of the game, a color is legal for an uncolored vertex if the vertex has no neighbors already colored with that color. Alice wins the game if all vertices of the graph are colored; otherwise, Bob wins. The least  $r$  such that Alice has a winning strategy for this game on  $G$  is called the game chromatic number of  $G$ . If the definition of legal color is altered so that at each step in the game, each color class must have maximum degree at most  $d$ , for some fixed integer  $d > 0$ , then the least  $r$  such that Alice has a winning strategy is called the  $d$ -relaxed game chromatic number of  $G$ . The classes of graphs for which the most is known are trees and forests. While upper bounds on the game chromatic number (and  $d$ -relaxed game chromatic number) are known for these classes, and the bounds are known to be achievable, it is not known what structural properties the graphs must have in order to achieve the bounds. This project examines these properties, and considers other variations of the game in which the definition of legal color is altered.

9:50–10:05

**Cut to the Chase: A Game of Cops and Robbers on Graphs**

Nikolas Schonsheck, Taylor Ball,

Madeleine Hanson-Colvin, and Jonathan Guzman

*Vassar College, Indiana University Bloomington,**Bryn Mawr College and California State University, Long Beach*

The game of Cops and Robbers is a perfect information, vertex-pursuit game where a set of “cops” and a “robber” occupy vertices in the graph  $G$ . Both players (the cops act as one player) alternate turns, either moving to adjacent vertices or passing. The cops win if they capture the robber by occupying the robber’s vertex whereas the robber wins if he perpetually avoids capture. The cop-number of a graph,  $c(G)$ , is the least number of cops needed to guarantee the cops can always capture the robber. We explore specialized classes of graphs and their associated strategies. Through this, we aim to better characterize the game played on these graphs and offer new bounds on the cop number.

10:10–10:25

**Graph Theoretic Models of Interdependent Preferences in Referendum Elections**

Brealyn Beals and Ian Calaway

*Grand Valley State University and Macalester College*

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when a voter’s preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. These kinds of interdependencies cannot be fully expressed in a simultaneous election, which forces voters to separate issues that may be linked in their minds. In such an election, the resulting outcomes can be unsatisfactory or even paradoxical. In this talk, we will use graph theoretic models to characterize, construct, and better understand interdependent voter preferences in referendum elections. We will also explore connections between our graph theoretic models and prior research on the structure of interdependent multidimensional preferences. This work was completed as part of the Grand Valley State University Summer Mathematics REU.

## MAA Session #4

3rd Floor, Studio Suite

8:30A.M. – 10:25A.M.

8:30–8:45

**Anomaly Detection Using Neural Networks**

Preston Thompson

*British Columbia Institute of Technology*

Using only vibration data generated from a simplified model of a bridge and random traffic on it, we explore the efficacy of various neural network algorithms in detecting subtle structural damages to the model. We also explore the impact of adding more complexity to the model and how it affects the ability of the algorithms to detect anomalies.

8:50–9:05

**New Upper Bounds on the Broadcast Domination Numbers of Grids**

Armando Grez

*Florida Gulf Coast University*

In graph theory a dominating set is a subset of vertices  $S$  such that all vertices in the graph is either in  $S$  or adjacent to some vertex in  $S$ . The domination number is the cardinality of the smallest dominating set of  $G$ . This study examines grid graphs given a broadcast center that has a transmission of weight 4 and we wish for all the vertices in the graph have a weight no less than 1 or be covered by more than a single center. We go further to reduce the lowest upper bound for  $(4, 1)$  dominating sets that was discovered in 2013. This is accomplished by a determined construction from a perfectly dominated grid in  $Z \times Z$ . Then, from this construction we predict the period with the diamond number proof (cardinality of the neighborhood), we derive the mapping from  $Z \times Z$  to  $Z_{period}$  as a homomorphism as well as several other properties for  $(4, 1)$  graph grids. We also discuss consequences of always being able to determine the construction for  $(t, 1)$  broadcast domination and finding smaller dominating sets to better approximate the domination number for large grids.

9:10–9:25

**Cluster Numbers and Percolation Models**

Michael Ackerman

*Georgia Southern University*

Percolation theory was officially introduced in 1957 to model the flow of fluids through porous material. This topic has since been extensively studied and generalized to many applications in industry, physics and related areas. In this presentation I will demonstrate the types of questions that are typically asked in percolation theory while discussing elements of graph theory, probability and difference equations. I will propose questions about higher dimensions while highlighting a few graphical methods and established theorems for two-dimensional lattices.

9:30–9:45

**Mathematical Approaches Nonlinear Spring with Complementarity Conditions**

Jay Mayfield

*Arkansas State University*

A mass on a nonlinear spring is considered. One end of the spring is attached to a wall, and the mass may come into contact with a rigid obstacle. We assume that friction between the mass and a surface is negligible. The stiffness of the spring will decrease with time due to damage on the spring. The equations of motion are formulated by differential inclusions. We use complementarity conditions to set up their corresponding time discretization. We propose numerical schemes and show the convergence theories. Numerical results are presented and discussed.

9:50–10:05

**Mathematical Modeling of Crack Formation in a Circular Plate**

Tram Le

*University of North Georgia*

When a point force (indenter) is applied to the center of a circular plate, symmetrical crack formation can take place. How many cracks will be formed? Assuming that the heat loss is negligible, the total energy spent in this process is used only to form a new surface of newly formed cracks and to bend the newly formed wedges. The energy used to form new surfaces of the cracks and the bending energy depend on the number of cracks,  $n$ , and the length of the cracks,  $L$ . Therefore, the total energy  $W$  is a function of two variables,  $n$  and  $L$ . Since all the processes in nature go in the direction of the minimum of energy spent, to find  $n$  and  $L$  we need to find a minimum of  $W(n, L)$ . This procedure is not simple since the expression for  $W(n, L)$  is complicated. Besides, during the minimization process, the final equation to find critical points,  $n$  and  $L$ , is transcendental. Therefore, we use special mathematical techniques which I will discuss in my presentation. We come up with an intriguing result that the number of cracks does not depend on the material's characteristics. This result has unique meanings to material science, especially to brittle materials.

10:10–10:25

**Modeling and exploring chain fountain dynamics**

Ksenia Zakirova and Michael Cork,

*Harvey Mudd College and Pomona College*

The chain fountain, an interesting phenomenon popularized by a viral YouTube video, occurs when the end of a chain contained in a beaker is drawn out. Gravity then continues to pull out the chain while forcing it above the side of the beaker into a curved shape. We model the steady state and dynamical behavior of the chain fountain and examine the shape of the chain based on boundary conditions.

## MAA Session #7

3rd Floor, Forum Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Double Bubbles in Hyperbolic Surfaces**

Alyssa Loving

*University of Hawaii at Hilo*

We seek the least-perimeter way to enclose and separate two prescribed areas in certain hyperbolic surfaces.

2:20–2:35

**Equal Circle Packing on a Square Flat Klein Bottle**

Brenna Baker and Julia Dandurand

*Mills College and California State University Northridge*

The study of maximally dense packings of disjoint equal circles is a problem in Discrete Geometry. The optimal densities and arrangements are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of equal circles onto a boundaryless container called a Klein bottle. Using numerous figures we will introduce all the basic concepts (including the notion of a Klein bottle, an optimal packing and the graph of a packing), illustrate some maximally dense arrangements, and outline the proofs of their optimality. This research was conducted as part of the 2014 REU program at Grand Valley State University.

2:40–2:55

**The Isoperimetric Problem in  $\mathbb{R}^n$  with Density  $r^p$** 

Sarah Tammen

*University of Georgia*

The isoperimetric problem with a density or weighting seeks to enclose prescribed weighted area with minimum weighted perimeter. According to Gregory Chambers' recent proof of the Log-Convex Density Conjecture, for a certain class of density functions on  $\mathbb{R}^n$ , isoperimetric regions are balls centered at the origin. We use similar methods of analyzing planar curves to investigate another conjecture that if  $\mathbb{R}^n$  has density  $r^p$ , where  $r$  is distance from the origin and  $p > 0$ , then isoperimetric regions are bounded by spheres that pass through the origin. This research is a joint project conducted by the Geometry group in the 2014 Williams College "SMALL" program and Gregory Chambers.

3:00–3:15

**On the Convex Body Isoperimetric Conjecture**

Bryan Brown

*Pomona College*

The Convex Body Isoperimetric Conjecture says that the least perimeter to enclose given volume inside an open ball in  $\mathbb{R}^n$  is greater than inside any other convex body of the same volume. The two-dimensional case has been proved by Esposito et al. for the case of exactly half the volume. We give some further partial results.

3:20–3:35

**Spaces of Affine Tensors**

Isaac Ahern

*University of Alaska Anchorage*

Killing vectors are generators of symmetries in a spacetime. This paper discusses certain generalizations of Killing vectors, including Killing tensors, affine vectors, and the recently investigated affine tensors. We define, and give some possible applications of the affine tensors. Further, we explicitly calculate the valence  $1, \dots, 6$  affine tensors in the Minkowski spacetime  $\mathbb{M}^4$ , of which the valences  $\geq 2$  have never been exhibited. We also discuss current work on  $|\mathcal{K}^n|$  and  $|\mathcal{A}^n|$ .

3:40–3:55

**Symplectic Embeddings of 6-dimensional Polydiscs**

Daniel Irvine

*University of Notre Dame*

Symplectic geometry is the mathematical language of mechanics. The subject grew out of a desire to better understand mechanical systems by reformulating them as problems in geometry. More precisely, symplectic geometry asks whether there is a Hamiltonian vector field generating a particular motion. This is a symplectic embedding problem, and the subtleties of this question are still unknown. Symplectic embeddings are well-understood for some simple 4-manifolds, such as ellipsoids and polydiscs; but in higher dimensions, very little is known. Define a relation  $\prec$  on symplectic manifolds according to the following rule:  $U \prec V$  if  $U$  symplectically embeds into  $(1 + \epsilon)V$  for  $\epsilon > 0$  arbitrarily small. Next, define a symplectic polydisc of radii  $r_1, r_2$ , and  $r_3$  to be the 6-manifold given by  $P(r_1, r_2, r_3) = \{(x_1, y_1, \dots, x_3, y_3) \in \mathbb{R}^6 \mid \pi(x_j^2 + y_j^2) \leq r_j, j = 1, 2, 3\}$ . In this presentation, I discuss how this relation gives a partial order on symplectic polydiscs of dimension 6.

## MAA Session #8

3rd Floor, Council Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Sensitivity Analysis and Optimal Control Model for the Transmission of Environmental and Contagious Mastitis in Dairy Cow Populations**

Ian Coleman

*Tarleton State University*

Mastitis cost the U.S. dairy industry approximately \$1.7-2 billion annually. Cost effective treatment and prevention protocols are highly sought after by the dairy industry. We constructed an epidemic model with multiple infected states that describes the transmission dynamics of mastitis in the herd while taking environmental bacterial concentrations into effect. Optimal control theory is applied to the epidemic model to suggest low cost treatment and prevention protocols.

2:20–2:35

**Studying Brain Connectivity using Weighted Graph Comparison**

Justine Langman and Thierno Diallo

*Rutgers University and New York City College of Technology*

Studying brain components and its connectivity is an important field in neuroscience. While concepts of weighted graphs are widely used in many areas including computer, social, biological pathways, and air traffic networks, application of weighted graphs to study brain connectivity pattern is relatively new in the field of graph theory. This project focuses on anatomical connectivity that connects nodes representing regions of interest (ROIs) and weighted edges associated with structural connectivity i.e. the density fibers connecting ROIs. An adjacency matrix was created using connectivity weights between ROIs and then uses Laplacian spectrum analysis and spectral clustering method to study the connectivity strength between and within two cerebral hemispheres. The next step is to analyze the sex differences in structural connections in human brains. To the proposers best knowledge, although brain connectivity is studied through comparison of weighted matrices very recently, analyzing brain connectivity through Laplacian spectrum and spectral clustering method is a novel technique. Various core graph theory measures were explored including degree distribution, clustering coefficients, and characteristic path length along with other measures such as, local and global efficiency for weighted graphs. Finally, concepts of Laplacian, normalized Laplacian were studied and Spectral clustering method were used to analyze connectivity of human brains.



2:40–2:55

**The Effects of Population Density on the Spread of Disease—A Mathematical Model**

Hasan Sumdani

*University of Texas at Arlington*

A new coronavirus, MERS-CoV (Middle East Respiratory Syndrome-Coronavirus) which is related to the coronavirus that caused the SARS outbreak 10 years ago, has been discovered in Saudi Arabia and observed in several other countries since last year. A specialized mathematical model is constructed in order to theorize possible patterns of the spread of this emerging disease which seems to require an extended contact time for transmission to occur. In order to better understand the role of population density on the spread of disease, the population is split into two groups, one with contact rates that are independent of population density and another group that has contact rates that are dependent on population density. Conditions under which the disease will spread are analyzed, and specifically, we note how population density affects the observed, theoretical dynamics and transmission characteristics predicted by the epidemiology model which uses MERS-CoV as an example.

3:00–3:15

**The Measles Are Coming! The Measles Are Coming!**

Danielle King and Charles Payne

*Southwestern University*

There has been a recent resurgence of the contagious disease of measles in the United States after being declared eradicated fourteen years ago. We created mathematical models representing cases of the infectious disease and control efforts in the United States, other countries, and the world as a whole. The sources of our data are mainly the World Health Organization and the Centers for Disease Control and Prevention. Our modeling methods include the logistic function as well as logarithmic and other transformations.

3:20–3:35

**Understanding The Drug-Related Crimes In Chicago  
Over Time Using Differential Equations**

Joanna Sasara

*Dominican University*

The purpose of this project is to explore a model and to give a description of the spread of illicit drug possession and trafficking in Cook County over time. The model is based on the theory of differential association, which proposes that through social interaction with others, individuals learn the values, techniques, and motives for criminal behavior. In this project, Chicago's population is split into three distinct groups: those who are not interested at all in committing a drug crime, those who are susceptible to committing a crime, and those who already have committed a crime. The model consists of a system of differential equations that describe the interactions between these groups. The solutions of the differential equations represent the size of each sub-population over time. Our approach to understand the dynamics of crime is similar to the one used in mathematical biology to model the spread or containment of disease. The model is fitted to data collected from the Chicago Police Department from 2006-2010.

3:40–3:55

**Understanding the Scales of Locomotion for *Caenorhabditis elegans* in a Viscous Fluid**

Katie Sipes

*James Madison University*

Viscosity is a measurement of a fluid's resistance to the rate of deformation. An example of a liquid with high viscosity is tree sap, or a homogeneous mixture of mud. Both of these liquids run very slowly when acted upon by gravity. In contrast, water has a very low viscosity and flows readily. So how does a liquid's characteristics effect the locomotion of a swimming organism? Do higher viscosities change the dynamics that an organism implements in order to move in a solution? The Reynolds number is defined as the ratio of inertial to viscious forces and is given by  $\frac{\rho V L}{\mu}$  where  $\rho$  is the fluid density,  $V$  is characteristic velocity,  $L$  is the characteristic length of the system, and  $\mu$  is kinematic viscosity. In particular, I am interested in measuring these different scales in a system where the worm *C.elegans* is swimming in fluids of different viscosities. We will compare these measurements to different modes of locomotion.

## MAA Session #9

3rd Floor, Directors Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Gridline Graphs in Higher Dimensions**

Joshua Mireles and Adam Volk

*University of Nebraska-Lincoln and University of Dayton*

A gridline graph (also known as a graph of  $(0, 1)$ -matrix) in two dimensions is a graph whose vertices can be represented as points in the plane with integer coordinates so that two vertices are adjacent if and only if they agree in one coordinate. This graph also models the way the chess piece rook attacks on the chessboard corresponding to the vertices. In this talk, we will describe a generalization of the gridline graphs to three and higher dimensions based on a generalization of how rooks attack in higher dimensions, and will investigate the properties and characterizations of these graphs. This research was conducted as part of the 2014 REU program at Grand Valley State University.

2:20–2:35

**Jacobians of Finite Graphs**

Louis Mayer Gaudet, Theodore Weisman, and Nicholas Wawrykow

*Yale University*

In this talk we will discuss work related to the Brill-Noether theory of graphs. Classical Brill-Noether theory studies maps from algebraic curves to projective space, and can be reinterpreted in terms of the geometry of an abelian group. In recent years, a parallel theory has emerged for graphs. One associates to each graph a finite abelian group called its Jacobian. Further, there is a natural way to define a duality pairing – an inner-product-like structure – on a Jacobian. As there is a complete classification theorem for finite abelian groups with pairing, we explore which groups with pairing can appear as Jacobians of graphs.

This is joint work with A. Deveau, L. Gaudet, J. Kainic, D. Mitropolsky, N. Wawrykow, and T. Weisman of Yale University.

2:40–2:55

**Brill-Noether Theory of Random Graphs**

Jenna Kainic, Andrew Brian Deveau, and Daniel Mitropolsky

*Yale University*

In this talk we will discuss questions regarding the Brill-Noether theory of random graphs. First, what is the expected gonality and structure of the Jacobian associated with the graph? Second, what is the probability that such a graph is Brill-Noether general? These questions arise as natural analogs to the classical Brill-Noether theory of algebraic curves.

This is joint work with A. Deveau, L. Gaudet, J. Kainic, D. Mitropolsky, N. Wawrykow, and T. Weisman of Yale University.

3:00–3:15

**Visualizing Dessins D’Enfants**

Mary Kemp and Susan Maslak

*Occidental College and Ave Maria University*

In this talk, we will discuss the results of a summer research project motivated by the theory of dessins. Dessin is short for dessin d’enfant which means child’s drawing. Mathematically speaking, a dessin is a connected bicolored graph where the edges around every vertex are cyclically ordered. Dessins can be realized by Belyi maps which are meromorphic functions  $f : X \rightarrow \mathbb{P}^1(\mathbb{C})$  such that  $X$  is a Riemann surface and  $f$  is unramified outside  $\{0, 1, \infty\}$ . One of the goals of this project is to determine Belyi maps that realize a given loopless, connected bipartite graph on a compact Riemann surface  $X$ . We report on our considerations of certain classes of such graphs, explorations of computational methods for finding associated Belyi maps and related applications.

3:20–3:35

**Laplacian Quantum Walk on Graphs**

Benjamin Lovitz

*Bates College*

A continuous-time quantum walk on a graph  $G = (V, E)$  is given by the unitary matrix  $U_G(t) = e^{-itA(G)}$ , where  $A(G)$  is the adjacency matrix of  $G$ . We say that  $G$  exhibits *perfect state transfer* between vertices  $a, b \in V$  at time  $t$  if  $|U(t)_{a,b}| = 1$ . These notions have been studied in the context of developing efficient quantum algorithms and also in simulating universal quantum computation. Here, we study quantum walk when the Laplacian  $L(G) = D(G) - A(G)$ , where  $D(G)$  is the diagonal degree matrix of  $G$ , is used in the definition of  $U(t)$  in place of the adjacency matrix  $A(G)$ . We describe an infinite family of graphs where a Laplacian quantum walk exhibits perfect state transfer whereas the standard quantum walk with  $A(G)$  does not. Our observation generalizes a result of Bose et al (2009) on perfect state transfer on the complete graphs.

3:40–3:55

**REU-pedia**

Atl Arredondo, Jose Ramirez, David DeSimone,

Ram Hari Dahal, and Dylan Quintana,

*San Diego City College, San Diego City College, Rutgers University,**Howard University, and Carnegie Mellon University*

The purpose of our project is to take a graph based approach to data representation in a way that hasn’t been tried before. We plan to develop a system where we can input any data that will create a “card” abstraction that can be applied to any topic. By developing this system, we hope to establish a better method for users to find and visualize information about any data they desire. We will be testing our mechanism by creating a database of REU projects, and seeing the relationship between the different cards. In order to make our project more attractive, we will be developing a trading mechanism, where users can interact by collecting and trading these cards. Value to the cars will be assigned depending on popularity and demand of the card. Cards will be exchanged in a virtual system where coins will be the currency among the users.

## MAA Session #10

3rd Floor, Studio Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Quantifying Uncertainty of the Black-Scholes Model  
by Relaxing its Unrealistic Assumptions**

Zihao Wang

*George Fox University*

Of the many contributing factors to the 2008 financial crisis, the Black-Scholes model often took center stage as an example of the forecasting that fell apart during this financial recession. We hypothesized that the true uncertainty of the Black-Scholes model was hidden by its unrealistic assumptions of low and constant volatility, normally distributed random risk (Brownian motion), and the Gaussian Copula used in simplifying codependencies in its implementation. I will share our findings in how these assumptions are built into the Black-Scholes Model and how the distribution in output prices change upon relaxing these input assumptions. The computed uncertainties are compared to the “Greeks” which gauge uncertainty in financial hedging. The numerical method of lines with finite differences is used in solving all versions of the model and results are compared to exact solution where possible.

2:20–2:35

**Vibration Analysis of a Bridge Due to Traffic**

Gaston Beaucage

*British Columbia Institute of Technology*

Bridges and other infrastructure need to be maintained to remain safe for use. To find out if a bridge is in disrepair, engineers often apply an explicit excitation to the bridge to measure the frequencies that it produces to determine if there is any structural damage. This process is effective; however, it requires an experienced, highly skilled person to perform the task and it is extremely difficult to duplicate. We will present a spectrum analysis of the vibration data using a mathematical model of a bridge with cars going over. The resulting spectrum data will be used in our subsequent study on damage detection using machine learning.

2:40–2:55

**Equivalence Class of Virtual Knot theory**

Mark Lugo

*University of North Texas at Dallas*

We introduce virtual 4-plats and formulate table of virtual 4-plats and their Conway symbols,  $(c_1, c_2, \dots, c_n)$  upto 8 crossings. We conjecture that the minimal number of classical crossings of any virtual 4-plat is greater than or equal to its minimal number of virtual crossings. We compute and classify the rational number  $\beta/\alpha$  with  $0 < \beta < \alpha$  to formulate equivalence classes of virtual 4-plat modulo Reidemeister moves.

3:00–3:15

**Invariance of Virtual Operator On Involutory**

Delia Rojas

*University of North Texas at Dallas*

In this paper we extend the notion of virtual operator  $\mathfrak{S}$  as an invariant of virtual knots and links. We use the operator  $\mathfrak{S}$  to calculate virtual involutory quandle (VIQ) for knots and links. Moreover, we compare VIQ with the fundamental groups to show the invariance of operator  $\mathfrak{S}$  for virtual knots and links.

3:20–3:35

**Knot Permutations of Prime Knots**

Javier Mondragon

*University of North Texas at Dallas*

Knot theory is a promising field that can possibly answer many important scientific open problems. In this paper, we introduce new invariants, called *knot permutations* to distinguish prime knots. We show that the knot permutation is unique for every prime knots up to eleven crossings. Furthermore, we develop a new notion called the *knot set* and show that prime knots are uniquely regenerated by using the knot set.

3:40–3:55

**Knot Theory, Combinatorics, and Grid Diagrams**

Patricia Josephine O'Brien and Elliot Alexander Kaplan

*The University of Texas and Ohio University*

Every knot can be represented by a grid diagram, with all segments of the knot being shown as straight horizontal or vertical lines. These representations can each be given a unique ordered pair of two permutations of  $n$  elements (unique with respect to the representations but not necessarily the knot). In recent years, these grid diagrams have proven to be very fruitful. We discuss patterns within these permutations and analyze the relationship of these patterns to the knots themselves.

## MAA Session #11

3rd Floor, Forum Suite

4:00P.M. – 6:15P.M.

4:00–4:15

**Exploring properties of  $Cay(\mathbb{Z}, \{\pm 2^k\})$  and  $Cay(\mathbb{Z}, \{\pm 3^k\})$** 

Daniel Adams and Daniel Gulbrandsen

*Utah Valley University*

One of the open problems in geometric group theory, first posed by Richard E. Schwartz, asks: Are the Cayley graphs  $C_2 = Cay(\mathbb{Z}, \{\pm 2^k\})$  and  $C_3 = Cay(\mathbb{Z}, \{\pm 3^k\})$ , for  $k = 0, 1, 2, \dots$ , of the additive integer group quasi-isometric? Moon Duchin and Bryan White have shown that  $C_2$  and  $C_3$ , with the associated word metric, are not quasi-isometric under the identity map. In this presentation, other classes of maps are considered and also proved that they are not quasi-isometries between  $C_2$  and  $C_3$ . In addition, properties known to be quasi-isometric invariants, namely hyperbolicity and metric ends, are studied. It will be shown that these properties cannot be used to determine if  $C_2$  and  $C_3$  are quasi-isometric. Further work will also be discussed. Joint work with Kristen Smith.

4:20–4:35

**Further Development of the Distance between Finite Groups**

Eric Le Lai, Kim (Kelly) Nguyen Pham, Yukang Zhang, and Minda He

*University of California - Irvine*

Our research explores the notion of distance between finite cyclic groups, an interesting measure of how close two finite groups are to being structurally the same. Finding a precise formula for the distance and understanding its properties is one of our biggest questions. We are also interested in determining lower and upper bounds for the distance and verify the validity of the triangle inequality. Mark Lewers hypothesized an upper bound for the distance, which we have shown being strictly bigger than the actual distance in the case of  $\mathbb{Z}_6$  and  $\mathbb{Z}_{10}$ . This has motivated us to look for even stronger upper bounds for the distance. Proving the triangular inequality is work in progress; we will present several instances in which it holds.

4:40–4:55

**Graphs Associated with Certain Groups**

Rebecca Post

*Augustana College*

We will consider groups generated by elements of order 2. For such a group, we construct a graph whose vertices are the elements of the group and whose edges represent the generators. We will show all the possibilities for groups generated by two and three generators. We will discuss some of the common properties of these graphs. Different choices of generators for a given group may produce nonisomorphic graphs. We will also look at a different class of graphs to study the relationships within a given set of generators.

5:00–5:15

**Visualization of the Subgroup Structures of Dihedral Groups**

Kelly McCurry

*Augustana College*

We will look at group multiplication tables and the subgroup lattices for some dihedral groups of small order. We will show how different subgroup tables for the same group can highlight different substructures. Through the examination of patterns in the subgroup lattices, we will look at different subgroup structures embedded in higher order groups. We will then look at how these lattices are determined by the factors of the order of the group.



## MAA Session #12

3rd Floor, Council Suite

4:00P.M. – 6:15P.M.

4:00–4:15

**Knight's Tours on Boards with Odd Dimensions**

Baoyue Bi and Elizabeth Doebel

*Iowa State University*

A closed knight's tour of a board consists of a sequence of knight moves, where each square is visited exactly once and the sequence begins and ends with the same square. For boards of size  $m \times n$  where  $m$  and  $n$  are odd, we determine which square to remove to allow for a closed knight's tour.

4:20–4:35

**Spinpossible and Its Connection to the Pancake Problem**

Ethan Kowalenko

*Metropolitan State University of Denver*

Spinpossible is a game played on a  $3 \times 3$  board, where the natural numbers up through 9 are bijectively mapped with the possibility of inverted orientation. Given any starting board, the goal of the game is to rotate rectangles on the board  $180^\circ$  in order to arrive at the identity board. This game is easily played on any  $m \times n$  board as well. The set of all possible boards in a fixed dimension  $m \times n$  forms a group and a metric space, considering the spins as generators. Our goal is to find the minimum number of spins required to generate any board.

A problem similar to Spinpossible, first posed by Jacob Goodman in 1975, is the pancake sorting problem. We are interested in the "burnt" variation, where we have a stack of pancakes which have one burned side, and every pancake is a different size. We wish to sort the stack (and hide the burns), but may only do so by flipping all of the pancakes above a certain point with a spatula. We also wish to minimize the number of flips required. It turns out that both the burned and unburned variations are difficult, and not yet solved.

In this preliminary talk, we will explore both problems, discussing both the similarities and differences between them. We will also consider a variant of the burnt pancake problem which is solved, and discuss the possibilities of extending those results to Spinpossible.

4:40–4:55

**Young Tableaux and PBW Bases**

John Claxton

*Loyola University Chicago*

Crystal bases of  $U_q^-(sl_n)$  and the corresponding combinatorial crystals are used to study the highest weight representations of  $sl_n$ . PBW monomials, which depend a choice of reduced expression  $w_0$  for the longest word of the Weyl group, index a crystal basis of  $U_q^-(sl_n)$ , so a subset of them index the corresponding crystal for each highest weight representation. These crystals can also be realized using Young tableaux. In general, describing the correspondence between PBW monomials and Young tableaux is a difficult problem. However, for a particular choice of  $w_0$ , we give a simple crystal isomorphism between PBW monomials and Young tableaux. The main proof uses induction on rank, and makes use of the combinatorial reflections on crystals which correspond to braid group operators on  $U_q^-(sl_n)$ .

5:00–5:15

**On a Class of Partition Generating Functions**

Youkow Homma, Jun Hwan Ryu, Benjamin Tong, and Amanda Folsom

*Yale University*

In this talk, we will discuss work related to families of partition numbers, whose generating functions are often non-modular. In particular, our work applies to and is inspired by related works of G.E. Andrews, K. Mahler, N. Sloane, and others. We will discuss various combinatorial bijections, congruence properties, and the asymptotic growth of these partition numbers, which, in the non-modular setting, is not immediate.

5:20–5:35

**Hook Sets of Partitions Corresponding to Numerical Semigroups**

Hannah Constantin

*Yale University*

The hook set of a given partition is the set of hook lengths, where each hook length is determined by the Young diagram corresponding to the partition. Given any possible hook set  $H$ , we can find a partition with this hook set by constructing it from a numerical semigroup. This produces one partition, but more may exist. We discuss the relations between hook sets and the sizes of these partitions, as well as the number of partitions with a given hook set  $H$ .

5:40–5:55

**Numerical Semigroups and their Corresponding Core Partitions**

Benjamin Houston-Edwards

*Yale University*

Given a numerical semigroup, a Young diagram can be created based on the elements of the semigroup. This process yields a correspondence between numerical semigroups and a certain class of partitions. We investigate the special properties of partitions that arise from numerical semigroups. For coprime  $a$  and  $b$ , we also explore the total number of semigroups which contain  $\langle a, b \rangle$ , which is equivalent to the number of simultaneous  $a$ –,  $b$ – core partitions that arise from some numerical semigroup.

## MAA Session #13

3rd Floor, Directors Suite

4:00P.M. – 5:55P.M.

4:00–4:15

**Veto Interval Graphs**

Stephanie Jones, Jessica Kawana, and Dana Lapides

*Woodburn Wellness, Business, and Sports School, Willamette University, and Lafayette College*

Given a set  $S$  of intervals on the real line, we can construct a graph with a vertex for each interval in  $S$ , and an edge between two vertices if and only if their corresponding intervals intersect. Conversely, given a graph  $G$ ,  $G$  is an interval graph if it has such a representation. We define a variation of interval graphs called veto interval graphs, where each interval contains a veto mark, and two intervals do not count as intersecting if either contains the veto mark of the other. We explore properties of veto interval graphs and their relation to other classes of geometric intersection graphs.

4:20–4:35

**An Exploration of Various Subsequences of the Fibonacci Sequence  
and their Applications to Cryptography**

Yi Wang

*Milburn High School, New Jersey*

The subset-sum problem is to find a subset of a given set whose elements sum to a given number. In this research, I consider a superincreasing subsequence of the Fibonacci sequence and investigate which natural numbers can be expressed as the sum of certain terms of this sequence. An algorithm to calculate such expressible numbers is developed. I further explore applications of this technique to the design of knapsack cryptosystems.

4:40–4:55

**Can One Fake Randomness?**

Robert Weber

*University of Illinois at Urbana-Champaign*

If head/tail sequences obtained by tossing a coin two hundred times are lined up with similar “made up” sequences, can one distinguish the “fake” random sequences from the “real” ones? In most cases one can — humans are notoriously inefficient in mimicking true randomness, and there is a long history of psychological experiments that point towards this conclusion.

In this talk we report on an interactive online “fake randomness detector” we have developed, which uses a suite of statistical tests that is optimized to maximize its effectiveness. Possible applications range from educational uses at all levels to the testing of real-world data for randomness, and to fraud detection.

This is joint work with Chad Franzen and Rishabh Marya.

5:00–5:15

**Constructing Linear Codes with Record Breaking Parameters**

Nicholas Connolly

*Kenyon College*

Coding theory is the branch of mathematics interested in the reliable transfer of information. Error correcting codes are designed to detect and correct errors that occur during the transmission of a message due to noise. Linear codes have the mathematical structure of a vector space over a finite field, and they are defined by three fundamental parameters: length, dimension, and minimum distance. A code's minimum distance determines its error correcting capacity. For a given length and dimension, there exists an upper bound on the value of the minimum distance of a code; the best known codes are those with a minimum distance as close as possible to this bound. In this project, we attempt to construct new codes with a larger minimum distance than the previous best known codes by exploiting the algebraic structures of constacyclic codes. For a given length and finite field, we exhaustively construct all constacyclic codes and compare their parameters against the best known results with the goal of discovering new codes with better parameters.

5:20–5:35

**Design of Knapsack Cryptosystems Using Certain  $t$ -Superincreasing Sequences**

Rob Rexler Baello and Donald Coleman

*Montclair State University and Essex County College*

In this presentation, we report our result on designing knapsack cryptosystems using certain  $t$ -superincreasing sequences. Methods of creating  $t$ -superincreasing sequences and how to use them to construct knapsack cryptosystems are provided.

This research is funded by MAA NREUP through NSF grants DMS-1156582 and DMS-1359016.

5:40–5:55

**No Talk**

Speaker Canceled

NA

## MAA Session #14

3rd Floor, Studio Suite

4:00P.M. – 5:55P.M.

4:00–4:15

**A Numerical Analysis of the Non-Self-Adjoint Harmonic Oscillator**

Jeremy Meza

*Carnegie Mellon University*

We investigate numerically the spectral properties of the Non-Self-Adjoint Harmonic Oscillator. In particular, we describe the pseudospectrum of this operator.

4:20–4:35

**Constrained Optimal Transportation**

Wyatt Boyer

*Williams College*

Optimal transportation seeks the least-energy way to transport material (like a pile of sand) from initial sites to destinations. McCann et al. have recently considered a constraint on the flow from  $x$  to  $y$ . We provide some new examples and results.

4:40–4:55

**Maximal Contraction Ratio of Invariant Sierpinski Snowflakes**

Mathew Gomez and Byungik Kahng

*University of North Texas at Dallas*

We derive a formula to find the maximal contraction ratio without overlap for outward invariant fractals of regular  $N$ -gons which we called regular Sierpinski snowflakes. We use infinite geometric series and the quadratic formula to derive the formula for the maximal contraction ratio, and the maximal Hausdorff dimension.

5:00–5:15

**No Talk**

Speaker Canceled

NA

5:20–5:35

**Recurrence in Dispersive Equations with Piecewise Constant Initial Conditions**

Reed Haubenstock

*Harvey Mudd College*

Recent research by Olver and collaborators has shown that large classes of linear and nonlinear dispersive equations with piecewise constant initial conditions exhibit a return to piecewise constant shape, a phenomenon known as dispersive quantization. Moreover, at other times the solution will take the form of a continuous but nowhere differentiable curve. We use discrete Fourier transform methods to examine this recurrence behavior.

5:40–5:55

**Using Logarithmic Basis Functions to Solve Singular Differential Equations**

John Garwood

*Austin Peay State University*

Numerical methods based on polynomial approximation perform poorly when applied to singular initial value problems. Hence, we are motivated to derive and implement numerical methods involving non-polynomial basis functions such as logarithmic and rational functions. Specifically, by imbedding a constant into the logarithmic function, we are able to overcome any discontinuity issues with the natural logarithm approximant. An efficient method that can handle singular differential equations is developed by using the Taylor Series expansion to optimize the imbedded parameter. Numerical experiments performed show that the methods are more accurate than the Improved Euler's method. These methods are implemented as predictor-corrector methods.

## MAA Session #15

3rd Floor, Forum Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**A New Algorithm for Galois Groups of Quintic Polynomials**

Robin French

*Elon University*

Finding solutions of polynomial equations is a central problem in mathematics. Of particular importance is the ability to solve a polynomial “by radicals”; i.e., using only the coefficients of the polynomial, the four basic arithmetic operations (addition, subtraction, multiplication, division), and roots (square roots, cube roots, etc.). For example, the existence of the quadratic formula shows that all quadratic polynomials are solvable by radicals. In addition, degree three polynomials and degree four polynomials are also solvable by radicals, which was shown in the 16th century. However, the same is not true for all degree five polynomials. Therefore, we are left with the following question: how do we determine which degree five polynomials are solvable by radicals? To answer this question, we study an important object that is associated to every polynomial. This object, named after 19th century mathematician Evariste Galois, is known as the polynomial’s Galois group. The characteristics of the Galois group encode arithmetic information regarding its corresponding polynomial, including whether or not the polynomial is solvable by radicals. In this talk, we will discuss a new algorithm for determining the Galois group of a degree five polynomial.

8:50–9:05

**Distinguishing Sextic Curves via Syzygies**

Austin Rodgers

*University of Notre Dame*

Let  $C$  be a rational plane curve of degree 6. We consider the syzygies of the parametric equations defining  $C$ . Let  $(d, t)$  be the degrees of the generators of the syzygy module. A general curve with  $(d, t) = (3, 3)$  will have ten double points as singularities. The same is true for a general curve with  $(d, t) = (2, 4)$ . How can we distinguish these two sets of points geometrically? Algebraically the curves are very different and a general rational plane curve of degree 6 will have  $(d, t) = (3, 3)$ . However it is still an open question of how to geometrically characterize the closed set given by the curves with  $(d, t) = (2, 4)$ . This research will attempt to address this problem.

9:10–9:25

**Maximum Waring ranks of monomials**

Jeremy Siegert

*Boise State University*

The Waring rank of a homogeneous polynomial is the least number of terms needed to express the polynomial as a linear combination of powers of linear polynomials. For example,  $xy = (1/4)(x + y)^2 - (1/4)(x - y)^2$ , is a linear combination of two squares of linear polynomials. Waring rank can be viewed as a measure of how “complex” a homogeneous polynomial is with a higher Waring rank corresponding to higher complexity. It is surprisingly difficult to find polynomials with higher than generic Waring rank. In this work we show that monomials and sums of pairwise coprime monomials in four or more variables have Waring rank less than the generic rank, with a short list of exceptions. We asymptotically compare their ranks with the generic rank.

9:30–9:45

**Positive Polynomials for Ample Toric Vector Bundles**

Geoffrey Smith

*Yale University*

A weighted polynomial in Chern classes of degree  $n$  maps a vector bundle on a projective variety of dimension  $n$  to an integer. Such a polynomial is called *numerically positive for ample toric vector bundles* if it maps every ample toric vector bundle on a toric variety to a positive integer. For each  $n$  we show that the Schur polynomials  $S_\lambda$ , with  $\lambda$  a partition of  $n$  of the form  $(k, 1, \dots, 1)$ , generate rays of the cone of positive polynomials for ample toric vector bundles. This result comprises a partial analogue of Fulton and Lazarsfeld's classification of the numerically positive polynomials for arbitrary ample vector bundles; they demonstrated that the cone generated by Schur polynomials comprises all numerically positive polynomials for ample vector bundles. To establish this result, we describe a family of toric varieties with equivariant nef vector bundles via an associated family of fans with multivalued piecewise linear functions, allowing us to use a combinatorial approach.

9:50–10:05

**Two Methods for Determining Galois Groups of Cubic Polynomials**

Nicole Soltz

*Elon University*

Let  $f(x)$  be an irreducible polynomial over a field  $F$  with roots  $a$ ,  $b$ , and  $c$  (in some algebraic closure), and let  $K = F(a)$  be the extension obtained by adjoining one root of  $f$  to  $F$ . We present two methods for determining the Galois group of  $f(x)$ . One involves answering the question: is  $(a - b)(a - c)(b - c) \in F$ ? The other involves answering the question: how many roots of  $f$  are in  $K$ ? We end by discussing an application to computing Galois groups of cubic polynomials defined over an extension of the  $p$ -adic numbers.

10:10–10:25

**Visualizing the  $A$ -discriminant of Sparse Polynomials and their Zero Sets**

Joann Coronado

*Texas A&M University - Corpus Christi*

To any finite point set  $A$  in  $\mathbb{Z}^n$ , we can associate a family of polynomials with exponent vectors lying in the set  $A$ . The polynomials with singular zero sets form the  $A$ -discriminant,  $\nabla_A$ , and the real connected components of its complement are called  $A$ -discriminant chambers. Classical Morse Theory tells us that the topology of the real zero set of a polynomial is constant in any  $A$ -discriminant chamber. For this reason, we would like to visualize the  $A$ -discriminant and better understand its chambers. We will use our recently developed software to understand  $\nabla_A$ , when  $A$  has cardinality  $n + 4$ , and efficiently compute the topology of certain real algebraic surfaces. This research was conducted as part of the 2014 Algorithmic Algebraic Geometry REU at Texas A&M University."



10:30–10:45

**The Norm of the Resolvent of a Square Matrix**

Olivia Meyerson

*Williams College*

We investigate the norms of resolvents of a normal or non-normal square matrix  $A$ . We estimate the norm of  $A - zI$  in terms of the distance from  $z$  to the spectrum of  $A$ .

10:50–11:05

**Pseudospectra of Matrices**

Feixue Gong

*Williams College, SMALL*

We give several equivalent definitions for the pseudospectra of a square matrix. We investigate numerically the shape of the pseudospectrum of certain non-normal matrices.

11:10–11:25

**The Distribution of Eigenvalues of the Non-Hermitian Anderson Operators**

Abigail Ward

*The University of Chicago*

The spectrum of the Non-Hermitian Anderson operator consists of real eigenvalues and complex eigenvalues on smooth curves in the complex plane. We study the spacings between nearest eigenvalues: for eigenvalues on the real line we obtain Poisson statistics; for complex eigenvalues, we see a transition towards the picket fence distribution.

11:30–11:45

**Counting Polynomials with Given Root Multiplicities**

Ayah Almousa

*University of Wisconsin*

Let a partition  $P$  of some polynomial consist of the multiplicities of every root of that polynomial. We analyze the function  $\bar{w}$  of a partition  $P$ , which counts the number of polynomials with coefficients modulo a prime  $q$  and with root multiplicities given by  $P$  or a degeneration of  $P$ . We present a class of partitions with formula  $\bar{w}$  that is very simple, namely a power of  $q$ , and wonder what other cases yield the same formula. This work was done jointly with Melanie Matchett Wood and was supported by NSF grant DMS-1147782.

## MAA Session #16

3rd Floor, Council Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**Addition of Factorials and Digits**

Bradley Abramson and Rodney Wells

*Penn State Erie, The Behrend College*

Our research sheds light on some interesting mathematical problems that involves concepts from the area of Number Theory. In this talk we will discuss two such problems: The first deals with addition of factorials of integers from 1 to  $n$ , and predicting how many digits will remain constant for various  $n$ . The second is a verification check for addition of integers in terms of addition of their digits.

8:50–9:05

**Collatz Sequences mod  $m$** 

Micah Jackson

*Georgia Southern University*

The Collatz function is defined as  $n/2$  for  $n$  even, and  $3n+1$  for  $n$  is odd. By iterating this function, we generate a Collatz sequence. A well known problem known as the Collatz Conjecture states that each of these sequences beginning with a positive integer will end in a cycle of 4, 2, 1. We will explore what happens when you reduce these sequences by a modulus  $m$ , and look at special structures of the sequences.

9:10–9:25

**Emergent Reducibility in Iterates of Cubic Polynomials**

Amrei Oswald

*University of Puget Sound*

We study a class of recursive sequences where the  $k$ th term is the  $k$ -fold composition of a polynomial  $f$  with itself. Our goal is to determine when a term  $f^k$  is irreducible over  $\mathbb{Q}$  but  $f^{k+1}$  is reducible over  $\mathbb{Q}$ , i.e. emergent reducibility. In Chamberlin et al., elliptic curves are used to identify emergent reducibility for quadratic  $f$ . We wish to extend this approach to cubic polynomials and ultimately generalize it to polynomials of any degree. To this end, we have determined that if  $f$  is a polynomial of degree  $n$ ,  $f^k$  is irreducible over  $\mathbb{Q}$  and  $f^{k+1}$  is reducible over  $\mathbb{Q}$ , then  $f^{k+1}$  factors into irreducible polynomials whose degrees are integer multiples of  $n^k$ . In this talk, we will summarize the results of Chamberlin, et al. and present our progress in generalizing their work.

9:30–9:45

**Number of Non-Witnesses to an Odd Integer  $n$** 

Shyam Narayanan

*Blue Valley West High School*

Currently, even the fastest deterministic primality tests still run too slowly, with the Agrawal-Kayal-Saxena (AKS) primality test runtime being  $\tilde{O}(\log^6(n))$ , and probabilistic primality tests are still highly inaccurate. In this paper, we discuss the accuracy of the Miller-Rabin Primality Test and the number of Non-witnesses for a general composite odd integer  $n$ . We also extend the Miller-Rabin Theorem by determining when the number of nonwitnesses  $N(n)$  equals  $\frac{\varphi(n)}{4}$  and by proving that  $\forall n$ , if  $N(n) > \frac{5}{32} \cdot \varphi(n)$  then  $n$  must be of one of the following 3 forms:  $n = (2x + 1)(4x + 1)$ , where  $x$  is an integer,  $n = (2x + 1)(6x + 1)$ , where  $x$  is an integer,  $n$  is a Carmichael number of the form  $pqr$ , where  $p, q, r$  are distinct primes congruent to  $3 \pmod{4}$ . Finally, we present open questions about how to determine if  $n$  is one of these three forms, and whether the Miller-Rabin Primality test can become a fast deterministic primality test.

9:50–10:05

**Searching for and Characterizing Abundancy Outlaws**

Zach Weiner

*Kenyon College*

For a positive integer  $n$ , the abundancy index  $I(n)$  is defined to be the sum of its divisors divided by the number itself, or  $\sigma(n)/n$ . The function  $I : \mathbb{N} \rightarrow \mathbb{Q} \cap (1, \infty)$  is not onto; rationals not in the range of  $I$  are called “abundancy outlaws.” Identifying and characterizing abundancy outlaws could prove helpful to better understand the existence of odd perfect numbers, a question over 2000 years old. In my research, I consider rationals of the form  $(\sigma(n) + t)/n$ , where  $t$  is a positive integer, to produce and characterize as-yet undiscovered outlaws.

10:10–10:25

**Sums of  $2^n$ -th Powers in Quaternion Algebras**

Anna Cooke and Sam Whitfield

*McDaniel College*

Generalizations of Waring’s problem - that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number is the sum of at most  $g(k)$   $k$ -th powers - have been studied in a variety of contexts, from algebraic number fields to non-commutative groups. We study this problem in the setting of quaternion algebras, and give examples of  $g(2^n)$  for infinite families of these algebras.

10:30–10:45

**The Igusa Zeta Function of a Quadratic Form Over the  $p$ -adic Integers**

Lauren White and Raameon Cowan

*California State University, Northridge*

Let  $f(x_1, \dots, x_n)$  be a quadratic form over the  $p$ -adic integers,  $\mathbf{Z}_p$ , and  $N_i(f)$  be the number of zeroes of  $f$  modulo  $p^i$ . The Poincare series,  $P(t) = \sum_{i=0}^{\infty} \frac{N_i(f)}{p^{in}} t^i$ , is a power series that organizes these zero counts. From the Poincare series one can obtain the Igusa local zeta function using the relation  $Z(s) = p^s - (p^s - 1)P(p^{-s})$ . For  $s$  in the complex right half-plane, the Igusa zeta function is defined to be the integral  $Z(s) = \int_{\mathbf{Z}_p^n} |f(x_1, x_2, \dots, x_n)|_p^s dx_1 \dots dx_n$ , where  $|\cdot|_p$  is the  $p$ -adic absolute value and  $dx_1 \dots dx_n$  is a volume element with respect to the Haar measure. We calculate both the Poincare series and the Igusa Zeta function for an arbitrary quadratic form over the  $p$ -adic integers where  $p$  is an odd prime. After determining the number of zeroes modulo  $p$  using generating functions, we use Hensel lifting to recursively calculate the Poincare series of  $f$  in terms of the Poincare series of simpler quadratic forms. Eventually this recursion stabilizes making the tail of the series geometric so that we may express  $P(t)$  as a rational function.

10:50–11:05

**The Strange Case of the Moebius Power Series:  
Reconciling Theory and Numerical Evidence**

Daniel Hirsbrunner and Tong Zhang

*University of Illinois*

The Moebius function  $\mu(n)$  is one of the most important functions in number theory whose behavior is closely related to the distribution of primes and the Riemann Hypothesis. We investigate the power series  $f(z) = \sum_{n=1}^{\infty} \mu(n)z^n$ , where  $\mu(n)$  is the Moebius function and  $|z| < 1$ . A result of Delange from 2000 implies that  $f(r)$  is unbounded as  $r \rightarrow 1^-$ . Yet, numerical data strongly suggest that  $f(r)$  converges with limit  $-2$ . This apparent contradiction between the known theoretical behavior and numerical evidence was first pointed out in a little-known paper by Carl Froberg from 1966.

In this talk, we explain what lies behind this mystery, and we report on investigations on related phenomena. This is joint work with Yiwang Chen and Dylan Yang.

11:10–11:25

**Computing  $\mathcal{A}$ -discriminant Chambers and Faster Homotopy Algorithms**

Bithiah Yuan

*University of Hawaii at Hilo*

The  $\mathcal{A}$ -discriminant,  $\nabla_{\mathcal{A}}$ , is the unique irreducible algebraic hypersurface containing all polynomials (with exponent set contained in  $\mathcal{A}$ ) having singular zero set. While the defining polynomial for  $\nabla_{\mathcal{A}}$  is difficult to calculate,  $\nabla_{\mathcal{A}}$  is central in numerous applications, including homotopy algorithms for approximating the real solutions of polynomial systems. We are developing a software package, and related quantitative estimates, to fully understand  $\nabla_{\mathcal{A}}$  for  $\mathcal{A} \subset \mathbb{Z}^n$  of cardinality  $n + 4$ . Our main goal is to quickly compute which discriminant chamber contains a given polynomial system, in order to find homotopies preserving the number of real roots. This research was conducted as a part of the 2014 Algorithmic Algebraic Geometry REU at Texas A&M University sponsored by NSF.

11:30–11:45

**Computing the Shape of Configuration Spaces**

Derek Francour

*University of Wisconsin Madison*

We calculate the cohomology of objects known as configuration spaces using spectral sequences.

## MAA Session #17

3rd Floor, Directors Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**Chutes, Ladders, and Markov Chains**

Harris Greenwood

*St. Edward's University*

The 2011 *College Mathematics Journal* and George Pólya Award-winning article, “Chutes and Ladders for the Impatient,” defines a Markov chain to compute the expected number of turns to complete a game of Chutes and Ladders using uniformly distributed spinners of any range on a board with any number of squares and configuration of chutes and ladders. After determining the uniformly distributed spinner range that minimizes game length on the standard board of 100 squares and observing equal game lengths for uniformly distributed spinners of range 99 and 100, it poses this open question: are the expected game lengths on a board with  $n$  squares and any configuration of chutes and ladders always the same for uniformly distributed spinners of range of  $n - 1$  and  $n$ ? After reviewing the article’s methods and results, we answer this question affirmatively and discuss possible generalizations and ramifications of our solution. Maintaining the “impatient” spirit, we conclude by showing that non-uniformly distributed spinners can yield shorter games than uniformly distributed spinners but can also lead to some paradoxical results when the related Markov chain is not absorbing.

8:50–9:05

**Mixing times of Markov chains**

John Rahmani and Matthew Allen Sutter

*Virginia Tech and University of Michigan - Flint*

The mixing time is a measure of the convergence rate of irreducible, aperiodic Markov chains to their stationary distribution. More specifically, the mixing time is the minimal number of steps required in order for the Markov chain’s state distribution to be within a prescribed distance from its stationary distribution. In the modern theory of Markov chains, an important question is how the mixing times grow as the size of the state space of the chain increases. We will present our work on the mixing times of some particular Markov chains obtained during our participation in the Willamette Valley REU this summer.

9:10–9:25

**On the Numerical Solution of Coefficient Identification Problem in Heat Equation**

Chloe Ondracek

*Minot State University*

In this paper, we investigate the time dependent coefficient identification in heat equation subject to over-specification data at a point in the spatial domain  $u(x^*, t) = G(t)$ ,  $0 \leq t \leq T$  along with usual initial and boundary conditions. Finite difference method is used to determine the solution and time dependent coefficient. A number of numerical illustrations are given to justify the proposed computational scheme.

9:30–9:45

**Rational Fixed Points of Polynomial Involutions**

Stanislav Atanasov

*Yale University*

It is well-known that every involution  $F = (f_1, \dots, f_n)$  of  $\mathbb{C}^n$  with polynomial components  $f_i$ ,  $1 \leq i \leq n$  admits a fixed point. The proof of this fact, however, is indirect, yielding little further information about the fixed-point set  $\text{Fix}(F)$ . The nature of this proof, coupled with the well-known open question of whether  $\text{Fix}(F)$  should necessarily be a conjugate of an affine subspace  $\mathbb{C}^k \subset \mathbb{C}^n$ , led Serre to ask: if the involution  $F$  has rational coefficients, does it necessarily possess a rational fixed point?

We answer Serre's question affirmatively in the case that  $\deg f_i = 1$  for  $1 \leq i < n$ , as well as for  $n = 2$ . We also link the analysis of Serre's problem to that of the Linearization Problem and the Fixed Point Problem. Indicative of the importance of these questions is the fact that an affirmative answer to any of them is known to imply an affirmative answer to the elusive Cancellation problem.

In our quest of understanding better polynomial involutions, we also pose the question of classification of polynomial involutions of  $\mathbb{C}^n$  according to the degrees of their polynomial components. We introduce the notion *multidegree of a polynomial map*  $F = (f_1, \dots, f_n)$  and denote it by  $M(d_1, \dots, d_n)$ , where  $d_i = \deg f_i$ ,  $i = 1, \dots, n$ . We then construct nontrivial rational involutions with multidegrees  $M(d_1, \dots, d_{n-1}, d_{n-1})$  and  $M(1, d_1, \dots, d_{n-1})$  for positive integers  $d_i \in \mathbb{N}$ ,  $1 \leq i \leq n - 1$ .

9:50–10:05

**Strong Solution to Smale's 17th Problem for Strongly Sparse Systems**

Paula Burkhardt

*Pomona College and Texas A&M University*

Smale's 17th problem asks whether one can deterministically approximate a single root of a system of polynomials, in polynomial-time on average. The best recent results are probabilistic polynomial-time algorithms, so Smale's 17th Problem has not yet been fully solved. We give a much faster deterministic algorithm for the special case of binomial systems, and certain systems of binomials and trinomials. Our approach is also a stepping stone to harder variants of Smale's 17th Problem, such as approximating roots near a query point or approximating a single real root. This research was conducted as part of the 2014 REU at Texas A&M University.

10:10–10:25

**The Development and Implementation of the Optimal Strategy for a Chance Based Game**

Peter Mielcarek

*Dominican University*

The goal of this research project was to find and evaluate the optimal strategy for playing 4-Way Countdown, a game in which tiles are flipped based on repeated rolls of two dice. Most of the research involved developing and testing a variety of strategies for a single-player game and then extending those strategies to a multiplayer version of the game. The testing of the strategies was completed by using Monte-Carlo simulations to provide conclusive results on the optimality of each strategy. The research was conducted by Nikita Belyaev, Yanyan Chen, and Peter Mielcarek.

10:30–10:45

**Extension of the Four Numbers Problem to Planar Graphs**

Samuel Justice

*Kenyon College*

After giving a brief overview of the so-called Four Numbers Game, we propose a natural generalization of it to planar graphs. In our generalization, the steps of the game alternate between the graph and its planar dual. We proceed to characterize all games played with nonnegative integers on some specific planar graphs, paying particular attention to games involving the self-dual wheel graphs. We also offer some avenues for further exploration.

10:50–11:05

**Variations on the Heat Equation**

Mohamed Aymen EL Ghodhbane

*U.S. Air Force Academy*

This talk discusses three different temperature related experiments. The first two are based on an iron ring. The first case involves a constant outside temperature while the second case involves a constant initial ring temperature. The third case examines steady state temperature on an iron cylinder. Each case will be examined using numerical methods and analytical product solutions starting from the heat equation. A transform approach will also be used although the answer cannot be fully solved by the transform method. The results of the three methods will be compared and contrasted.

11:10–11:25

**Hybrid Compression Algorithm using M-Band Wavelet and Discrete Cosine Transforms**

George Trejo and Tate Rogers

*Western Connecticut State University*

The greatest obstacle currently to the multimedia revolution is digital obesity. This occurs when sound, pictures, and video are converted from their natural analog form into computer language for manipulation or transmission. In the present emergence of high quality data, the need to compress data with minimal distortion is crucial. Compression lowers the cost of storage and transmission by encoding data into a smaller size. Digitalized image data is widely transmitted through multimedia, medical, and military communication. Effective distribution within a limited band-width and restricted storage space is essential. In this research we introduce a new hybrid compression method on digital images to test whether a greater compression ratio without compromising significant reconstruction quality can be achieved. Our hybrid compression algorithm utilizes the Discrete Cosine Transform and 4-Band Wavelet Transform to establish a four-dimensional transform technique effective in compressing large sets of data. The combination of these two compression techniques results in an improved relationship between the final compression ratio to the mean square error and peak signal noise ratio.



11:30–11:45

**Spatial Filtering on Digital Images**

Taylor Huettenmueller

*Emporia State University*

The first usage of digital image processing came about in the 1920s with images sent to and from newspapers for printing. The goal was to clarify the image in order to be able to print the image. Today, digital image processing is used in all sorts of fields and studies, ranging from a host of medical topics, outer space imaging, and for personal use in our digital cameras.

Filtering is a technique that is used to smooth, blur, or sharpen an image, or to detect the edge of an image. There are two common types of filtering applied to images: spatial domain filtering and frequency domain filtering. A spatial filter is an image operation where each pixel value is changed by a function of the intensities of pixels in the neighborhood of that pixel. For this project we examined the mean filter, the median filter and the non-local means filter and we demonstrated how to use these filters to remove digital noise. Our poster includes a description of each one of these filters, as well as a representation of the mathematical concept behind each of the filters. We will explain how each filter works. There are also copies of an original image, a noisy image, and an image filtered by each of these three filters to enable the viewer to see how well each filter clarifies the image.

## MAA Session #18

3rd Floor, Studio Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**A Cross-cultural Evaluation of Whether Teaching Methods Affect  
Students' Problem Solving Abilities**

Kathryn Flaharty

*Canterbury School*

Today's students need to think critically and to problem solve in a global world where change is rapid and certain. It has been cited that traditional teaching methods do not adequately prepare the student to decipher an unfamiliar set of information. Furthermore, it is argued that most teaching efforts are focused on learning skills and concepts, and there is little emphasis on thought process for extrapolation into problem solving.

This observational research study evaluated mathematical teaching methods associated with problem solving skills in Independent College Preparatory schools in the United States and Turkey. All schools had highly motivated students of similar socioeconomic background. The hypothesis was that students would perform better at problem solving if they had meaningful prior experience with problem solving, if the instructional style of the teachers placed value and focus on problem solving skills and if they had a positive attitude toward math. A number of additional findings will be presented.

8:50–9:05

**Determining the Effectiveness of Peer-Led Supplemental Instruction  
from Undergraduate Mathematics Fellows**

Autumn McMunn

*University of North Alabama*

Many colleges offer individual tutoring for students who struggle with mathematics. Could peer-led supplemental instruction from mathematics fellows be an effective alternative? The University of North Alabama has piloted a Fellowship Program, and this paper studies the results of that program. The fellows in the program attend the assigned class and meet with the students once a week to provide peer-led supplemental instruction. This study seeks to determine if the Fellowship Program at the University of North Alabama is effective. The results of this study will contribute to peer-led learning literature.

9:10–9:25

**My Teacher is Using Calculator, Why Should I Learn Mental Math?**

Abdullah Khan

*University of Texas at Dallas*

As we reside in the age of information and technology, our brains have become less accustomed to saving phone numbers, remembering dates, etc, now that our smart phones can store all types of information. However, as this occurs, we tend to err more when it comes our mental physique. Checking our receipts, comparing insurance rates, or even getting to point  $B$  when our GPS fails are all examples of practical uses of a strong and quick mind. However, stimulating one's mind through techniques such as mental math cannot only help with these everyday problems, but also help one in the long run. In this talk I will present simple strategies to improve mental math at any age.

9:30–9:45

**No Talk**  
 Speaker Canceled  
 NA

9:50–10:05

**Patterns and Statistics in Set Partitions**

Robert Dorward, Carlin Purcell, Lindsey Reppuhn

*Oberlin College, Vassar College, and Kalamazoo College*

We consider partitions  $\pi$  of the set  $[n] = \{1, 2, \dots, n\}$  and denote the set of these partitions by  $\Pi_n$ . We use the notation  $\pi = B_1/B_2/\dots/B_k \vdash [n]$  to indicate that  $[n] = \uplus_i B_i$  and call the  $B_i$  blocks. If  $S \subseteq [n]$  and  $\sigma \vdash [n]$ , then the partition  $\sigma'$  of  $S$  obtained by intersecting the blocks of  $\sigma$  with  $S$  is called a subpartition of  $\sigma$ . We standardize  $\sigma'$  to a set partition  $\pi \vdash [k]$ ,  $k = |S|$ , by replacing the smallest element of  $\sigma'$  with 1, the next smallest with 2, and so forth. In this case we say that  $\sigma$  contains  $\pi$ . We say that  $\sigma$  avoids  $\pi$  if  $\sigma$  does not contain  $\pi$  and let  $\Pi_n(\pi) = \{\sigma \in \Pi_n : \sigma \text{ avoids } \pi\}$ .

To obtain the standard form of  $\sigma = B_1/B_2/\dots/B_k$  we list the blocks in the order so that  $\min B_1 < \min B_2 < \dots < \min B_k$ . Given  $\sigma$  in standard form, the restricted growth function (RGF) of  $\sigma$  is the word  $w = w(\sigma) = w_1w_2\dots w_n$  where  $w_i = j$  if  $i \in B_j$ . Wachs and White introduced four fundamental statistics on set partitions via their RGFs. For example, one such statistic is  $lb(w) = \sum_i lb_i(w)$  where  $lb_i(w)$  is the number of integers to the left of  $w_i$  which are also bigger than  $w_i$ . We let  $lb(\sigma) = lb(w(\sigma))$ .

Letting  $q$  be a variable, we study the generating functions defined by  $LB_n(\pi) = \sum_{\sigma \in \Pi_n(\pi)} q^{lb(\sigma)}$  for all  $\pi \vdash [3]$  as well as the analogous polynomials for the other statistics. In particular, we determine their coefficients, degrees, and other properties.

10:10–10:25

**Patterns and Statistics in Restricted Growth Functions**

Jonathan Gerhard and Thomas Grubb

*James Madison University and Michigan State University*

We consider partitions  $\pi$  of the set  $[n] = \{1, 2, \dots, n\}$  and denote the set of these partitions by  $\Pi_n$ . We use the notation  $\pi = B_1/B_2/\dots/B_k \vdash [n]$  to indicate that  $[n] = \uplus_i B_i$  and call the  $B_i$  blocks. If  $S \subseteq [n]$  and  $\sigma \vdash [n]$ , then the partition  $\sigma'$  of  $S$  obtained by intersecting the blocks of  $\sigma$  with  $S$  is called a subpartition of  $\sigma$ . We standardize  $\sigma'$  to a set partition  $\pi \vdash [k]$ ,  $k = |S|$ , by replacing the smallest element of  $\sigma'$  with 1, the next smallest with 2, and so forth. In this case we say that  $\sigma$  contains  $\pi$ . We say that  $\sigma$  avoids  $\pi$  if  $\sigma$  does not contain  $\pi$  and let  $\Pi_n(\pi) = \{\sigma \in \Pi_n : \sigma \text{ avoids } \pi\}$ .

To obtain the standard form of  $\sigma = B_1/B_2/\dots/B_k$  we list the blocks in the order so that  $\min B_1 < \min B_2 < \dots < \min B_k$ . Given  $\sigma$  in standard form, the restricted growth function (RGF) of  $\sigma$  is the word  $w = w(\sigma) = w_1 w_2 \dots w_n$  where  $w_i = j$  if  $i \in B_j$ . Wachs and White introduced four fundamental statistics on set partitions via their RGFs. For example, one such statistic is  $lb(w) = \sum_i lb_i(w)$  where  $lb_i(w)$  is the number of integers to the left of  $w_i$  which are also bigger than  $w_i$ . We let  $lb(\sigma) = lb(w(\sigma))$ .

Letting  $q$  be a variable, we study the generating functions defined by  $LB_n(\pi) = \sum_{\sigma \in \Pi_n(\pi)} q^{lb(\sigma)}$  for all  $\pi \vdash [3]$  as well as the analogous polynomials for the other statistics. In particular, we determine their coefficients, degrees, and other properties.

10:30–10:45

**Statistical and Topological Data Analysis, Part 1**

Mitchell Krock and Joshua Kiers

*Bradley University and Taylor University*

Recently, topological ideas and techniques have been applied to the study of data through a tool called persistence homology. We compare and contrast information gleaned from persistent homology with that obtained from the established techniques of clustering and principal component analysis. We investigate partnerships between these techniques that will lead to new insight and more effective modeling of data.

10:50–11:05

**Statistical and Topological Data Analysis, Part 2**

Christopher Cericola and Hanna Torrence

*Seattle University and University of Chicago*

Recently, topological ideas and techniques have been applied to the study of data through a tool called persistence homology. We compare and contrast information gleaned from persistent homology with that obtained from the established techniques of clustering and principal component analysis. We investigate partnerships between these techniques that will lead to new insight and more effective modeling of data.

## PME Session #1

3rd Floor, Executive Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Combination Intervention Mathematical Model for Malaria**

Taylor McClanahan

*University of Arkansas at Little Rock*

Every year up to about 300 million people are infected by Malaria, an infectious disease caused by Plasmodium spp. parasites. Consequently, nearly 660,000 deaths occur. Infected female Anophele mosquitoes transmit the parasite to humans through their saliva. Although there are other species that cause Malaria, P. falciparum infects and is transmitted by humans. Mosquito elimination, avoidance of mosquitoes, sleeping nets, and spraying insecticides are a few methods to retard the spread of Malaria. There are several known medications to cure this disease, however, no form of vaccination exists. However, due to the intense effort being applied to the problem, it seems likely that a vaccine for Malaria will eventually be produced. Under the assumption that a vaccine exists, a mathematical model is described that simulates the combined effects of vaccination and conventional strategies, such as sleeping nets, upon the spread of the disease.

2:20–2:35

**The Effect of a Time-Dependent Genetic Mutation Rate on Cancerous Growth**

Krista Kernodle

*University of California, Irvine*

One of the principal effects of genetic instability is an increase in the probability for a cell to experience a strong malignant mutation resulting in a much faster proliferation rate. Since genetic instability plays a very important role in Carcinogenesis, we are interested in finding the optimal malignant mutation rate. This would make the cancer progress in the fastest way. We are developing algorithms to determine the optimal mutation rate for a mathematical model of the growth of cancer cell populations when an oncogene is activated. Previously, we were able to find the optimal control by solving an initial value problem. For the current case, we are using the Richardson Extrapolation with Runge-Kutta 4 to solve a boundary value problem in order to obtain a numerical solution. Knowledge of the most efficient way to cancer would provide insight and information on how we may intervene clinically and provide a lower bound for the time available for clinical intervention.

2:40–2:55

**Safeguard Fair Voting: Mathematically Diagnosing Gerrymanders**

Eric A. Shehadi

*Youngstown State University*

Mathematical measures of ‘gerrymandering’ across political units such as congressional districts and city wards will be discussed. Concepts of compactness and convexity and their effectiveness at ensuring “one person, one vote” will be examined. Current measures of gerrymandering focus heavily on the geometric shape of a political unit. In contrast, the concept of population compactness of political units as a measure of gerrymandering will be elaborated upon.

3:00–3:15

**Computational Models of Congressional Redistricting**

Shawn Doyle

*Youngstown State University*

We will analyze a graph-theoretical model that generates congressional districts and promotes fair voter representation in the U.S. House of Representatives. The model employs free graph partitioning software to eliminate the need for biased committees in redistricting.

3:20–3:35

**Classifying Youngstown State University's Peer Institutions: An Analytical Approach**

Daniel P. Catello

*Youngstown State University*

Institutions of higher education across the nation use peer group comparisons in their strategic planning. From policies to budgets, a university's goals are benchmarked to their peers. Thus, it is important that this peer group is defined correctly. Youngstown State University's peer group is derived using factor analysis and clustering on key characteristic variables. This method generates a best peer list based on institutional characteristic data. This methodology is adaptable to any university and can be implemented by institutional researchers.

3:40–3:55

**Examining a Least Squares Method for Valuing American Options**

Ashley Orr

*Youngstown State University*

The challenge in pricing and profiting from American options is knowing when to stop—do we exercise now or wait until expiration? We examine a least squares regression valuation algorithm for pricing derivatives proposed by Longstaff and Schwartz and compare the value of such options under different basis functions.

## PME Session #2

3rd Floor, Senate Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Squaring the Circle in the Hyperbolic Disk**

Noah Davis

*Aquinas College*

Before squaring the circle was proved impossible in Euclidean geometry in 1882, Bolyai described a strategy for constructing a hyperbolic square and circle with the same hyperbolic area. In this talk, we will see how to construct such a pair of objects in both hyperbolic and elliptic geometries. The two non-Euclidean cases differ in both construction method and target sizes. Yet both geometries have a surprising property in common: neither the circle nor the square can be constructed directly from each other.

2:20–2:35

**Using Complex Analysis to Integrate Real Functions**

Dorothy Klein

*Kent State University*

Applying Cauchy's Theorems, Jordans Lemma, and Residue Theory, I will show how one can evaluate seemingly impossible real integrals. It is possible that one can shift the function from the real line into the complex plane and, with some manipulation, reach a finite solution to certain integrals.

2:40–2:55

**A Particular Polarity, Part I**

Marissa Hartzheim

*St. Norbert College*

In projective geometry, a polarity is a type of correlation between points and lines that preserves incidence. One such correlation associates points  $(a, b)$  in the Cartesian plane with non-vertical lines  $y = ax - b$ . We will discuss this polarity and explore some of its properties.

3:00–3:15

**A Particular Polarity, Part II**

Taylor Miller

*St. Norbert College*

A special polarity in the Cartesian plane associates points  $(a, b)$  with non-vertical lines described by the equations  $y = ax - b$ . In this talk, we will look into the possibility of extending this correlation to a three-dimensional version.

3:20–3:35

**A Proposed Algorithm for Finding the Congruence Criteria Between Two  $n$ -sided  
Convex Polygons in Euclidean Geometry.**

Shakil Rafi

*Troy University*

The research paper proposes an algorithm to find congruence criteria between two convex polygons in Euclidean Geometry. It begins with a review of triangles, then extends to quadrilaterals and eventually generalizes the case to  $n$ -sided polygons. It attempts to prove said algorithm using a method of induction and a case-by-case analysis. It also states a corollary to said algorithm.

3:40–3:55

**Domain Representability and Topological Completeness**

Matthew DeVilbiss

*University of Dayton*

A topological space  $X$  is domain representable if there exists a continuous directed complete partially ordered set  $D$  such that  $X$  is homeomorphic to  $\max D$ . Domain representability is a topological completeness property that is weaker than subcompactness and stronger than Baire. In this presentation, we explore the relationship between some of these completeness properties and domain representability. Specifically, we show that the box product of a collection of domain representable spaces is domain representable and if  $Y$  is compact and  $X \times Y$  is domain representable, then  $X$  is domain representable.



**PME Session #3****3rd Floor, Executive Suite****4:00P.M. – 6:15P.M.**

4:00–4:15

**Cantor Set Sums**

Alphonso Lucero

*Iowa State University*

The Cantor set can be described as all numbers between 0 and 1 whose digits in ternary expansion are not 1. The Cantor Set can be generalized to form Cantor type sets which exclude a different number in their base 3 representation. This project takes numbers from this the Cantor Set and a Cantor type set and sums them to form the real numbers between 0 and 1. We consider which of these real numbers can be written as two different sums of elements of the Cantor sets. For some numbers this is impossible, while for others there may exist infinitely many different ways to write it as a sum of two elements from Cantor sets. Considering base 4 Cantor sets yields different interesting results.

4:20–4:35

**Not Just a Problem**

Elliot Golias

*Kent State University*

After solving two related competition problems, we will go beyond their solutions to find the smallest integer  $k$  for which there exist distinct integers  $m_1, m_2, \dots, m_n$  such that the polynomial  $p(x) = (x - m_1) \cdots (x - m_n)$ ,  $n \geq 1$  has exactly  $k$  nonzero coefficients.

4:40–4:55

**Commutators of Upper Triangular Matrices Over Unital Rings**

Michael Kaufman

*Kent State University*

Let  $R$  be a unital ring and let  $X \in M_n(R)$  be any upper triangular matrix of trace zero. Then there exist matrices  $A$  and  $B$  in  $M_n(R)$  such that  $X = [A, B]$ .

5:00–5:15

**An Extension of Classical Derangements**

William Linz

*Texas A&M University*

Given any collection of objects, a permutation is an ordering of these objects. Choose any permutation of a collection of objects as an initial permutation. A derangement (from that initial permutation) is defined to be a distinct permutation such that no object is placed in the same position as it was in the initial permutation. This is the classical derangement case that was studied extensively by de Montmort in the early 18th century. De Montmort was able to provide a formula to compute the number of derangements from any permutation of a collection of any given size independent of any initial permutation of the collection.

My research extends the classical derangement question by considering the same question on objects known as Ferrers boards. A Ferrers board is a (square) chessboard with a missing section in the lower right corner. Although the enumeration of these derangements is not necessarily independent of the initial permutation, as it is in the classical case, it is shown that in certain cases (in particular, when the missing section of the Ferrers board is rectangular) the derangement numbers do satisfy some properties that also hold in the classical case.

This research is theoretically interesting as a particular case of the fundamental combinatorial structures of noncrossing and nonnesting partitions. It has direct applications to computer science and computational mathematical biology, for example in the elucidation of the pseudoknot structure of RNA.

5:20–5:35

**The Distance Between Finite Groups**

Eric Lai

*University of California - Irvine*

In his undergraduate research project at Virginia Tech, Mark Lewers defines a distance as the number of differing elements between the multiplication tables of two finite cyclic groups and mentions some interesting conjectures about this distance. In this project, we have generalized Lewers' definition of distance for general finite groups. This distance gives us a measure of how close two finite groups are from having the same algebraic structure. The concept of the distance relies heavily on what we call matching pairs as the number of matching pairs gives us the measure of closeness between two finite groups. However, it is still unclear what some properties of the distance are as well as how the distance is computed when considering different finite groups. To address this question, we explored different properties of the distance and the validity of the conjectures made by Lewers. Throughout our research, we have been able to prove one of Lewers essential conjectures, find many different properties of the distance, and give explicit formulas for computing the distance of finite cyclic groups of small order. It turns out that the distance has almost all of the properties of a metric, and we have been able to find an upper bound for the distance for all finite cyclic groups.

5:40–5:55

**The Cayley Graph of the Braid Group on Four Strands and its Geodesics**

Aaron Calderon

*University of Nebraska-Lincoln*

Braid groups  $B_n$  are a special finitely presented class of groups which can be represented visually as strings braided together. Interest in braid groups comes from areas such as modeling symmetries of hyperplane arrangements and knot theory. Braid groups have an efficiently solvable word problem, but nonetheless contain very complicated computational problems. We examine the Cayley graph (a geometric construction that allows us to visually represent the algebraic structure of a group) of the braid group on four strands. Lucas Sabalka proved the geodesics of this group over three strands with the Artin generating set form a regular language, so the paths of shortest length on the Cayley graph can be modeled relatively easily. The answer for four or more strands is heretofore unanswered. Through an examination of simpler subgroups and quotients as well as direct investigation, we describe the Cayley graph and discuss its geodesics. This is joint work with Susan Hermiller.

6:00–6:15

**Classifications of Simple Lie Algebras**

Opal Graham

*University of North Florida*

The classification of semisimple Lie algebras is presented. We show that irreducible root systems can be represented by Dynkin diagrams. These diagrams are then classified and since irreducible root systems are in bijective correspondence with semisimple Lie algebras, the classification of the latter follows.

## PME Session #4

3rd Floor, Senate Suite

4:00P.M. – 6:15P.M.

4:00–4:15

**Establishing the Connection Between Stability Theories  
in Feynman's Operational Calculus**

William Graft

*Creighton University*

In this paper, we will discuss aspects of the abstract formulation of Feynman's operational calculus that was originated by B. Jefferies and G. W. Johnson in the late 1990's. This formulation of the operational calculus allows Feynman's heuristic 'rules' for the formation of functions of several noncommuting operators to be applied in a mathematically rigorous fashion.

In particular, given bounded linear operators  $A_1, \dots, A_n$  on a Banach space  $X$ , we associate to each operator  $A_j$ ,  $j = 1, \dots, n$ , a Borel probability measure  $\mu_j$  on  $[0, T]$ . Given a sufficiently well-behaved function  $f(z_1, \dots, z_n)$  of  $n$  complex variables, we then compute the function  $f_{\mu_1, \dots, \mu_n}(A_1, \dots, A_n)$  of the operators  $A_1, \dots, A_n$  according to the time-ordering by the probability measures  $\mu_1, \dots, \mu_n$  on  $[0, T]$ . The operator  $f_{\mu_1, \dots, \mu_n}(A_1, \dots, A_n)$  is called the *disentangling* of the function  $f$ .

The problem that will be addressed in the research I will be pursuing is associated with the stability theory of the operational calculus. More precisely, given *continuous* Borel probability measures  $\mu_1, \dots, \mu_n$  on  $[0, T]$ , select sequences  $\{\nu_{j,k}\}_{k=1}^{\infty}$ ,  $j = 1, \dots, n$ , of *purely discrete finitely supported probability measures* on  $[0, T]$  which converge *weakly* to  $\mu_j$ . Using the version of Feynman's operational calculus for purely discrete measures, compute the *disentangled operator*  $f_{\nu_{1,k}, \dots, \nu_{n,k}}(A_1, \dots, A_n)$ . The stability theory asks if  $\{f_{\nu_{1,k}, \dots, \nu_{n,k}}(A_1, \dots, A_n)\}_{k=1}^{\infty}$  of linear operators on  $X$  converges to  $f_{\mu_1, \dots, \mu_n}(A_1, \dots, A_n)$ ; ie, we wish to establish

$$\lim_{k \rightarrow \infty} f_{\nu_{1,k}, \dots, \nu_{n,k}}(A_1, \dots, A_n) = f_{\mu_1, \dots, \mu_n}(A_1, \dots, A_n).$$

In view of the stability theory as it now exists, this limit must hold; it is a question of working out the details. Indeed, the connection between the stability theory for the setting of continuous measures and the stability theory for arbitrary measures is not understood. Finding this connection is the goal of my research program.

4:20–4:35

**Reflections of Lines and Curves across Curves**

Jake Kearley

*Elmhurst College*

For a planar curve  $f(t) = (x(t), y(t))$ , let  $P(t)$  and  $Q(t)$  be the two points a fixed distance  $r$  from  $f(t)$  along the normal line at  $f(t)$ . The curves  $P(t)$  and  $Q(t)$  are mutual reflections across  $f(t)$ . Self-intersection properties of the reflections are expressed in terms of the curvature of  $f(t)$  and the distance  $r$ . Extensions and generalizations are made to reflections of lines and curves across curves.

4:40–4:55

**Spectral Theory for Expanding Maps with  
Countable Markov Partitions and Holes**

Lisa Naples

*Fairfield University*

We study expanding maps of the unit interval that admit countable Markov partitions after the introduction of a hole. We prove that the transfer operator associated with such open systems is quasi-compact and admits a spectral gap on an appropriate function space. In this context, the eigenfunction corresponding to the maximal eigenvalue represents a physically relevant quasi-invariant distribution for the open system. As a by-product, we prove that the spectrum of the transfer operator scales precisely after the introduction of the hole so that the spectral gap does not deteriorate as the hole gets larger as long as a mixing condition is satisfied. This work was completed as part of the Fairfield University REU during Summer 2013.

5:00–5:15

**Expandability of Discrete Analytic Functions**

Fernando Roman

*Kansas State University*

Discrete function theory is a branch of mathematics which deals with functions defined on a discrete set of points, such as the vertex set of a graph. The class of discrete analytic function is the discrete counterpart of the classical complex analytic functions. A noticeable peculiarity of discrete complex analysts is that the point wise product of discrete analytic functions is not discrete analytic in general; for example, on the integer lattice in the complex plane the functions  $z$  and  $z^2$  are discrete analytic, but the function  $z^3$  is not. Thus it is not an easy task to describe even the simplest algebraic discrete analytic functions, such as polynomials, rational functions or, even less, power series. Over the Years, there have been developed bases of discrete analytic polynomials to approximate discrete analytic functions (the analogue of Weierstrass approximation), and “formal” power series associated to discrete analytic functions, but there has not been success developing a polynomial basis that allows power series expansion of discrete analytic functions (which would be the discrete analogue of Taylor’s expansion). In this project, we take a new approach to the theory developing such a basis that not only allows approximation but also allows power series expansion of discrete analytic function. We then use this expansion to characterize discrete analytic functions that are rational with respect to a suitable, non-point-wise, product of discrete analytic functions which preserves analyticity.

5:20–5:35

**Brownian Motion in the Complex Plane**

Elizabeth Greco

*Kenyon College*

This project explores Brownian motion, a model of random motion, in the plane. Given a domain in the complex plane and a basepoint in the domain, start a Brownian traveler at that basepoint. The  $h$ -function of the domain gives us information about where the Brownian traveler first hits the boundary of the domain. After defining the  $h$ -function of a domain, we present several examples of domains and their corresponding  $h$ -functions. We also explore properties of these functions as well as the relationship between the geometries of domains and their  $h$ -functions.

5:40–5:55

**Approximating the Volume of a Solid of Revolution**

Robin Belton

*Kenyon College*

In Calculus, one learns how to approximate a definite integral with left, right, midpoint, and trapezoid Riemann Sums. The error these sums produce can be described in terms of the first and second derivatives of the integrand,  $f$ . The definite integral describes the area under the graph of  $f$ . Our project explores numerical techniques for approximating the volume obtained when  $f$  is rotated about the  $x$  axis. We define left, right, midpoint, trapezoid, and Simpson approximations for this setting. Then we examine the error with these methods and create bounds.

6:00–6:15

**Exploring Leibniz's Infinitesimals**

Johnathan Bush

*The University of Montana*

Leibniz used the intuitive notion of infinitesimal numbers to describe instantaneous rate of change. But the age-old question is: “Do Leibniz’s infinitesimals exist?” Of course, in the 1960’s Abraham Robinson proved that the answer was “yes.” He used classical axiomatic set theory with the axiom of choice to build an extension of the real numbers to include infinitesimal and infinite numbers using Skolem’s 1930’s ultrapower construction. Infinitesimals and their number system are now called the hyperreals. In the opinion of mathematician H. Jerome Keisler, “Robinson solved a three hundred year old problem by giving a precise treatment of infinitesimals. Robinson’s achievement will probably rank as one of the major mathematical advances of the twentieth century.” The hyperreal number system allows for differential and integral calculus to be developed without the use of limits and provides the rigorous foundation that infinitesimal-based calculus had been lacking for nearly three hundred years. In this presentation, we explore the concept of an ultrafilter, the ultrapower construction of the hyperreals as an ordered field of equivalence classes of real valued sequences with an equivalence relation given by an ultrafilter, and the relationship between the field of real numbers and the field of hyperreal numbers.

## PME Session #5

3rd Floor, Executive Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**Solving Systems of Equations Using Groebner Bases**

Kelsey Swerdfeger

*Concordia University Irvine*

In linear algebra, we solve linear systems of equations using matrices. However, solving a general system of equations is a more difficult problem. We can use Groebner bases, an algebraic tool, to simplify this problem. Groebner bases can be used to solve systems of equations in many different applications such as sudoku puzzles, graph colorings, and robotics.

8:50–9:05

**Finding 10 a Friend**

Matthew Larson

*Hendrix College*

Let us define  $\bar{\sigma}(n) = \sigma(n)/n$  to be the index function for any positive integer,  $n$ , where  $\sigma(n)$  is the divisor function. In number theory, a positive integer,  $a$ , is said to be *friendly* if there exists a different positive integer,  $b$ , such that  $\bar{\sigma}(a) = \bar{\sigma}(b)$ . An integer whose index value is unique is said to be *solitary*. This paper focuses on integers who have yet to be characterized as friendly or solitary. The smallest element of this set of unknown integers is 10. Although a friend of 10 has not yet been discovered, a lower bound for the value of 10's friend can be established.

9:10–9:25

**The RSA Public-Key Cryptosystem**

Samantha Sprague

*Marist College*

Data is encrypted and sent over the Internet (i.e. when you use your credit card to make a purchase) thousands of times a day. How do two entities communicate using a cipher when they never meet? One solution is public-key cryptography, the most famous being the RSA public-key cryptosystem. By using modular arithmetic and concepts from number theory and abstract algebra, we will give an in-depth analysis of the mathematics behind one of the worlds most widely used public-key cryptosystems.

9:30–9:45

**Pattern Avoiding Permutations and Tribonacci Tableaux**

Sara Tandon

*Pepperdine University*

Fibonacci tableaux have recently been used to study the inversion polynomials for permutations that avoid certain subsets of consecutive patterns of length three. In this talk, I will give a generalization of Fibonacci tableaux to Tribonacci tableaux and use these tableaux to give recursive formulas for the inversion polynomials on permutations that avoid subsets of consecutive patterns of length three and four.

9:50–10:05

**Binomial Character Equations Modulo Two**

Joseph Sheppard

*Kent State University*

I was working with chi character equations as seen in the research paper, which is related to Number Theory in mathematics. Chi character equations usually deal with primitive primes; we wanted to know how the powers of primitive prime two would affect the character equations. This research was built from previous research that was dealing with odd primitive primes with chi character equations completed before by Vincent Pigno and Christopher Pinner. Because this deals with two as the only even primitive prime, we assumed it would behave differently than the odd primitive prime, which turned out to be true.

10:10–10:25

**Integer Compositions Applied to the Probability Analysis of Blackjack and Infinite Deck Assumption**

Jonathan Marino

*Roanoke College*

Composition theory can be used to analyze and enumerate the number of ways a dealer in Blackjack can reach any given point total. The rules of Blackjack provide several restrictions on the number of compositions of a given number. We present a constructive approach to enumerate the number of ways the dealer can reach any point total. Our results cover all possible cases and also generalize to changes to the rules of Blackjack. Using the infinite deck assumption, we also find the approximate probability that the dealer reaches that point total.

10:30–10:45

**Representations by Ternary Quadratic Forms**

Edna Jones

*Rose-Hulman Institute of Technology*

Let  $Q(\vec{x})$  be a positive definite diagonal ternary quadratic form such that  $Q(\vec{x}) = ax^2 + by^2 + cz^2$ , where  $a, b, c$  are positive integers and  $\vec{x} = (x, y, z)^T$ . An integer  $m$  is (globally) represented by  $Q$  if there exists  $\vec{x} \in \mathbb{Z}^3$  such that  $Q(\vec{x}) = m$ . An integer  $m$  is locally represented everywhere by  $Q$  if for every positive prime integer  $p$  and every nonnegative integer  $k$  there exists  $\vec{x} \in \mathbb{Z}^3$  such that  $Q(\vec{x}) \equiv m \pmod{p^k}$  and there exists  $\vec{x} \in \mathbb{R}^3$  such that  $Q(\vec{x}) = m$ .

When is  $m$  locally represented everywhere by  $Q$ ? Are there integers locally represented everywhere by  $Q$  but not globally represented by  $Q$ ? Methods involving quadratic Gauss sums and the fast Fourier transform can be used to help answer these questions.



10:50–11:05

**An Irrational Decomposition of Generalized Fibonacci Numbers**

Robert Lehr

*Southwestern University*

Every natural number can be decomposed uniquely as a sum of distinct, non-consecutive, integral powers of the golden ratio (Bergman, 1950). Burger et. al. generalized and extended Bergman's result in 2013 by replacing the golden ratio with an arbitrary real quadratic irrational number. However, there was no closed formula for finding such decompositions of a given natural number without knowing the decomposition of all previous natural numbers. We introduce an explicit formula for decomposing the Fibonacci numbers in the Bergman decomposition and extend our formula to generalized Fibonacci numbers decomposed in terms of the corresponding generalized golden ratio.

11:10–11:25

**The Collatz Conjecture**

Marcus Elia

*SUNY Geneseo*

Choose any positive integer. If it is even, divide by two. If it is odd, multiply by three and add one. Repeat this with the new number, and so on. Most numbers eventually reach one. Can you find a number that does not? This tantalizing question, although simple to state, has stumped thousands of mathematicians over the years. We will talk about patterns we have found in how long it takes numbers to reach one and a theorem that guarantees certain trajectories will coincide.

11:30–11:45

**Quantum Calculus and Fermat's Christmas Theorem**

Shyam Vasandee

*University of North Florida*

Starting on the basis of  $q$ -calculus and proving Ramanujan Product Formula, this presentation will introduce a connection between Quantum Calculus and Number Theory, as well as its correlation to ordinary calculus. This framework will present a new way of counting the possibilities of expressing a number as a sum of two squares. We conclude showing the proof of Fermat's Christmas Theorem as an application.

## PME Session #6

3rd Floor, Senate Suite

8:30A.M. – 11:45A.M.

8:30–8:45

**A Quantitative Analysis of SIR-type Malaria Models**

Elisha Hall

*Austin Peay State University*

Malaria is a parasitic infection transmitted by female *Anopheles* mosquitos which can be fatal if not treated. The CDC reports Malaria had killed more than 627,000 people and created 207 million clinical episodes in 2012 alone. With approximately 1,500-2,000 cases reported every year, America is far from being isolated from this disease.

Sir Ronald Ross was the first to create an analytical model of Malaria and received a Nobel Prize for his work in 1902. Others such as Macdonald and Lotka have built upon this basis to give a refined view of the behavior of Malaria and treatment options. Ross model, a general Kermack-McKendrick SIR-type model, is still being used as a basis to create further models of the Malaria epidemic.

This project is a comprehensive analysis of Ross and Ross-Lotka models including determining the equilibrium points, reproduction numbers and stability of the equilibrium point. Further study is done to compare/contrast both models.

8:50–9:05

**Qualitative Dynamics of MDR-TB and XDR-TB with Isolation**

Justin Cook

*Austin Peay State University*

We present a deterministic system of ODE for the transmission dynamics of three mycobacterium tuberculosis (TB) strains, active TB, multi-drug-resistant TB (MDR-TB), and extensively-drug-resistant TB (XDR-TB). The conditions for elimination or persistence of the disease is qualitatively studied via stability analysis of the model. Using sensitivity analysis, response of the system parameters to their associated reproduction numbers are discussed. The impact of isolation of infected individuals on control efforts is determined through simulation.

9:10–9:25

**Resonance Inside A Cigar Box Guitar**

Greg Leclerc

*Fitchburg State University*

The main purpose of this project is to model a simple acoustic guitar. Specifically it will model a cigar box guitar, which is an acoustic guitar with a rectangular box for a body rather than the classic “hourglass” shape. Modeling the movement of sound inside the body can be found by solving the wave equation. In order to find the initial conditions for the wave function, other aspects must be found. These conditions rely on the vibrating string which is attached to the body. The string will set the initial conditions for the wave equation on the top of the body. In turn, the vibrations from the top of the body will provide initial conditions for the chamber of the guitar.

9:30–9:45

**A Linear Analysis of a Straight Rod Under Tension  
Both With and Without Drag**

Victoria Kelley

*James Madison University*

Starting with a straight rod under tension, we are studying the perturbations in twist and bend using the Kirchhoff Rod Model. Additionally, we can include the effects of drag approximated by resisted force theory. Finally, this model can inform us about the response of internal forces compared to external ones. This work has applications to the study of worm locomotion, bacterial flagella, and DNA.

9:50–10:05

**Dynamics of Light Refracted in Tilings of Metamaterials**

Jenny Rustad

*Luther College*

Media with negative indices of refraction, called metamaterials, have been recently discovered by physicists. These materials may someday be used in such applications as perfect lenses and invisibility shields. However, the behavior of light in scenarios involving metamaterials is unintuitive and has not yet been extensively studied. To improve understanding of the behavior of light in metamaterials, we focus on the predicted behavior of light in tilings of two materials with equal and opposite indices of refraction. In particular, we study the properties of orbits of light in three families of such tilings: divisions of the plane by a finite number of lines, triangle tilings, and the trihexagonal tiling. We demonstrate the existence of periodic orbits and unbounded orbits in each family of tilings, as well as the stability or instability under perturbation of each type of orbit. This work was done during the Summer@ICERM 2013 program at the Institute for Computational and Experimental Research in Mathematics.

10:10–10:25

**Agent-Based Modeling of Pandemic Influenza**

Roger Estep

*Marshall University*

A striking characteristic of influenza pandemics is the multiple peaks of infection. For example, the United States has experienced two peaks of infection in each of the past four influenza pandemics, one peak during the summer months and a second peak during the typical flu season. In contrast, the number of infected individuals peaks only once during a seasonal flu. The mechanisms that cause the multiple peaks of infection during pandemic influenza seasons are not well understood. The goal of this project is to use agent-based modeling to investigate mechanisms that can generate two peaks of infection. Here I will describe the susceptible-exposed-infectious-recovered (SEIR) agent-based model developed in NetLogo for simulating the 2009 H1N1 influenza pandemic. The incubation and infectiousness periods are drawn from gamma distributions. The model is calibrated by matching known average daily contacts and key epidemiological quantities, such as the basic reproduction number, the number of new infections generated from one infectious person at the beginning of the outbreak. Also, I will discuss the results of model simulations that include waning immunity, two-weakly interacting sub-populations, and a periodic transmission rate, mechanisms that can result in multiple peaks of infection. Pharmaceutical and social control strategies are considered as well.

10:30–10:45

**Protecting Confidentiality and Scientific Integrity Through  
Synthetic Data and Mediator Servers**Samantha Parsons  
*Roanoke College*

How can scientists perform statistical tests on sensitive data and preserve confidentiality at the same time? Synthesizing data is a method for providing confidentiality of data given to the public while simultaneously providing access to information contained in the data to scientists. A model is built from the raw data that captures statistical information then simple random samples are taken from this model and released to the public instead of the raw data itself. This method is powerful because inferences made on synthetic data will have similar results to inferences performed on the raw data and no sensitive information is released. However, some may not be confident in synthetic data results since the validity relies on the accuracy of the model. One solution is a server that will mediate between the raw data and scientists to verify results. The public can input synthetic data results which the server will compare to raw data results and provide feedback. This server must be designed to defend against ill-intentioned persons who will use the server to extract sensitive information about the raw data. This research project has two main focuses. We look into how synthetic data is created in order to design more accurate models and we investigate how people could use this mediator server maliciously so that we can create a server that is proactive against attacks.

10:50–11:05

**Computing Cophylogenetic Invariants**Heather Gronewald  
*Southwestern University*

Phylogenetics is the study of evolutionary relationships among organisms. One method for reconstructing phylogenetic trees involves establishing phylogenetic invariants (polynomial relationships that vanish when expected pattern frequencies are substituted for variables) and then comparing expected invariants to observed invariants. Cophylogeny is the study of concomitantly evolving organisms (e.g. a host and parasite). We compute phylogenetic invariants in the context of cophylogeny.

11:10–11:25

**The Flour Beetle: A Discrete Mathematical Model**

Matthew Buhr

*University of South Dakota Pi Mu Epsilon*

Some recent experimental studies of flour beetles (*Tribolium Castaneum*) have indicated a possibility of behavior in the laboratory that appears to be chaotic. I describe and attempt to analyze a model for such behavior, taking note of the properties of the life cycle of the flour beetle. The life cycle consists of larval and pupal stages, each lasting approximately two weeks, followed by an adult stage. Both larvae and adults are cannibalistic, consuming eggs and thus reducing larval recruitment. In addition, there is adult cannibalism of pupae. I will take two weeks as the unit of time and formulate a discrete mathematical model describing the larval population, pupal population, and adult population at two-week intervals. I can begin with a linear model as if there were no cannibalism, then I can modify my model to assume that cannibalistic acts occur randomly as the organisms move through the container of flour that forms their environment. This suggests a metered model with cannibalism rates proportional to the original size of the group being cannibalized. Finding the equilibria of this basic model, I can find solutions corresponding to the extinction and also a solution corresponding to survival for some sets of additional parameter values. I can then manipulate parameter values to find if the dynamics are very sensitive to any changes in the cannibalism rate, and then determine any possible chaotic behavior.

11:30–11:45

**Data Analysis: Predicting a Treatment Region  
for Subclinical Hypothyroidism**

Emily Paulson

*University of Wisconsin Whitewater*

Subclinical hypothyroidism is a thyroid dysfunction caused from having Hashimotos thyroiditis. Hashimotos thyroiditis is an autoimmune disease in which the thyroid is being attacked as if it were a foreign tissue. Subclinical hypothyroidism is where there is an increased level of thyroid-stimulating hormone (TSH) but the free thyroxine (FT4) level is within the “normal” range. Thyroid diseases, such as hypothyroidism result when the two hormones the thyroid produces, T) and T3 and the TSH levels become unbalanced. Diagnosing these diseases relies on determining TSH levels and depending on the levels present, doctors may or may not treat the patient. To determine these levels, doctors take blood and perform tests. The main tests include anti-thyroid antibodies test, the TSH diagnostic test and FT4 diagnostic test. The anti-thyroid antibodies test measures the levels of thyroid antibodies to detect the cause for thyroid problems. The TSH and FT4 diagnostic tests measure the amount of respective hormone to see if it falls within the designated reference range. Together these tests determine if a patient has a thyroid problem and potentially what the problem is. My research consisted of a data analysis to predict a treatment and therapeutic region, along with a risk analysis to create a risk function associated with this particular data set. This included a receiver operating characteristic test to determine the best FT4 level, which is used to calculate the best TSH level. These were then graphed and the result was a treatment region for patients and a therapeutic region.

## PME Session #7

23rd Floor, Skyline 2

8:30A.M. – 11:45A.M.

8:30–8:45

**The Chaotic Dynamics and Fractal Dimension of Newton's Method**

Andrew Evans

*Hendrix College*

Past research has shown that Newton's Method produces chaotic behavior in certain regions of the complex plane for some simple classes of polynomials. We expand on this research and introduce symbolic dynamics in order to describe chaotic properties of Newton's Method over general polynomials with real coefficients. In addition, we explore fractal properties of the Julia set generated by Newton's Method and estimate the dimension of the Julia set, both analytically and with computer-generated box-counting.

8:50–9:05

**Modulus of Families of Walks on Graphs**

Max Goering

*Kansas State University*

We introduce the notion of modulus of families of walks on graphs. We demonstrate that the modulus is a generalization of both effective resistance and capacitance. We show Beurling's famous criterion for extremality, that was formulated in the continuous case, can be interpreted on graphs as an instance of the Karush-Kuhn-Tucker conditions. We then develop an algorithm to numerically compute the modulus using Beurling's criterion as our guide.

9:10–9:25

**Assessment of Water Quality in the Chesapeake Bay by Parameter Estimation**

Heather Cook

*Roanoke College*

Algal blooms have been on the rise in the past few years within the Chesapeake Bay including its tributaries such as the Lafayette River. These blooms are caused by an abundance of nutrients within the water coming from the waste treatment plants and runoff of fields on which dinoflagellates thrive. Virginia Estuarine and Coastal Observing System (VECOS) has been monitoring, at 15 minute intervals via stationary sondes and via boats variably, levels of chlorophyll within the Lafayette River. For this talk, we will decide how often the boat measurements are needed based on the distribution of the 15 minute data since we want to be thoughtful about the amount of data while still obtaining accurate readings.

9:30–9:45

**Mathematical Manipulatives from 3D Printing**

James Ford

*SUNY Geneseo*

This project stems from the obstacles that arise when teaching students about three-dimensional mathematical objects with only the use of two-dimensional media. Generally, students have difficulties with visualization when they encounter functions of multiple variables for the first time in Multivariable Calculus. The same issues occur in topology courses, where we study shapes of various dimensions. Whiteboards and chalkboards are excellent for describing cross-sections of such shapes, but they are not ideal for depicting the full range of subtleties that occur in the third dimension. Software that renders three-dimensional images is an excellent tool to help students visualize multivariable functions, as such software allows the mathematical objects to be viewed from a wide variety of different perspectives. However, the current methods employed for aiding students understanding fail to provide any tangible entities that is, the students lack physical objects to provide the tactile foundation for the abstract concepts they study.

This project has successfully used three-dimensional printing technologies to provide students with tangible representations of mathematical objects. Many of these objects could be time-consuming to create accurately without the use of 3D printing technology. I have been able to render graphs of mathematical functions, surfaces, and objects for 3D printing using software that is either free online or made available to students through campus software licenses. The final 3D printed objects results are being presented to students in conjunction with standard Multivariable Calculus and Topology curricula.

9:50–10:05

**Using Independent Bernoulli Random Variables  
to Model Gender Hiring Practices**

Kimberly Hildebrand

*University of Mary Washington*

Gender bias is a problem in the workforce at large. In order for society to progress it is important that hiring practices do not use gender as a competitive factor. Hiring practices based on gender can be represented statistically using Bernoulli Random Variables and the Beta and Binomial Distributions. Using the moment generating function (MGF) of the Bernoulli and Binomial Distributions, it is possible to calculate the expected value (mean) and variance for the number of women hires for  $n$  positions. The probability generating function (PGF) of a sample size  $n$  can be used to find the probability of hiring a specific number of women ( $X$ ). The PGF when solved for  $P(X = 0)$  reveals the probability of no women hired for  $n$  positions, while  $P(X \leq 1)$  gives the probability that one or no women were hired. A computer program was used to run trials to simulate different male/female distributions using recent data on the proportion of women earning a PhD in a variety of disciplines. The simulations were used to represent hiring results for seven faculty positions. Situations where the female proportion is centered at 0.3, 0.5, and 0.7 were studied. Trials that included random proportions of women for each position were run as well. Results revealed that it is actually unusual for employers to hire one or fewer women for seven positions, which could provide evidence of gender bias.

10:10–10:25

**Factorization Theory of Numerical Monoids**

Crystal D. Mackey

*Youngstown State University*

A monoid is a set with an operation on the elements in the set, and it has three properties which are closure, associative, and identity. Consider the natural numbers including zero. We will discuss a submonoid known as a numerical monoid. A numerical monoid is an additive submonoid of the natural numbers including zero. This means that we can write a number in a numerical monoid using the generators of the monoid in several different ways. The different ways to write the number in the monoid is called factorization. We can understand this by using linear algebra, modular arithmetic and a small amount of number theory. We will also take a look at monoids in Sage.

10:30–10:45

**Elliptic Curves and Their Applications in Cryptography**

Timothy R. Shaffer

*Youngstown State University*

Promising greater security with shorter key sizes, elliptic curve cryptography shows exciting potential but is generally less understood than its traditional counterparts. Elliptic curves have become one of the major foci of modern number theory, yielding deep results and novel approaches to longstanding problems in mathematics, most notably the recent proof of Fermat's Last Theorem by Andrew Wiles. This talk highlights the cryptographic applications of elliptic curves, including a survey of the general study of these curves, foundations of their cryptographic use, and analogs of existing cryptosystems using elliptic curves.

10:50–11:05

**Everywhere Continuous, Nowhere Differentiable Functions: An Introduction**

Camron A. Bagheri

*Youngstown State University*

Every student of calculus covers, at some point in their studies, the well-known theorem which proves that differentiability implies continuity. While it is understood today that the converse of this statement is false, the pioneers of modern analysis generally believed continuous functions to be differentiable on most of their domains. In 1872, Weierstrass presented the first published counter-example by constructing an everywhere continuous, nowhere differentiable function defined on the whole real line. In this presentation, we investigate the construction of such functions and work through an example by Van der Waerden.



11:10–11:25

**Approaches to Addressing Overfitting in Averaging Classifiers**

James D. Munyon

*Youngstown State University*

Bayesian Model Averaging is theoretically the optimal method for combining learned models, but its application in machine learning remains an open problem due to its tendency to overfit. We will explore some alternative averaging methods in classification that aim to address this problem.

11:30–11:45

**Minimal Weakly Connected Dominating Sets**

Jenna L. Wise

*Youngstown State University*

Self-stabilization is a paradigm for distributed systems that allows the system to achieve a desired global state, even in the presence of faults. A fundamental idea of self-stabilizing algorithms is that no matter what global state the system finds itself in, after a finite amount of time the system will reach a correct and desired global state. In a self-stabilizing algorithm, each node maintains its local variables, and can make decisions based only on its local variables and the contents of its neighbor's local variables. I will be talking about a self-stabilizing algorithm for minimal weakly connected dominating sets, called MWCDS. For an arbitrary connected graph with  $n$  nodes, this algorithm terminates in  $O(n)$  steps.

**PME Session #8****3rd Floor, Executive Suite****2:00P.M. – 3:55P.M.**

2:00–2:15

**Investigating the Benefits of Utilizing Mobile Devices as Student Response Systems in Freshman Level Mathematics Courses**

Garrett Shontrice

*Jackson State University*

Mobile devices have become increasingly more accessible to students and instructors at the post-secondary level. With the implementation of Jackson State University's iPad Initiative, all first time freshmen now have personal iPads at their disposal. This study is designed to investigate how iPads and other mobile devices can be used in undergraduate mathematics courses to engage students in mathematical dialogue, which has been shown to facilitate conceptual understanding. Specifically, the mobile devices will serve as components of a student response system to engage students in meaningful classroom discussions. The investigation will be conducted at Jackson State University, an urban minority-serving university and the participants will be enrolled in a freshman level mathematics course. The following research questions will be addressed:

1. What are benefits of utilizing mobile devices as student response systems to (a) facilitate mathematical discourse, (b) encourage student participation in class activities, and (c) manage class time?
2. In what ways do students believe the use of mobile devices as student response systems enhances their learning experience(s)?
3. What instructional practices facilitate successful implementation of mobile devices as student response systems?

2:20–2:35

**A Modified Random Walker Ranking System for NCAA Mens Division I Basketball Teams**

Nakhila Mistry

*Elon University*

Every year around mid-March millions of people in America fill out a bracket for the NCAA Mens Division I Basketball Tournament. This tournament, also known as March Madness, is played each spring in order to determine the National Champion. The bracket is set up so that lower ranked teams play the higher ranked ones. Because there are 68 teams in the tournament, there are  $2^{67}$  ways that the tournament could play out. That number of pennies would carpet the surface of the earth twice! No one so far has been able to pick a perfect bracket mostly due to the likelihood of upsets. Lower ranked teams beat higher ranked teams because rankings fail to appropriately take into account current win trends of lesser-known teams. Our research aims to determine a more reliable method of ranking the NCAA Mens Division I Basketball Teams that better reflects the outcome of the national championship tournament. Our methodology is based on a relatively new ranking system created for Division I Football teams. This ranking system, the random walker method, uses a Markov process where every team has fans, but each fan is constantly changing allegiance according to randomly picked games played by their current favorite team. When the fan picks a game, they will decide whether or not to change allegiance, weighting their decision towards the team that won. We modified this ranking system by both taking into consideration score difference and how long ago the game was played. These considerations were parameterized and optimized using past seasons.

2:40–2:55

**An Individual-Based Model of Chaparral  
Vegetation Response to Frequent Wildfires**

Dayna Mann

*Pepperdine University*

The increasing fire frequency in the Santa Monica Mountains has become a major cause for concern for the native chaparral species that are the dominant vegetation. Chaparral species are classified into three life history types by their response to wildfires. Nonsprouters are completely killed by a fire, but their seeds germinate in response to fire cues. Obligate sprouters survive by resprouting from their root crown after a wildfire and only recruit seedlings during fire free intervals. Facultative sprouters both resprout and release seeds that germinate post-fire. This project is based on data collected since 1986 at a biological preserve adjacent to the Malibu campus of Pepperdine University that has an average fire return interval of just over 7 years. We present a spatial model that simulates the growth, seed dispersal and resprouting behavior of individual shrubs that compete for space and resources in a domain similar to our study site. The model also incorporates varying rainfall and fire frequency as well as the invasion of exotic grasses. Our simulation is capable of reproducing the change in plant community structure at our study site such as the local extinction of *Ceanothus megacarpus* due to shortened fire return intervals. A mathematical model of this system is important because a reduction of vegetation cover can cause a decrease in slope stability that leads to rock and mudslides.

3:00–3:15

**Maybe God Does Throw Dice? Einstein, Quantum Logic,  
and the Kochen and Specker Theorem**

Sharat Chandra

*University of California, Irvine*

The mid-1920's ushered in a revolution in the world of physics and mathematics - the development of Quantum Mechanics. However, unlike Newtonian Mechanics, Quantum Mechanics provides probabilities that a measurement on a system will return one particular result of many possibilities rather than a deterministic description of the system for all time. Due to this, and because Quantum Mechanics admits violations of Special Relativity described as the "Einstein-Podolsky-Rosen Paradox", Albert Einstein was never satisfied with Quantum Mechanics as it stood. Einstein always believed that Quantum Mechanics was incomplete - that one could find a theory which incorporated "hidden variables" that would ease his concerns. This sort of theory is known as a Hidden Variables Theory of Quantum Mechanics.

It was proven in 1967 by Simon Kochen and E. P. Specker that in any Hidden Variables Theory at least one of the following cannot hold:

- All observables for a Quantum system have definite values at all times
- If a Quantum system has a property, then it possesses this property independently of how the property is measured

This was done by showing that quantum mechanical observables form a partial algebra, and a necessary condition for the existence of Hidden Variables is to be able to embed this partial algebra into a commutative algebra. It is further shown that this embedding problem is equivalent to determining if the logic of Quantum Mechanics is equivalent to Classical logic. It is found that this is not the case. This talk seeks to present and explore these proofs.

3:20–3:35

**The Equivalence of the Theorems of Helly, Caratheodory, and Radon**

Adam Robinson

*Portland State University*

We present a new proof of Helly's theorem, an important result of Convex Geometry, using some tools of Convex Analysis and Optimization. We also provide a complete proof of the equivalence of Helly's theorem with two other important results of Convex Geometry: Caratheodory's theorem and Radon's theorem.

3:40–3:55

**Data-Driven Forecasting of Available and Required Energy  
for a Solar Water Heating System**

Daniel Miller

*Texas A&M University*

A central problem in renewable energy is matching supply with demand. Statistical models may be used to forecast the energy a renewable system will generate, as well as the energy required by end-users. I apply such models specifically to a solar water heating control system I designed and implemented. The predictions of such models are of interest to both energy consumers and producers, doubly so to those homeowners who function as both by implementing renewable systems to offset their consumption. By only generating or converting enough energy to meet demand, minimizing excess energy, it is possible to enhance the efficiency of many types of generation systems. While demonstrated here with a small residential system, such methods are equally relevant on a larger scale.

## PME Session #9

3rd Floor, Senate Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Solving Elliptic PDE Using Polynomial Basis Functions via Perturbed Collocation**

Dodji Kuwonu

*Austin Peay State University*

In this presentation, we propose a method for solving elliptic Partial Differential Equations (PDEs) using polynomial basis functions via perturbed collocation. The method is implemented without requiring starting values or predictors. Numerical examples such as Poisson Equation, Laplace Equation are solved to show the accuracy of the method.

2:20–2:35

**Using Differential Equations to Solve the Lion and Man Game**

Martin Dean

*Gonzaga University*

Childhood games, the animal kingdom, and dogfights all have a common theme: they can be modeled as a pursuit evasion game (PEG). A PEG is a differential game in which there are two teams of players: pursuers and evaders. The pursuer's goal is to minimize the time in which the distance between the players is 0 (i.e. catching the evader), while the evader's goal is to maximize this time. We provide a background of game theory, its origin, and a brief discussion on the various forms of PEGs, where the two teams have different speed, maneuverability, and number of players. Our main focus is the Lion and Man game in which the lion is the pursuer, the man is the evader, and they both have similar maneuverability. Tools from ordinary differential equations provide simple solutions for this game, which depend on the speed of the man relative to that of the lion. We also discuss how the Hamilton-Jacobi-Isaacs partial differential equation arises and how it may be used to find optimal strategies for both participants. Several practical and real-world applications of the Lion and Man game as well as other differential games are then discussed.

2:40–2:55

**On the Disjunctive Rado Number of the Equations  $ax_1 = x_2$  and  $bx_1 + x_2 = x_3$** 

Adam Purcilly

*Saint Peter's University*

For a given system of linear equations  $L$ , the Rado number of the system is the least integer  $n$  for which every  $t$ -coloring of  $\{1, \dots, n\}$  contains a monochromatic solution of one of the equations in  $L$ , if such an integer exists. In this presentation, the 2-color disjunctive Rado numbers for the equations  $ax_1 = x_2$  and  $bx_1 + x_2 = x_3$  will be determined for more than half of all values of  $a$  and  $b$ .

3:00–3:15

**Simultaneous Generation of a Simple Basis B and a Corresponding B-CZDS**

Andre Bunton

*University of Alaska Southeast*

In an ongoing attempt to keep our zeros real, or at least to decrease our “complex” zeros, an application of a special Complex Zero Decreasing Operator leads to the characterization of a new class of Complex Zero Decreasing Sequences with respect to simple sets of polynomials that solve an interesting differential equation.

3:20–3:35

**A Simplified Approach to Hamiltonian Decompositions of Hypercubes**

Elizabeth Field

*University of Illinois at Urbana-Champaign*

A Hamiltonian decomposition of a graph  $G$  is a collection of disjoint Hamiltonian cycles which partition the edges of  $G$ . When  $G$  is a hypercube, such decompositions can be useful in the design of fault tolerant communication protocols among the processors of supercomputers with a hypercube network topology. Hamiltonian decompositions are known to exist for all even-dimensional hypercubes. We present a novel approach to constructing such decompositions.

3:40–3:55

**Tiling  $m$ -deficient Mutilated Chessboards with  $m$ -polyominoes**

Kimberly Phillips

*Washington & Jefferson College*

A chessboard is called *mutilated* if a single square is removed from the board. We define a *square root of unity mod  $m$*  as a natural number  $n$  such that  $n^2 \equiv 1 \pmod{m}$ . If  $n$  is a square root of unity, then the  $n \times n$  chessboard is  $m$ -deficient. For natural numbers  $m$  and  $n$ , an  $n \times n$  chessboard is called  *$m$ -deficient* if  $m$  divides  $n^2 - 1$ . We analyze tiling  $m$ -deficient mutilated chessboards with  $m$ -polyominoes where an  *$m$ -polyomino* is a geometric figure with  $m$  congruent squares placed edge to edge. We focus on the values of  $n$  where  $n$  is a square root of unity, such as 1 and  $-1 \pmod{m}$ ,  $2k + 1$  and  $2k - 1$  when  $m = 4k$ , and others.

## PME Session #10

3rd Floor, Council Suite

2:00P.M. – 3:55P.M.

2:00–2:15

 **$C_4$ -Factorizations with Two Associate Classes, the Exception**

Rachel Bass

*Armstrong Atlantic State University*

Let  $K = K(a, p; \lambda_1, \lambda_2)$  be the multigraph with: the number of vertices in each part equal to  $a$ ; the number of parts equal to  $p$ ; the number of edges joining any two vertices of the same part equal to  $\lambda_1$ ; and the number of edges joining any two vertices of different parts equal to  $\lambda_2$ . Necessary and sufficient conditions for the existence of  $z$ -cycle decompositions of this graph have been found when  $z \in \{3, 4\}$ . The existence of  $C_4$ -factorizations of  $K$  has been settled when  $a$  is even, and when  $a \equiv 1 \pmod{4}$  with one exception. In this paper, we prove the case of the exception.

2:20–2:35

**When Things get a Little Edgy: Finding the Grundy Number of Line Graphs**

Ryan Matzke

*Gettysburg College*

A Grundy edge-coloring of a graph is a proper (adjacent edges get different colors) assignment of positive integers to the edges of the graph such that if an edge is colored (assigned)  $c > 1$ , then the edge is adjacent to edges of all the colors  $1, \dots, c - 1$ . The Edge Grundy number of a graph is the largest positive integer appearing on an edge of the graph, among all the colors appearing on all the Grundy edge-colorings of the graph. The practical significance of this number is its function as an index of a worst possible outcome during online or greedy proper colorings of the graph; such colorings are useful in scheduling. In this work we determine the Edge Grundy numbers of various commonly known graphs.

2:40–2:55

**Some Results on Path Localities of Completed Bipartite Graphs**

Tung Hoang

*Millsaps College*

In everyday life, many people encounter the need of arranging objects from high-dimensional space into low-dimensional space while best preserving the relationships between those objects. For instance, grocery store owners want to arrange their products into online column lists such that the nearby products in their store are close to each other in the lists; computer developers want to improve the burst-mode by optimally arranging data into linear addresses; or engineers want to best order the tasks for space robots so that the robots can work efficiently and productively. Motivated by these needs, in 2012, Dr. Yan Wang and David Poliakoff from Millsaps College built a graph theory model and introduced the concept of locality - a parameter measuring how well the distance relationships between vertices in one graph are preserved while those vertices are arranged into another graph. Continuing their research, I apply the idea of locality to find the best arrangement of the vertices in completed bipartite graphs into paths. In my presentation, I will introduce the results of the best arrangement for two types of completed bipartite graphs:  $K_{1,n}$  and  $K_{2,n}$ .

3:00–3:15

**Inversion Polynomials for Pattern Avoiding Permutations**

Erika Ordog

*Pepperdine University*

Pattern avoiding permutations have long been an important area of enumerative combinatorics and more recently, people have begun to study the generating functions for various statistics on pattern avoiding permutations. In this talk, I will give the inversion polynomials for permutations that avoid subsets of consecutive patterns of length three and four. I will use Fibonacci tableaux and a more general notion of strip tableaux to prove the recursive formulas for these polynomials.

3:20–3:35

**Hypercubes: A Quest for the Minimum Number of Deleted Edges  
Resulting in  $r$  Connected Components**

Meagan Benavides

*Texas State University*

This is work in progress on the following open problem in graph theory: determine the  $r$ -edge connectivity of the  $n$ -dimensional hypercube upon removal of a subset of edges. In other words, determine the number of connected components in the resulting subgraph when a particular subset of edges is removed from the  $n$ -dimensional hypercube graph. Hypercubes are a widely studied interconnection network model. Recently, researchers have discovered the minimum number of vertices in the  $n$ -dimensional hypercube whose deletion produces a graph with  $r$  connected components. The goal of the work presented here is to establish the minimum number of edges whose removal from the  $n$ -dimensional hypercube results in a graph with  $r$  connected components. The problem is important, for example, in determining the state of a network modeled by the  $n$ -dimensional hypercube when links fail.

3:40–3:55

**Intersection Graphs of Polygons**

Andrew Bishop

*Willamette University*

We explore the properties of intersection graphs of translated copies of polygons in the plane. The talk focuses primarily on regular polygons and star graphs. We show among other results that the graphs representable with regular  $n$ -gons for odd  $n$  are exactly those representable with regular  $2n$ -gons, and that the star graph  $S_5$  is representable with translated copies of every regular polygon other than squares.



## PME Session #11

3rd Floor, Directors Suite

2:00P.M. – 3:55P.M.

2:00–2:15

**Chord Recognition Using Wavelets**

Erlan Wheeler

*Carthage College*

Wavelets are a great tool for approximating a function or a set of discrete data points. We will begin by explaining the idea of a wavelet, with special attention being given to the Haar Wavelet. We will use the wavelet representation of musical chords to create representative chords which will be used to classify other, unknown, chords. This will be repeated using the Fourier series that best approximates the data, and using the raw data itself. We will determine which method has the greatest accuracy in identifying chords.

2:20–2:35

**Functional Programming as Logic and Mathematics**

Barton Yadowski

*Kent State University*

We will discuss Curry-Howard isomorphism, calculus of constructions and correspondence between natural deduction and typed  $\lambda$ -calculi; your programs as proofs with a practical demonstration using Coq; and categories, functors and endofunctors ( $M : C \rightarrow C$  or monads) as they appear in Haskell programming language.

2:40–2:55

**When Should You Check the Twitter Trends?**

Maria Glenski

*Saint Mary's University of Minnesota*

Twitter, a popular microblogging site that maintains a dynamic list of trends, has millions of users who are active monthly and the number of users grows daily. Sentiment analysis of postings on popular microblogging websites like Twitter can be used in the fields of marketing and social sciences to understand public opinions on products, policies, companies, and more. Twitter's users can also take advantage of the same analysis. This paper focuses on finding the best time to check Twitter trends using percentages of subjective, objective, and positive tweets containing the trending topics in the United States gathered over two non-consecutive weeks. Identifying a time when users can check Twitter trends most efficiently can be used to create a more positive and efficient user experience for users who prefer to view subjective, objective, or positive tweets. Users who wish to view more subjective, objective, and positive tweets should check Twitter trends from Tuesday morning through Thursday morning, from Thursday evening through Saturday evening, and in the evening respectively.

3:00–3:15

**Making Use of Quantum Computing: Adapting Optimization  
Problems to Adiabatic Quantum Computers**

Sam Blakely

*Saint Michael's College*

In recent months, NASA has acquired a *D*-Wave Quantum System. Beyond the classical computing paradigm where bits may only hold values of 0 or 1 but not both, a quantum computer's qubits may hold a 0, 1, or both at once. This is based on the spectacular properties of quantum mechanics, and allows for an exponential increase in a machine's computing power. However, in order to input a problem into a *D*-Wave quantum computer, a problem must be re-phrased in terms of the Ising Objective Function (Quadratic Unconstrained Binary Optimization form). This mapping is then minimized as the quantum system simultaneously examines all qubit configurations and selects the one with the lowest energy state, giving the value of the solution. The current state of quantum computation, and the ability to input optimization problems, will be explored.

3:20–3:35

**Quantum Computing and Prime Factoring**

Robert Hoffman

*University of California, Irvine*

Quantum computing offers the hope of drastically increasing computational speeds. This would mean a significant decrease in the amount of time required for certain computationally difficult problems. A fast solution to prime factorization could have drastic consequences, e.g., in digital security. In this talk, we discuss an early quantum algorithm for prime factorization developed by Peter Shor. After an introduction to the basic tools of quantum computing, including quantum notation, unitary operators, and the quantum Fourier transform, we will illustrate how Shors algorithm reduces the problem of factorization to one of order finding. Finally, we will present the error bounds and running time of this algorithm.

3:40–3:55

**Restoration and Analysis of Apollo Lunar Data**

Melissa Gaddy

*Wofford College*

The Lunar Ejecta and Meteorites (LEAM) Experiment on Apollo 17 was designed to measure hypervelocity particles that collide with the surface of the Moon. Original data analyses from the LEAM experiment yielded unusually high numbers of low-velocity impacts during the passage of the terminator. Recently, a question about electrical noise on the LEAM instrument has arisen, and there is some speculation that noise generated by the heaters and other instrumentation may have influenced the results. The current analysis examines the data to see if there are correlations between the unexpected number of events and the times that the power levels were fluctuating. Analyses have revealed unusual patterns in the accumulators of the UP, EAST, and WEST sensor.

## J. Sutherland Frame Lectures

2014	Keith Devlin	<i>Fibonacci and the First Personal Computing Revolution</i>
2013	Gilbert Strang	<i>Matrices I Admire</i>
2012	Melanie Matchett Wood	<i>The Chemistry of Primes</i>
2011	Margaret H. Wright	<i>You Can't Top This: Making Things Better with Mathematics</i>
2010	Nathaniel Dean	<i>Incomprehensibility</i>
2009	Persi Diaconis	<i>The Mathematics of Perfect Shuffles</i>
2008	John H. Conway	<i>The Symmetries of Things</i>
2007	Donald E. Knuth	<i>Negafibonacci Numbers and the Hyperbolic Plane</i>
2006	Donald Saari	<i>Ellipses and Circles? To Understand Voting Problems??!</i>
2005	Arthur T. Benjamin	<i>Proofs that Really Count: The Art of Combinatorial Proof</i>
2004	Joan P. Hutchinson	<i>When Five Colors Suffice</i>
2003	Robert L. Devaney	<i>Chaos Games and Fractal Images</i>
2002	Frank Morgan	<i>Soap Bubbles: Open Problems</i>
2001	Thomas F. Banchoff	<i>Twice as Old, Again, and Other Found Problems</i>
2000	John H. Ewing	<i>The Mathematics of Computers</i>
1999	V. Frederick Rickey	<i>The Creation of the Calculus: Who, What, When, Where, Why</i>
1998	Joseph A. Gallian	<i>Breaking Drivers' License Codes</i>
1997	Philip D. Straffin, Jr.	<i>Excursions in the Geometry of Voting</i>
1996	J. Kevin Colligan	<i>Webs, Sieves and Money</i>
1995	Marjorie Senechal	<i>Tilings as Differential Gratings</i>
1994	Colin Adams	<i>Cheating Your Way to the Knot Merit Badge</i>
1993	George Andrews	<i>Ramanujan for Students</i>
1992	Underwood Dudley	<i>Angle Trisectors</i>
1991	Henry Pollack	<i>Some Mathematics of Baseball</i>
1990	Ronald L. Graham	<i>Combinatorics and Computers</i>
1989	Jean Cronin Scanlon	<i>Entrainment of Frequency</i>
1988	Doris Schattschneider	<i>You Too Can Tile the Conway Way</i>
1987	Clayton W. Dodge	<i>Reflections of a Problems Editor</i>
1986	Paul Halmos	<i>Problems I Cannot Solve</i>
1985	Ernst Snapper	<i>The Philosophy of Mathematics</i>
1984	John L. Kelley	<i>The Concept of Plane Area</i>
1983	Henry Alder	<i>How to Discover and Prove Theorems</i>
1982	Israel Halperin	<i>The Changing Face of Mathematics</i>
1981	E. P. Miles, Jr.	<i>The Beauties of Mathematics</i>
1980	Richard P. Askey	<i>Ramanujan and Some Extensions of the Gamma and Beta Functions</i>
1979	H. Jerome Keisler	<i>Infinitesimals: Where They Come From and What They Can Do</i>
1978	Herbert E. Robbins	<i>The Statistics of Incidents and Accidents</i>
1977	Ivan Niven	<i>Techniques of Solving Extremal Problems</i>
1976	H. S. M. Coxeter	<i>The Pappus Configuration and Its Groups</i>
1975	J. Sutherland Frame	<i>Matrix Functions: A Powerful Tool</i>

## MAA Lectures for Students

2014	Jack Graver	<i>The Founding of Pi Mu Epsilon 100 Years Ago</i>
2013	Frank Morgan	<i>Optimal Pentagonal Tilings</i>
2012	Ivars Peterson	<i>Geometreks</i>
2011	Roger Nelson	<i>Math Icons</i>
2010	Sommer Gentry	<i>Faster, Safer, Healthier with Operations Research</i>
2009	Colm Mulcahy	<i>Mathemagic with a Deck of Cards on the Interval Between 5.700439718 and 806581751709438785716606368564037 6697528950544088327782400000000000</i>
2008	Laura Taalman	<i>Sudoku: Questions, Variations and Research</i>
2007	Francis Edward Su	<i>Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes</i>
2006	Richard Tapia	<i>Math at Top Speed: Exploring and Breaking Myths in Drag Racing Folklore</i>
2005	Annalisa Crannell & Marc Frantz	<i>Lights, Camera, Freeze!</i>
2004	Mario Martelli	<i>The Secret of Brunelleschi's Cupola</i>
2004	Mark Meerschaert	<i>Fractional Calculus with Applications</i>
2003	Arthur T. Benjamin	<i>The Art of Mental Calculation</i>
2003	Donna L. Beers	<i>What Drives Mathematics and Where is Mathematics Driving Innovation?</i>
2002	Colin Adams	<i>"Blown Away: What Knot to do When Sailing" by Sir Randolph "Skipper" Bacon III</i>
2002	M. Elisabeth Pate-Cornell	<i>Finding and Fixing Systems' Weaknesses: The Art and Science of Engineering Risk Analysis</i>
2001	Rhonda Hatcher	<i>Ranking College Football Teams</i>
2001	Ralph Keeney	<i>Building and Using Mathematical Models to Guide Decision Making</i>
2000	Michael O'Fallon	<i>Attributable Risk Estimation: A Tale of Mathematical/Statistical Modeling</i>
2000	Thomas Banchoff	<i>Interactive Geometry on the Internet</i>
1999	Edward G. Dunne	<i>Pianos and Continued Fractions</i>
1999	Dan Kalman	<i>A Square Pie for the Simpsons and Other Mathematical Diversions</i>
1998	Ross Honsberger	<i>Some Mathematical Morsels</i>
1998	Roger Howe	<i>Some New and Old Results in Euclidean Geometry</i>
1997	Aparna Higgins	<i>Demonic Graphs and Undergraduate Research</i>
1997	Edward Schaefer	<i>When is an Integer the Product of Two and Three Consecutive Integers?</i>
1996	Kenneth Ross	<i>The Mathematics of Card Shuffling</i>
1996	Richard Tapia	<i>Mathematics Education and National Concerns</i>
1995	David Bressoud	<i>Cauchy, Abel, Dirichlet and the Birth of Real Analysis</i>
1995	William Dunham	<i>Newton's (Original) Method - or - Though This Be Method, Yet There is Madness</i>
1994	Gail Nelson	<i>What is Really in the Cantor Set?</i>
1994	Brent Morris	<i>Magic Tricks, Card Shuffling and Dynamic Computer Memories</i>
1993	Richard Guy	<i>The Unity of Combinatorics</i>
1993	Joseph Gallian	<i>Touring a Torus</i>
1992	Peter Hilton	<i>Another Look at Fibonacci and Lucas Numbers</i>
1992	Caroline Mahoney	<i>Contemporary Problems in Graph Theory</i>
1991	Lester Lange	<i>Desirable Scientific Habits of Mind Learned from George Polya</i>

## **PI MU EPSILON**

### **President:**

Angela Spalsbury  
Youngstown State University

### **President Elect:**

Paul Fishback  
Grand Valley State University

### **Past-President:**

Eve Torrence  
Randolph-Macon College

### **Secretary-Treasurer:**

Stephanie Edwards  
Hope College

### **Councillors:**

Chad Awtrey  
Elon University

Jennifer Beineke  
Western New England University

Darci L. Kracht  
Kent State University

Ben Ntatin  
Austin Peay State University

### **Editor, PME Journal:**

Brigitte Servatius  
Worcester Polytechnic Institute

## **MAA Committee on Undergraduate Student Activities and Chapters**

Dora Cardenas Ahmadi, Chair  
Morehead State University

Jennifer Bergner  
Salisbury University

George R. Bradley  
Duquesne University

Joyati Debnath  
Winona State University

Benjamin J. Galluzzo  
Shippensburg University

Kristina Cole Garrett  
St. Olaf College

William J. Higgins  
Wittenberg University

Theron J. Hitchman  
University of Northern Iowa

Lisa E. Marano  
West Chester University

Pamela A. Richardson  
Westminster College

Jennifer Schaefer  
Dickinson College

Daluss Siewert  
Black Hills State University

Pete Skoner, KME Representative  
St. Francis University

Eve Torrence, PME Representative  
Randolph-Macon College

Gerard A. Venema, Ex Officio Associate Secretary  
Calvin College

Jiehua Zhu  
Georgia Southern University

# 100 YEARS OF PI MU EPSILON

by Darci L. Kracht

The year 2014 marks the 100<sup>th</sup> anniversary of the founding of Pi Mu Epsilon at Syracuse University. It grew out of the Mathematical Club formed in 1903. By 1913, Professor Edward Drake Roe felt that the club focused too much on social activities at the expense of the scholarly aspects. At a meeting marking the 10<sup>th</sup> anniversary of the club, he proposed the formation of a “professional fraternity” with membership restricted to students with strong academic records and a serious interest in mathematics.

Roe led a committee charged with developing proposals for the organization of such a society. A convention in the spring of 1914 adopted a constitution and by-laws. They chose the name Pi Mu Epsilon, an acronym for a Greek phrase meaning “to promote scholarship and mathematics.” Officers were elected, including Roe as the first Director General. Finally, on May 25, 1914, the Pi Mu Epsilon Fraternity was incorporated in the State of New York.

Despite the “fraternity” designation, PME welcomed women as members from its inception. Three of the original eight “incorporators” were female. Women presented mathematical papers at many of the early meetings. Furthermore, when the prestigious committee on scholarship was established in 1916, the by-laws stipulated explicitly that both sexes be represented. The name of the organization was legally changed from “Pi Mu Epsilon Fraternity, Inc.” to “Pi Mu Epsilon, Inc.” in 1990.

The inauguration of a chapter at Ohio State University in 1919 started PME down the road to becoming a national organization. This was followed soon after by chapters at the University of Pennsylvania, the University of Missouri, and the University of Alabama. However, Syracuse University still ran the show. That changed in 1922 when a national executive board was democratically elected from all chapters. Roe was again elected Director General, along with W. V. Houston (Ohio State) as Vice-Director General, Warren G. Bullard (Syracuse) as Secretary General, Louisa Lotz (Pennsylvania) as Treasurer General, and Mabel G. Kessler (Pennsylvania) as Librarian General.

On the occasion of the fifteenth anniversary of PME in 1929, the society collected \$75 to purchase a memento for founder Roe. They presented him with a scarf pin set with an aquamarine and two diamonds. While deeply appreciative of the gesture, Roe apparently didn’t care much for the pin. He exchanged it for a PME pin encrusted with small diamonds and rubies. On his death, Roe’s widow gave the pin to Alan D. Campbell, the president of the Syracuse Chapter of PME. The pin was rediscovered by Campbell’s widow in 1951. She sent it to the editor of the **PME Journal**, Ruth W. Stokes, who turned it over to Director General C. C. MacDuffee. In the November 1951 **Journal** article describing this series of events, Stokes wrote,

We decided that no more fitting disposition could be made of the pin than that it be worn by each succeeding director-general of the fraternity while in office.

The current whereabouts of the Founder’s Pin are a mystery.

Meanwhile, PME continued to add chapters. By the middle of the twentieth century the count had reached 50. A constitutional amendment paved the way for the first and only industrial affiliate chapter at the Evandale plant of the General Electric Corporation in 1957. That same year, PME donated \$300 for the establishment of Mu Alpha Theta, a mathematics honor society for high school and junior college students founded by Richard V. and Josephine P. Andree. R. V. Andree was then PME Secretary-Treasurer General, a position he held for 21 years before serving as Vice President and President. It was also during the mid-century that PME established two of its cornerstone activities: an undergraduate student journal and student paper sessions at its summer national meeting.

The **Pi Mu Epsilon Journal** was launched in 1949 under the editorship of Ruth W. Stokes. Its primary objective was to publish mathematics articles aimed at and written by undergraduate students. Finding good mathematical content challenged the first several editors. Chapter reports and lists of new members comprised a substantial portion of the **Journal** in its early years.

To lure submissions to the **Journal**, an annual prize for the top paper authored by an undergraduate student was established in 1966. This was expanded in 1971 to first, second, and third prizes for papers by undergraduate or beginning graduate students. These awards were named in honor of R. V. Andree in 1987.

A Problem Department appeared in the very first issue and has been a popular presence ever since. To celebrate the anniversary, current Problems Editors Steven J. Miller, James M. Andrews, and Avery T. Carr, have created 100 additional Centennial Problems, one corresponding to each year of the society's existence. These are being published in four groups, according to the congruence of the year modulo 4.

In the April 1952 issue of the **Journal**, Secretary-Treasurer General J. Sutherland Frame published a call for student speakers at the upcoming national meeting of PME. The meeting was to be held at his home institution of Michigan State College on Labor Day, in conjunction with the summer meetings of the MAA and AMS. Frame urged each chapter to send a student speaker or delegate, writing,

This is a real opportunity for students interested in mathematics to become acquainted with each other and their future colleagues in the profession. The cooperation of each of you is requested to make this meeting a success.

Frame provided a tentative schedule, with a two-hour time slot for “[f]our or five 20 minute talks, including at least two by students.” As it turned out, there were six student speakers and PME had to add an evening session to accommodate them. In the November 1952 issue of the **Journal**, Frame reported,

These six student papers . . . were the unusual feature of the national meeting, and were of much higher caliber than one might have expected from relatively inexperienced speakers.

Student paper presentations again featured at the national summer meeting in 1955, 1957, and annually from 1960, excepting only 1962 and 1974 (when no national meeting took place). The meetings continued to be held in conjunction with the AMS-MAA joint summer meetings until the AMS discontinued its summer national meeting. Since then they have occurred at MathFest with the generous support and cooperation of the MAA. While the number of student presentations at the PME sessions has varied from year to year, the general trend has been upward. The count reached 46 in 1989 for the 75<sup>th</sup> anniversary and a record 80 this year for the 100<sup>th</sup>.

The society has provided travel support for speakers and student delegates from the start. It has also provided incentives in the form of monetary prizes for the best talks. Since 1989, the NSA has provided a generous grant for subsistence support and the AMS has donated prize money. (A complete list of this year's sponsors is found below.)

Most PME summer meetings included a plenary address by a noted mathematician. This practice was formalized as the J. Sutherland Frame Lecture Series. Frame himself delivered the inaugural lecture, “Matrix Functions: A Powerful Tool,” in 1975.

PME established the C. C. MacDuffee Award for Distinguished Service in 1965 in honor of its seventh president. The highest award offered by the society, it is to be bestowed “often enough to be recognized and seldom enough to be meaningful.” The first two MacDuffee Awards were given to J. S. Frame and R. V. Andree at the Rutgers meeting in 1966. At that time, PME had 120 chapters, 51 of which had been installed by Frame.

A 1965 constitutional revision updated the titles Director General and Vice Director General to President and Vice President, respectively. Another revision in 1972 gave the national council its present form. The office of Vice President was renamed President-Elect. The President-Elect automatically succeeds to President, and then to Past President, serving one three-year term in each office. The Secretary-Treasurer, **PME Journal** Editor, and four Councillors round out the council.

Meanwhile, the society continued to grow. By the 75<sup>th</sup> anniversary in 1989, there were 257 chapters and close to 90,000 members. Recognizing that the chapters form the bedrock of the society, PME established a lectureship grant program. This enables chapters to bring a member of council to their institution to give a talk and offer advice on how to build and maintain a vibrant chapter. Richard A. Good, a four-term

Secretary-Treasurer and two-term Councillor, funded this program anonymously for many years. A bequest made this funding permanent at his death in 2008. The program was then renamed the Richard A. Good Pi Mu Epsilon Lectureship and Chapter Enhancement Program, or Good Lectureship for short. Chapters are also eligible for Prize Grants and Conference Grants for hosting a meeting.

By the end of the twentieth century, PME had established a web page. President J. Douglas Faires wrote in the Spring 2000 issue of the **PME Journal** that “we want to make [the web site] the primary vehicle for communication for the organization.” Chapter reports and other announcements were moved to the web, enabling the **Journal** to take on a more professional tone under the guidance of Editor Brigitte Servatius. The society also embarked on a project to computerize its membership records. In 2008, after six years of work, all members of all chapters had been entered into the database. A major redesign of the web site was launched in the spring of 2014. The first eleven volumes of the **PME Journal** are posted on the site along with a searchable index through 2001. In addition, there are forms for applying for grants, submitting chapter reports, enrolling members, and so on.

In 2008, PME introduced a triennial Advisor’s Award to recognize outstanding chapter advisors. The first recipient was Michelle Ghrist, advisor of the Colorado Gamma Chapter at the US Air Force Academy.

And still, the society grows. Under the leadership of President Angela S. Spalsbury, Pi Mu Epsilon will embark on its second century with 382 chapters, representing 47 states and the District of Columbia. The states without chapters are Hawaii, Idaho, and Wyoming. Our goal is to reach 400 chapters, covering all 50 states by the end of 2014.

*The above brief history was compiled from documents available on the Pi Mu Epsilon web page <http://pme-math.org/>. The author regrets any errors (which she hopes are few) and omissions (which she knows are many). Please send corrections and additions to [darci@math.kent.edu](mailto:darci@math.kent.edu).*



# **THANK YOU TO OUR DONORS AND SUPPORTERS**

## **PME CENTENNIAL FUND DONORS**

**Robert Devaney  
Joe Diestel  
Stephanie Edwards  
Barbara Faires  
Paul E. Fishback  
William T. Fishback  
John Frohlinger  
Jack Graver  
Richard Holford  
Robert Vallin and Jacqueline Jensen-Vallin  
Darci L. Kracht  
Thomas S. K. Kung Trust  
Thomas Maniscalco  
Stephen Pennell  
Eileen Poiani  
Rick and Cindy Poss  
Brigitte Servatius  
Robert Sefton Smith  
Angela Spalsbury  
Colin Starr  
David and Pebble Sutherland  
Eve and Bruce Torrence  
Ronald Wasserstein  
Joan Weiss  
George Yates  
Ohio Xi Chapter**

To donate to the PME centennial fund  
contact Paul Fishback, [fishbacp@mail.gvsu.edu](mailto:fishbacp@mail.gvsu.edu)  
or visit [pme-math.org](http://pme-math.org) to learn more.

Pi Mu Epsilon would like to express its appreciation to the American Mathematical Society, the American Statistical Association, the Committee for Undergraduate Research, the Society for Industrial and Applied Mathematics, Budapest Semesters in Mathematics, the SIGMAA-Environmental Mathematics and BioSIGMAA for the sponsorship of the Awards for Outstanding Presentations. A special thanks is extended to Dover Publications and Princeton University Press for donating book prizes. It would additionally like to thank the National Security Agency Mathematical Sciences Program, Grant number H98230-14-1-0155, for its continued support of the student program by providing subsistence grants to Pi Mu Epsilon speakers.

**Notes:**

**Notes:**

**Notes:**

**Notes:**

**Notes:**

**Notes:**

**Notes:**