Proof Without Words: Fibonacci Tiles

\[ F_{n+1}^2 = 2F_{n+1}F_n - F_n^2 + F_{n-1}^2 \]
\[ = 2F_{n+1}F_n^2 + F_{n-1}^2 - F_n^2 \]
\[ = 2F_nF_{n-1} + F_{n-1}^2 + F_{n-1}^2 \]
\[ = F_{n+1}F_n + F_nF_{n-1} + F_{n-1}^2 \]
\[ = F_{n+1}F_{n-1} + F_{n-1}^2 + F_nF_{n-1} \]

\[ F_n^2 = F_{n+1}F_{n-1} + F_{n-1}F_{n-2} - F_{n-2}^2 \]

\[ F_{n-1}^2 = F_n^2 + 3F_{n-1}^2 + 2F_{n-1}F_{n-2} \]

\[ F_n = F_{n+1}F_{n-2} + F_{n-1}^2 \]

\[ F_n \] denotes the \( n \)th Fibonacci number, where \( F_{n+1} = F_n + F_{n-1}, \) \( F_0 = 0, \) \( F_1 = 1. \) Obvious assumptions concerning the least value of \( n \) in each identity should be made as required. Further visual proofs of Fibonacci identities may be found in:

A. Brousseau, Fibonacci numbers and geometry, Fibonacci Quarterly 10 (1972) 308–318.

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