

**Abstracts for the MAA  
Undergraduate Poster Session**

**San Diego, CA  
January 12, 2018**

**Organized by**

Eric Ruggieri  
*College of the Holy Cross*

and

Chasen Smith  
*Georgia Southern University*



## Dear Students, Advisors, Judges and Colleagues,

If you look around today you will see over 400 posters (a record number!) and more than 650 student presenters, representing a wide array of mathematical topics and ideas. These posters showcase the vibrant research being conducted as part of summer programs and during the academic year at colleges and universities from across the United States and beyond. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new Ph.D.s, and administrators. Many of the judges are acknowledged here in this booklet; however, a number of judges here today volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by the National Science Foundation, Tudor Investments, and Two Sigma. We are also helped by the members of the Committee on Undergraduate Student Activities and Chapters (CUSAC) in some way or other. They are: Dora C. Ahmadi; Benjamin Galluzzo; TJ Hitchman; Cynthia Huffman; Aihua Li; Sara Louise Malec; May Mei; Stacy Ann Muir; Andy Niedermaier; Peri Shereen; Angela Spalsbury; Violetta Vasilevska; Gerard A. Venema; and Jim Walsh. There are many details of the poster session that begin with putting out the advertisement in FOCUS, ensuring students have travel money, making sure the online submission system works properly, and organizing poster boards and tables in the room we are in today that are attributed to Gerard Venema (MAA Associate Secretary), Kenyatta Malloy (MAA), and Donna Salter (AMS).

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Kenyatta Malloy, for her work setting up and managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi for doing the final production work on the abstract book.

We would also like to thank James P. Solazzo (Coastal Carolina University), Peri Shereen (California State University Monterey Bay), Aihua Li (Montclair State University), and Dora Ahmadi (Moorehead State University) for organizing an orientation for the judges and authoring the judging form.

Thanks to all the students, judges, volunteers, and sponsors. We hope you have a wonderful experience at this years poster session!

Eric Ruggieri  
*College of the Holy Cross*

Chasen Smith  
*Georgia Southern University*

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Their generosity on behalf of the 2018 Undergraduate Student Poster Session enables students to interact with peers and role models in the mathematical sciences during the largest mathematics meeting in the world.

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487. V. Thomas, *IISER Thiruvananthapuram*
488. Benjamin Thompson, *Boston University*
489. Elizabeth Thoren, *Pepperdine University*
490. Jianjun Tian, *New Mexico State University*
491. Emilio Toro, *University of Tampa*



492. Denise Rangel Tracy, *CCSU*
493. Khang Tran, *California State University, Fresno*
494. Trang Tran, *University of California, Riverside*
495. Rodrigo Trevino, *University of Maryland, College Park*
496. Henry Tucker, *UC San Diego*
497. Chris Tweddle, *Governors State Univeristy*
498. Monica Van Dieren, *Robert Morris University*
499. Kevin Vander Meulen, *Redeemer University College*
500. Brian Vargas, *California State University, San Marcos*
501. Violeta Vasilevska, *Utah Valley University*
502. Oscar Vega, *California State University, Fresno*
503. Mikael Vejdemo Johansson, *CUNy College of Staten Island*
504. Salvador Venegas-Andraca, *Tecnologico de Monterrey*
505. John Villalpando, *California Lutheran University*
506. Maria Viola, *Lakehead University*
507. Karl Voss, *Bucknell University*
508. Henry Walker, *Grinnell College*
509. Stefanie Wang, *Trinity College*
510. Xiang-Sheng Wang, *University of Louisiana at Lafayette*
511. Xingting Wang, *Temple University*
512. Zhaoxia Wang, *Louisiana State University*
513. Yajni Warnapala, *Roger Williams University*
514. Margaret Watts, *Doane University*
515. Thomas Wears, *Longwood University*
516. Darrin Weber, *University of Evansville*
517. Noah Weiss, *Carthage College*
518. Beverly H West, *Cornell University*
519. Gretchen Whipple, *Warren Wilson College*
520. Ursula Witcher, *Mathematical Reviews (AMS)*
521. Ryan White, *Florida Institute of Technology*
522. Tad White, *IDA Center for Computing Sciences*
523. Hays Whitlatch, *University of South Carolina*
524. Joshua Whitlinger, *Virginia Commonwealth University*
525. Esther Widiasih, *University of Hawai'i - West O'ahu*
526. Brandy Wieggers, *Central Washington University*
527. Don Wilathgamuwa, *MSU-Billings*
528. David Wildstrom, *University of Louisville*
529. Jennifer Williams, *Drake University*
530. Nakeya Williams, *United States Military Academy*
531. Devin Willmott, *University of Kentucky*
532. Benjamin Wilson, *Stevenson University*

533. Celestine Woodruff, *James Madison University*
534. Lina Wu, *Borough of Manhattan Community College*
535. Yinshu Wu, *Alabama A&M University*
536. Cynthia Wyels, *CSU Channel Islands*
537. Zhifu Xie, *The University of Southern Mississippi*
538. Nina Xu, *Bucknell University*
539. Xiaoqian Xu, *Carnegie Mellon University*
540. Carolyn Yackel, *Mercer University*
541. Dina Yagodich, *Frederick Community College*
542. Jihyeon Yang, *Marian University - Indianapolis*
543. Philip Yates, *DePaul University*
544. Andrea Young, *Ripon College*
545. James Young-Nastally, *University of Northwestern Ohio*
546. Petr Zemanek, *Masaryk University, Czech Republic*
547. Ryan Zerr, *University of North Dakota*
548. Yunkai Zhang, *University of California, Santa Barbara*
549. Ju Zhou, *Kutztown University of Pennsylvania*
550. Jiuyi Zhu, *Louisiana State University*
551. Yingxian Zhu, *Utah Valley University*
552. Yan Zhuang, *Brandeis University*
553. Lori Ziegelmeier, *Macalester College*
554. Jay Zimmerman, *Towson University*
555. Scott Zinzer, *Aurora University*
556. Elizabeth Zollinger, *St. Joseph's College, Brooklyn*

# Titles, Authors, Advisors and Abstracts

## 1. Withdrawn

## 2. Multiplier Sequences Over Finite Fields

**Miguel Baza** CSU Fresno

**Advisor(s):** Oscar Vega, CSU Fresno

Abstract: Craven and Csordas' 1977 paper, Multiplier Sequences for Fields outlined essential properties, theorems and definitions which aided in the study of multiplier sequences over any given field. In particular, they studied these sequences over finite fields, transforming a typically analytical subject into an algebraic one. Their paper was not left without questions to be answered; having characterized all multiplier sequences over fields of order at least five, they were unable to do so for fields containing 2, 3, or 4 elements. In this poster, we explore the properties of sequences and polynomials over the smallest three fields in an attempt to complete the work Craven and Csordas started forty years ago.

## 3. Towers of Algebras Related to Symmetric Groups and Hecke Algebras

**Dale Bigler** Michigan Technological University

**Advisor(s):** Jie Sun, Michigan Technological University

In a recent paper, Savage and Yacobi defined a tower of algebras to be a graded algebra which satisfies four axioms. Under additional conditions the Grothendieck groups of the categories of finitely generated left modules and finitely generated projective left modules over a tower of algebras can form a dual pair of Hopf algebras. Examples of such towers of algebras include the tower of group algebras of symmetric groups  $\oplus \mathbb{C}\mathfrak{S}_n$  and the tower of 0-Hecke algebras  $\oplus H_n(0)$ . We prove that the graded algebra  $\oplus H\mathfrak{S}_n$ , introduced by Hivert and Thiéry, also satisfies the axioms of a tower of algebras. In particular,  $H\mathfrak{S}_{m+n}$  is a two-sided projective  $H\mathfrak{S}_m \otimes H\mathfrak{S}_n$ -module.

## 4. On Inverse Semigroups of Self-Similar Graph Actions

**Alexander Black** Cornell University

**Trevor Arrigoni** Westminster College

**Yansy Perez** University of Texas at Tyler

**Jessica Harter** University of Wisconsin Stevens Point

**Advisor(s):** David Milan, University of Texas at Tyler

We discuss a recently discovered class of inverse semigroups called the inverse semigroups of self-similar graph actions. To describe this class, we characterized the idempotents, ideals, Green's Relations, and other properties common to inverse semigroups in general. Furthermore, we provide conditions that ensure a self-similar graph action is fundamental or 0-disjunctive and, more generally, when it satisfies properties of inverse semigroups relevant to the study of  $C^*$ -algebras.

## 5. Estimating the Effect of Entanglement on the Entropy of Quantum Channels Arising from Quantum Groups

**Josiah Blaisdell** Texas A&M University

**William Ogletree** Texas A&M University

**Nathan Mehlhop** Texas A&M University

**Advisor(s):** Michael Brannan, Texas A&M University

We investigate the potential of using highly entangled subspaces of tensor products from quantum groups to produce violations of the additivity of the minimal output entropy for quantum channels. Using MATLAB and computational methods (Brannan-Collins, 2016) we determine the maximum singular values that can exist in the irreducible components of tensor product representations of free orthogonal quantum groups. Using these singular values we identify a lower bound on the minimum output entropy for the quantum channels associated to these Hilbert spaces. Our ultimate goal was to find a non-random example of a quantum channel for which the additivity of the minimum output entropy did not hold (Hastings, 2009). While we did not succeed, we were able to come up with data that can be used to

describe the relationship between the subrepresentations that you choose to tensor and the minimum output entropy of the channels in these subrepresentations.

## 6. Periodicity and Complexity of Words on Groups

**Eben Blaisdell** Bucknell University

**Advisor(s):** Van Cyr, Bucknell University

A bi-infinite word is a string of characters that is infinite in both directions. This can be conceptualized as a function from  $\mathbb{Z}$  to a finite alphabet  $\Sigma$ . For a bi-infinite word  $f$ , the block complexity at scale  $n$  is the number of distinct subwords of length  $n$ . For example, the scale 4 block complexity of  $\dots 0101010101\dots$  is 2 since only 0101 and 1010 appear as length 4 subwords. The Morse-Hedlund theorem, a foundational theorem in symbolic dynamics, states that a word is periodic with a period at most  $n$  if and only if the block complexity at scale  $n$  of that word is less than or equal to  $n$ . If  $G$  is a group, it is natural to ask whether a version of the Morse-Hedlund theorem holds for functions  $f : G \rightarrow \Sigma$ . The pattern complexity of a word  $f : G \rightarrow \Sigma$  is defined, for patterns  $N \subseteq G$ , to be the number of ways to color translations of  $N$  in  $G$ , i.e.,  $|\{w_g \in \Sigma^N : w_g(n) := f(gn), g \in G\}|$ . The word  $f$  is called periodic with period  $v \in G$  if  $\forall g \in G, f(g) = f(vg)$ . Our research gives an optimal complexity bound that implies periodicity (with certain periods) in an arbitrary group. Additionally, in the case where  $G := \mathbb{Z}^d$  and  $N$  is a hyperbox, we provide an optimal complexity bound for implying periodicity in a principle direction. We also closely examine the effects of complexity bounds on periodicity in the specific case where  $G := \mathbb{Z}^2$  and  $N$  is a box, which is known as the Nivat conjecture, proposed at ICALP in 1997.

## 7. (Duplicate)

## 8. Completions of Noncatenary Local Domains and UFDs

**Caitlyn Booms** University of Notre Dame

**Chloe Avery** UC Santa Barbara College of Creative Studies

**Alex Semendinger** Williams College

**Advisor(s):** Susan Loepp, Williams College

We find necessary and sufficient conditions for a complete local ring to be the completion of a noncatenary local domain, as well as necessary and sufficient conditions for it to be the completion of a noncatenary local UFD. This allows us to find a larger class of noncatenary UFDs than was previously known. We also discuss how noncatenary these rings can be.

## 9. Error-Correcting Capabilities of Various Algebraic Geometric Codes

**Lia Bozzone** Vassar College

**Advisor(s):** Gretchen Matthews, Clemson University

In the past, researchers have predominantly looked at Hermitian codes for use in distributed storage. Algebraic geometric codes (AG-codes) have also been studied, and the generation algorithm of such codes is known. Because AG-codes have many advantages over Hermitian codes, our goal was to extend beyond the Hermitian case to explore the novel idea of using generalized AG-codes specifically in the context of distributed storage. We investigated their structure, approximated the maximum number of total correctable errors, and compared our codes to the Hermitian case. Based on our discoveries about AG-codes, we expanded and improved an already existing decoding algorithm from Wang, Kan, and Shum that may enable decoding of an otherwise non-decodable code, and explored what occurs if certain pieces of information cannot be retrieved.

## 10. The Sortability of Graphs and Matrices Under Context Directed Swaps

**Colby Brown** University of Arizona

**Claudia Sofia Carrillo Vazquez** University of Rochester

**Rashmika Goswami** University of Michigan – Ann Arbor

**Sam Heil** Washington University in St. Louis

**Advisor(s):** Marion Scheepers, Boise State University

The context directed swap, cds, is a sorting operation performed on permutations. It models a process used by ciliates during gene decryption. While cds is the most efficient type of block swap for sorting some permutations, it can fail on

others by reaching fixed points — unsorted permutations on which cds cannot be performed. Prior work characterized which permutations are sortable by the cds operation. We present extensions of prior results for permutations through a novel graph theoretical and linear algebraic framework for cds. This framework applies also to graphs and matrices that do not correspond to any permutations. Our results include sortability criteria for graphs and matrices for these generalizations of cds, implying the prior findings for the special case of permutations.

## 11. Cycle Transformations and CDR — A Unifying Concept

**Cecily Chase** Brown University  
**Olivia Dennis** University of Utah  
**Luke Guatelli** Western Carolina University  
**Jaroor Modi** Rutgers University

**Advisor(s):** Marion Scheepers, Boise State University

Past research on the topic of sorting signed permutations by context directed reversals (CDR) has focused primarily on the algorithmic properties of CDR sorting. Our work emphasized an algebraic study of signed permutations to work toward a better understanding of reachability and sortability under CDR. By defining a cycle transformation on signed permutations, we were able to study how a particular CDR operation affected the dynamics of the sorting process. Our findings include specific conditions under which signed permutations exhibit various types of sortability via CDR sorting and how sortability is intertwined with reachability.

## 12. Refined Inertia of Sign Patterns

**Derek DeBlieck** St. Olaf College  
**Deepak Shah** St. Olaf College  
**Advisor(s):** Adam Berliner, St. Olaf College

Motivated primarily by the Inverse Eigenvalue Problem, the mathematical community has invested large amounts of time and energy into the study of the relation between matrices and their eigenvalues. We contribute to that study by examining which  $3 \times 3$  sign patterns, matrices with non-numeric  $0/+/-$  entries, allow certain types of eigenvalues. In particular, we are interested in patterns that allow refined inertia  $\mathbb{S}_3 = \{(0, 3, 0, 0), (0, 2, 1, 0), (1, 2, 0, 0)\}$ . In other words, we seek patterns that have one realization with all negative real-part eigenvalues, another with all negative real-part except for one zero eigenvalue, and a third with all negative real-part eigenvalues except for one positive real-part eigenvalue. The presence of this property is of particular interest, as it signals the presence of a saddle-node bifurcation in the study of dynamical systems. We classify all 19,683  $3 \times 3$  sign patterns, and determine which of these patterns allow this specific set of eigenvalues. We also present a theorem for the extension of our work beyond  $3 \times 3$  patterns.

## 13. Evaluating Kostant's Multiplicity Formula

**Elise Eckman** Youngstown State University  
**Advisor(s):** Alicia Prieto Langarica, Youngstown State University

Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a wellknown formula of Kostant that consists of an alternating sum over a finite group and involves a partition function called Kostant's partition function. There are two major obstacles in the use of this formula. First, the number of terms arising in the sum grows factorially as the rank of the Lie algebra increases and, second, the value of the partition function is often unknown. The focus of the research is to convert all of the known values contributing nontrivially to Kostant's weight multiplicity formula in two dimensions into three dimensions. The graph of these contributing elements in three dimensions and a program to run the calculations for any weight space is the final goal.

## 14. Enumerating $2 \times 2$ and $3 \times 3$ Diagonalizable Matrices over $\mathbb{Z}_p^k$

**Catherine Falvey** Eastern Connecticut State University  
**Rico Vicente** California State University Long Beach  
**Advisor(s):** Brian Sittinger, California State University Channel Islands

In a standard approach to linear algebra, one works with matrices over fields (usually  $\mathbb{R}$  and  $\mathbb{C}$ ). Here, we consider matrices over the commutative ring  $\mathbb{Z}_p^k$ . Avni provided insight into similarity classes of matrices over such a ring,

and Kaylor enumerated diagonalizable matrices over the field  $\mathbb{Z}_p$ . Using these ideas, we give the different similarity classes of diagonalizable matrices in  $\mathbb{Z}_{p^k}$ . Then, we enumerate the  $2 \times 2$  and  $3 \times 3$  matrices with entries in  $\mathbb{Z}_{p^k}$  that are diagonalizable over  $\mathbb{Z}_{p^k}$  and thus find the probability that a matrix is diagonalizable in the ring  $\mathbb{Z}_{p^k}$ .

## 15. Superpatterns of Companion Matrices

**Jonathan Fischer** Redeemer University College

**Advisor(s):** Kevin Vander Meulen, Redeemer University College

Given the polynomial  $p(x) = x^n + \sum_{i=1}^n a_i x^{n-i}$ , a *companion matrix* to  $p(x)$  is an  $n \times n$  matrix  $A$  with  $n^2 - n$  entries in  $\mathbb{R}$  and the remaining entries variables  $-a_1, -a_2, \dots, -a_n$  such that the characteristic polynomial of  $A$  is  $p(x)$ . If  $A$  has  $2n - 1$  nonzero entries, we say that it is *sparse*. The most well-known sparse companion matrix is the *Frobenius companion matrix*. In this poster we explore patterns and properties of *non-sparse* companion matrices of small order and illustrate our research results by way of examples and digraphs. We also show why there does not exist a non-sparse companion matrix that can be obtained from the Frobenius companion matrix by changing some of the zero entries to nonzero entries.

## 16. Spectral Analysis of Graphs Related to the Basilica Julia Set

**Courtney George** Western Kentucky University

**Samantha Jarvis** Trinity College

**Advisor(s):** Luke G. Rogers, University of Connecticut

We analyze the spectra of a sequence of graphs constructed from the Schreier graphs of the Basilica group. Our analysis differs from earlier work of Grigorchuk and Zuk in that it is based on a macroscopic decomposition of the graphs. This method gives precise information about the multiplicities of eigenvalues and consequently good information about the spectral measures of large graphs. It also permits a proof of the existence of gaps in the spectrum of limiting graphs.

## 17. Classifying Graph Lie Algebras

**Michael Gintz** Homeschooled

**Advisor(s):** Tanya Khovanova, MIT

A Lie algebra is a linear object which has a powerful homomorphism with a Lie group, an important object in differential geometry. All semisimple Lie algebras can be classified by Dynkin diagrams, for which the A, D, and E types are simple graphs. In previous work a construction was proposed that builds a Lie algebra for Dynkin diagrams of these types from the properties of the graphs themselves. We expand this definition to construct a Lie algebra given any simple graph, and consider the problem of determining its structure. We determine sufficient conditions that allow the Lie algebra of a graph to be decomposed into two Lie algebras of graphs with smaller size, and construct these graphs explicitly. We also define an isomorphism between graph Lie algebras, and use this to prove that every graph Lie algebra is isomorphic to that of some tree. We then describe work done towards further classifying graph Lie algebras.

## 18. Non-commutative Massey-Omura Encryption with Symmetric Groups

**Shannon Haley** University of Mary Washington

**Advisor(s):** Randall Helmstutler, University of Mary Washington

This research introduces two non-commutative variations on the original Massey-Omura encryption system using conjugations in the symmetric group  $S_n$ . Patented in 1986, the original system was based on the cyclic group  $\mathbb{F}^*$  of units in a finite field  $\mathbb{F}$ . In place of the abelian group  $\mathbb{F}^*$ , we will work in the non-abelian group  $S_n$  using disjoint permutations as well as maximal abelian subgroups in order to potentially create a more secure system. Introducing the non-abelian group  $S_n$  presents the need to create a keyspace of commuting permutations and abelian subgroups of sufficient size. We analyze the security of our modified systems by examining the bit-level security of each and susceptibility to standard message attacks. We find that the first modification is not more or less vulnerable to standard attacks than the original system. Additionally, we find that the keycount for the first system grows factorially with  $n$ . We show that the keycount for the second variation grows exponentially with  $n$  while improving on the first modification by allowing any number of users to participate in communication.

## 19. Realizing Symmetric Hypergroup of Rank 4

**Bingyue He** Grinnell College

**Advisor(s):** Christopher French, Grinnell College

In Group theory, Cayley proved that any group can be realized as a permutation group, if we view each element of  $G$  a permutation of the set  $G$ . Correspondingly, we have definition for hypergroups, but the operation between elements is not binary, and rather the product of two elements could contain more than one element. A parallel definition of permutation group, in this case, is association scheme. However, it is not the case that every hypergroup can be realized as an association scheme. This project mainly focuses on the realization of symmetric hypergroup of rank 4.

## 20. Seminormal Representations of the Partition Algebra

**Theo Jacobson** Macalester College

**Advisor(s):** Tom Halverson, Macalester College

The partition algebra  $P_k(n)$  is an algebra of diagrams with a basis consisting of partitions of the set  $\{1, 2, \dots, 2k\}$ , where  $n \geq 2k$ . These set partitions are drawn as two-row diagrams with  $k$  vertices in each row and edges connecting vertices that are in the same block of the set partition. Multiplication in the algebra is given by diagram concatenation, and the irreducible matrix representations of the algebra are labeled by integer partitions of  $n$ . The partition algebra has many subalgebras including the group algebra of the symmetric group  $S_k$ . We study an orthogonal form of these representations which generalize Young's orthogonal representation of  $S_k$ , taking a new approach to the problem by using combinatorial objects called set-partition tableaux.

## 21. Algebra associated with the Hasse graph of the 600-cell

**Jared Johnson** University of Wisconsin-Eau Claire

**Geoffrey Glover** University of Wisconsin-Eau Claire

**Advisor(s):** Colleen Duffy, University of Wisconsin-Eau Claire

We can construct a graded algebra associated to a directed Hasse graph of a regular polytope by taking the quotient of the free algebra on the set of edges of the graph by the relations given by equating two directed paths having the same initial and final vertices. Previous work has studied the finite Coxeter groups  $A_n$ ,  $B_n$ ,  $D_n$ ,  $I_2(p)$  and  $H_3$  and their related polytopes. The current goal of our project is to determine the structure of a graded algebra,  $A(\Gamma_{H_4})$ , that is associated to the Hasse graph,  $\Gamma_{H_4}$ , obtained from the 600-cell. The symmetry group of the 600-cell, and thus the automorphism group of the graphs, is isomorphic to the  $H_4$  Coxeter group. For each unique symmetry, we consider the Hasse subgraph consisting of fixed  $k$ -faces of the polytope under the action. From each Hasse subgraph, we determine the graded dimension of the related subalgebras of  $A(\Gamma_{H_4})$  by counting the directed paths between each pair of levels in the graph. We have created programs to produce the generating functions that will in turn describe  $A(\Gamma_{H_4})$  under the action of each symmetry class. This then allows us to describe the complete algebraic structure using representation theory.

## 22. On the Creation of Centrosymmetric Matrices

**Ashley King** Anderson University

**Christina Coats** Anderson University

**Emily Miller** Anderson University

**Advisor(s):** Lee Van Groningen, Anderson University

For any given matrix  $A$ , we can create a centrosymmetric matrix  $B = A + JAJ$  where  $J$  is the skew identity matrix. If the matrix  $A$  is created as the outer product of two vectors  $\mathbf{v}$  and  $\mathbf{h}$ , the resulting centrosymmetric matrix has a maximal rank of 2. However, not all such rank 2 matrices can be written in this form. In this work, we fully examine when a  $3 \times 3$  centrosymmetric matrix can be created from two vectors and generalize our results to larger matrices.

## 23. Modular Arithmetic and Cryptography

**Catherine King** University of Texas at Tyler

**Advisor(s):** Emily Witt, University of Kansas

Construction of a cryptosystem generally requires the system to be relatively easy to solve in one direction but difficult to solve in the reverse. For example, modular multiplication is easy but modular division can be difficult. To assist in modular arithmetic, factoring methods such as Pollard's  $p - 1$  factoring algorithm can be used. An overview for

the types of encryption methods are given to introduce a basic cryptosystems. With this foundation, we will present a cryptosystem based on the theory of elliptic curves.

## 24. Ordered Multiplicity Inverse Eigenvalue Problem on Six Vertices: Unattainability

**Haley Knox** Eastern Connecticut State University

**Michael Wigal** West Virginia University

**John Ahn** Bowdoin College

**Advisor(s):** Steve Butler, Iowa State University

For a graph  $G$  we can associate a family of real symmetric matrices,  $S(G)$ , where for any  $M \in S(G)$ , the location of the nonzero off-diagonal entries of  $M$  are governed by the adjacency structure of  $G$ . The ordered multiplicity inverse eigenvalue problem of a graph is concerned with finding all attainable ordered lists of eigenvalue multiplicities for matrices in  $S(G)$ . We completely solve the ordered multiplicity inverse eigenvalue problem for connected graphs on six vertices partly by using tools and theorems in this poster to prove a graph does not attain a particular ordered multiplicity list. Using these tools, we found that while  $K(m, n)$  with  $\min(m, n) \geq 3$  attains a particular ordered multiplicity list, it cannot do so with arbitrary spectrum.

## 25. Economical Generating Sets of the Monoid of Order-Preserving Partial Permutations

**Saul Lopez** California State University Fullerton

**Advisor(s):** Scott Annin, California State University Fullerton

Semigroups and monoids are important in such fields as computer science, cryptography, abstract algebra and other branches of mathematics. In this research project we seek to determine economical generating sets for the monoid of order-preserving injections of an  $n$ -element set,  $\text{POI}(n)$ . A generating set for any semigroup or monoid is a collection of elements  $S$  such that every element of the semigroup or monoid can be expressed as a product of elements from  $S$ . Generating sets are of fundamental importance in fields across math and science, and mathematicians have great interest in studying generating sets of a variety of algebraic structures. By an economical generating set, we refer to a generating set containing as few elements as possible. Using the algebra software package Groups, Algorithms, and Programming, GAP, we examine the key features of various generating sets of  $\text{POI}(n)$  including their sizes, the domains and ranges of the injections arising in them, and more. We confirm that every generating set of  $\text{POI}(n)$  contains at least  $n$  elements, a fact that follows from another theorem characterizing these generating sets according to the domain and ranges of the elements of the generating sets. Also, we make a conjecture about the number of generating sets for  $\text{POI}(n)$ .

## 26. A partial characterization of the symmetric spaces of $U(2)$

**Johnathan Marquardt** University of St. Francis

**Advisor(s):** Angela Antonou, University of St. Francis

This work presents a characterization of specific symmetric spaces of the 2nd degree unitary group,  $U(2)$ , as well as a characterization of all matrices in  $U(2)$ . Moreover, the generalized and extended symmetric spaces of  $U(2)$  given by inner automorphisms are characterized. Structure and conditions for unitary matrices to be in these symmetric spaces are established through a parameterization of  $U(2)$ . This work is contributes to the ongoing process of characterizing these symmetric spaces.

## 27. Centralizer-like Subgroups Associated with the n-Engel Word Inside of Direct Products

**John McClellan III** University of Wisconsin-Milwaukee

**Advisor(s):** Dandrielle Lewis, University of Wisconsin-Eau Claire

We introduce what centralizer-like subgroups associated with the  $n$ -Engel word are and we examine what they look like inside of direct products.

## 28. The Embeddability of Partial Latin Squares in the Cayley Table of Dihedral Groups

**Grace McMonagle** Grand Valley State University

**Advisor(s):** Lauren Keough, Grand Valley State University

Latin squares are  $n \times n$  square matrices with  $n$  symbols, where each symbol appears exactly once in each row and column. Partial latin squares (PLS) can have missing symbols, and we are examining if a copy of these structures exist



in the Cayley table of a dihedral group. We refer to the number of filled entries in the PLS as the size. Our goal was to determine the maximum size of PLS such that each PLS of that size could be embedded in a given dihedral group.

## 29. Self-Contained Zelmanov Algebras

**Alma Pineda** Fullerton College

**Fabbiha Khan** Fullerton College

**Advisor(s):** Dana Clahane, Fullerton College

We will present a new rigorous formulation of a still-open question due to E. Zelmanov, presenting it for the first time in a way that is self-contained and accessible to anyone with a background no higher than intermediate algebra. The question is as follows: If  $R$  is an associative algebra with a non-zero derivation  $d$  such that  $d$  composed with itself is local, then is  $R$  necessarily abelian? We will present examples for each required definition, along the way, in our attempt to search for an answer to this question.

## 30. Computing maximal genetic distance in terms of signed permutations.

**Tanner Rosenberg** Northern Arizona University

**Advisor(s):** Dana Ernst, Northern Arizona University

One can model a configuration of genes as a permutation of the numbers 1 through  $n$ , where each number can be right-side-up or upside-down. In this model, one type of mutation corresponds to performing  $180^\circ$  reversals of consecutive subsequences of the permutation. The genetic distance between two configurations of genes is the minimum number of reversals needed to convert one permutation to the other. While there exist algorithms for computing genetic distance between two given permutations, our goal is to determine the maximum genetic distance between any permutation of 1 through  $n$  and the identity permutation. This maximum determines an upper bound for the evolutionary distance between any two gene sequences of the same length. In this presentation, we will discuss our current progress on the gene sorting problem.

## 31. Generation of 2-Generator Groups by Alternation

**James Shade** California State University, Fullerton

**Christian Do** California State University, Fullerton

**Advisor(s):** Adam Glessner, California State University, Fullerton

A variation on the 1969 Lovász Conjecture states that all finite connected Cayley graphs contain a Hamiltonian cycle. A consistent and algorithmically simple method of traversing an entire group from a small subset of its elements would improve running time and storage requirements for computer programs that work with groups. We examine the case of generating 2-generator groups by repeatedly multiplying two group elements in alternating order, specifically metacyclic 2-groups.

## 32. The Mathematics of Fair Isle Knitting

**Alison Sifuentes** Carthage College

**Advisor(s):** Mark Snively, Carthage College

The knitting style of Fair Isle knitting can be traced back for generations to a small island located just between the Atlantic Ocean and the North Sea called Fair Isle. Some of the characteristic traits of fair isle knitting include its use of patterns containing no more than two colors in a single row, where each row is stacked upon the other to form mirrored images of geometric shapes. These shapes are also characterized by small repeats in the pattern's structure. Since fair isle knitting is a style containing simple color changes and forming geometric shapes by means of these color changes, it is possible to consider each row of a pattern and to examine stitch by stitch the color changes occurring to construct the completed geometric shapes involved in the pattern. This examination raises an interesting question. If each row is an important component in constructing a pattern and contains stitches of only two different colors, is it possible to discover the total possible number of row patterns, and to further build upon this idea to find the total possible number of patterns created through the fair isle knitting style? This study will investigate knitting patterns within knitting patterns, beginning with an examination of the number of possible patterns within singular rows of various stitch length.

### 33. Factorizations of $k$ -nonnegative matrices

**Ewin Tang** University of Texas at Austin

**Neeraja Kulkarni** Carleton College

**Joe Suk** Stony Brook University

**Advisor(s):** Pavlo Pylyavskyy, University of Minnesota Twin Cities

A matrix is *totally nonnegative* if all of its minors are nonnegative. Totally nonnegative matrices have long attracted attention because of their applications in combinatorics, dynamics and probability, as well as their interesting topological structure. In particular, the semigroup of invertible totally nonnegative matrices can be partitioned, based on their factorizations into Chevalley generators and diagonal matrices, into cells that form a regular CW-complex. The closure poset of this CW-complex is described by the Bruhat order, as seen by considering subwords of factorizations. Our work considers  $k$ -nonnegative matrices, in which all minors of order at most  $k$  are nonnegative. We give a minimal set of generators in two special cases:  $(n - 1)$ -nonnegative invertible matrices, and  $(n - 2)$ -nonnegative triangular matrices with 1s on the diagonal. We describe how these semigroups can also be partitioned into cells, to which we extend the Bruhat order by describing the subwords that arise naturally from our new generators. As a result, the topological closure of the cells of  $k$ -nonnegative matrices induces an easily describable, analogous poset, and in the case of  $n - 2$ -nonnegative unitriangular matrices, a CW complex.

### 34. Stability Switches of a Square Matrix

**Gabrielle Tauscheck** College of William and Mary

**Advisor(s):** Junping Shi, College of William and Mary

In 1952, Alan Turing created a partial differential equation model for the chemical mechanism of biological morphogenesis formation. In his analysis, certain stable biological systems can become unstable with the addition of diffusion of chemicals. A linear system of differential equations  $u' = Au$  is stable if any solution  $u$  converges to zero as  $t \rightarrow \infty$ . From the theory of linear differential equations, this is equivalent to that each eigenvalue of  $A$  has negative real part. Turing's question is equivalent to: given two matrices  $A$  and  $P$ , how does the stability of  $A - tP$  change when the parameter  $t$  varies? In this work, we give a complete answer for  $2 \times 2$  matrices  $A$  and  $P$ . We show that there are at most three stability switches; we classify all possible sign patterns of  $A$ ; and we also identify the eigenvector sign pattern for the unstable eigenvalue.

### 35. Applying Discriminant Chambers to Structured Polynomials

**Diane Tchuindjo** University of Maryland, College Park

**Amy Adair** Louisiana State University

**Advisor(s):** Robert Walker, University of Michigan, Ann Arbor

One way to prove the algorithmic hardness of a function is to consider its circuit complexity: the minimal size of a circuit computing the function. Recent work in circuit complexity has revealed that it is enough to restrict to depth-4 circuits, inspiring the study of certain structured polynomials called sum-of-products-of-sparse (SPS) polynomials. In particular, showing sufficiently good upper bounds for the number of real roots of SPS polynomials would imply the hardness of the permanent. If  $f$  and  $g$  are univariate polynomials with real coefficients and  $t$  terms, then does  $fg-1$  have a number of real roots that grows at most linearly in  $t$ ? We consider the first non-trivial approach towards such a bound via  $A$ -discriminants, leading to interesting questions involving sparse polynomial systems.

### 36. On the realizability of the critical points of a realizable list

**Amber Thrall** University of Washington Bothell

**Sarah Hoover** Hamilton

**Daniel McCormick** Florida Institute of Technology

**Advisor(s):** Pietro Paparella, University of Washington Bothell

The nonnegative inverse eigenvalue problem (NIEP) is to characterize the spectra of entrywise nonnegative matrices. A finite multiset of complex numbers is called realizable if it is the spectrum of an entrywise nonnegative matrix. Monov conjectured that the  $(k)$ th moments of the list of critical points of a realizable list are nonnegative. Johnson further conjectured that the list of critical points must be realizable. In our work, Johnson's conjecture, and consequently Monov's conjecture, are established for a variety of important cases including Ciarlet spectra, Suleĭmanova spectra, and spectra realizable via complex Hadamard similarities. Additionally we have proven a result on differentiators and

trace vectors, and make use of it to provide an alternate proof to a theorem of Malamud and a broad generalization of a theorem in Kushel and Tyaglov's work on circulants. We then used these results to provide a new proof of a classical result on the relationship between a polynomial and its critical points. Several ramifications of the validity of Johnson's conjecture are discussed — in particular, a majorization result due to Schmeisser that relates the critical points of a polynomial to its zeros.

### 37. Generalizations of Collatz Functions to Geometric Algebras

**Jason Turner** Union College

**Advisor(s):** Alejandra Alvarado, Eastern Illinois University

The original Collatz mapping, attributed to Lothar Collatz of the University of Hamburg, maps odd positive integers  $x$  to  $\frac{3x+1}{2}$  and even positive integers  $x$  to  $\frac{x}{2}$ . The Collatz Conjecture states that, regardless of the initial value, every iteration trajectory of the Collatz mapping will eventually yield 1. Despite its concise statement, the conjecture has yet to be solved in over 75 years. In the summer of 2016, Steffen Kionke of Heinrich-Heine-Universität Düsseldorf defined generalized Collatz mappings on free abelian groups of finite rank and used geometric arguments to find cones of points with divergent trajectories. We further generalized his functions to mappings on geometric algebras of finite-dimensional real vector spaces, and investigated examples in  $C\ell_1$ .

### 38. The Moduli Space of 3|2-dimensional complex associative algebras

**Jory Wagner** University of Wisconsin- Eau Claire

**Tyler Gonzales** University of Wisconsin-Eau Claire

**Advisor(s):** Michael Penkava, University of Wisconsin-Eau Claire

We have recently constructed the moduli space of all non nilpotent 3|2-dimensional complex associative algebras, that is the  $\mathbb{Z}_2$ -graded algebras with 3 even and 2 odd generators. In this poster we explain the construction, which uses the Fundamental Theorem of Finite Dimensional Algebras to generate these algebras as extensions of semisimple algebras by nilpotent algebras. There are some differences in the  $\mathbb{Z}_2$  graded case; for example the notion of a division algebra needs to be modified.

### 39. An extension of cleaning for the Dessin d'Enfant

**Austin Wei** Pomona College

**Gabrielle Melamed** University of Hawaii at Manoa

**Advisor(s):** Naomi Cameron, Lewis and Clark College

The Dessin d'Enfant (hereafter dessin) is in correspondence with permutation triples  $(\sigma_0, \sigma_1, \sigma_\infty) \in S_n^3$  where  $\sigma_0\sigma_1\sigma_\infty = \text{Id}$  and the subgroup  $\langle \sigma_0, \sigma_1 \rangle \leq S_n$  is transitive: this group is called the monodromy group (hereafter MG). We define the composition of two dessins, by inserting a copy of the first dessin in place of each edge in the second dessin. The cleaning of a dessin is a canonical example of this composition process in which all original white vertices are colored black and a white vertex is inserted for each edge so that each new white vertex has degree two. The MG of the cleaned dessin embeds nicely into the group  $G \wr C_2$  where  $G$  is the MG of the pre-cleaned dessin. We define and study a generalization of cleaning, called  $k$ -cleaning, which similarly induces a nice embedding into the group  $G \wr C_k$ . In particular we show that given a dessin with monodromy group  $G \geq A_n$  the MG of the corresponding  $k$ -cleaned dessin must contain  $A_n \wr C_k$  for  $k > 3$ , and  $k > 2$  provided that  $|\sigma_0| \neq |\sigma_1|$ . This result is used to compute the MG for the two families of trees of diameter six with passport size 1.

### 40. Representations of Lie Algebras in Graph Theory

**Junho Won** Columbia University

**Jia Wan** Columbia University

**Yi Wang** Columbia University

**Alejandro Buendia** Columbia University

**Advisor(s):** Daniel Litt, Columbia University, Department of Mathematics

Representations of Lie algebras have found many applications in combinatorics following work by Richard Stanley. Inspired by the study of two-colored diagonal Ramsey numbers, our first project constructs linear operators acting on graphs by recoloring complete monochromatic subgraphs. We try to determine the Lie algebras generated by

these operators and their representations, finding  $(sl_n)$ -actions for special cases. We present computational evidence in small dimensions for semisimplicity and stabilization of the Lie algebras that arise, study the space of graphs as their representations, and outline further directions to pursue. In our second, closely related project, we use edge coloring of complete graphs to construct a linear representation of  $sl_n$ , and use an explicit description of this representation to study the number of isomorphism classes of complete multicolored graphs without vertex labelling. Counting unlabelled graphs is generally a difficult problem, and we find that representation theory can give a lot of information about these numbers. We relate the number of colored edges in a graph to certain coefficients of the “roots” of the Lie algebra. This approach allows us to deduce nice combinatorial relations.

#### 41. The Deformations of 5-dimensional Complex Associative Algebras

**Haotian Wu** UW Eau Claire

**Grant Keane** UW Eau Claire

**Advisor(s):** Michael Penkava, UW Eau Claire

We study the deformations of all 5-dimensional complex non-nilpotent associative algebras. We have recently constructed all of the algebras in the moduli space, and now we study isomorphisms that occur after a small perturbation to the multiplication rules of the algebras. In this poster, we show how we compute the deformations of algebras such that the associativity condition is conserved. Furthermore, we will show how we can determine whether or not an algebra is able to deform by studying the cohomology in degree 2. Finally, we show examples of specific algebras and the various ways in which the algebras deform.

#### 42. The Adjacency Operator on Graphs with Infinite Tails

**Astrid Berge** University of Washington

**Aahan Agrawal**

**Advisor(s):** Ruben Martinez-Avendano, Autonomous University of the State of Hidalgo

Given a countably infinite, locally finite, and connected graph  $G$ , one can define functions that assign vertices to members of a field. One can then study the action of operators that transform these functions in the function space  $L^p(G)$ . The operator we focus on is the adjacency operator  $S : L^p G \rightarrow L^p G$ , defined by the adjacency matrix of  $G$ . The last three years have seen progress on studying the spectra of the adjacency operator on graphs that adjoin finite graphs with the one-sided infinite path. The adjacency matrices of these graphs are finite-rank perturbations of the discrete Laplacian. Previous work on finding the spectra of these graphs has employed non-trivial methods involving a canonical form of the adjacency matrix. We offer a different approach using linear recurrence relations and matrix algebra, giving an associated eigenvector for each element of the point spectra. This is joint work with Dr. Ruben Martinez-Avenidaño, Elyssa Sliheet and Seth Colbert-Pollack.

#### 43. Spectrum of $\square_b^t$ on $\mathbb{S}^3$

**Madelyne Brown** Bucknell University

**Tawfik Abbas** Michigan State University

**Advisor(s):** Yunus Zeytuncu, University of Michigan-Dearborn

When is a CR-manifold CR-embeddable into  $\mathbb{C}^n$ ? The Whitney embedding theorem states that any manifold is embeddable into Euclidean space at most twice its dimension, but this may not preserve the CR-structure. The question was partially answered by Boutet de Monvel in 1975, who stated that if the CR-manifold has real dimension at least 5, then it can always be CR-embedded into  $\mathbb{C}^N$  for some  $N$ . It turns out that if a CR-manifold has real dimension 3, this fails. Rossi showed in 1965 that the CR-manifold  $(\mathbb{S}^3, \mathcal{L}_t)$  is not CR-embeddable, where  $\mathcal{L}_t$  is a slight perturbation of the normal CR-structure on  $\mathbb{S}^3$ . Later in 1986, Kohn showed that CR-embeddability is equivalent to showing that the tangential Cauchy-Riemann operator  $\bar{\partial}_{b,t}$  does not have closed range. We tackle the same problem of embeddability, but from the perspective of spectral analysis. In particular, we show that if the Kohn Laplacian  $\square_b^t$  contains 0 in its essential spectrum, then  $(\mathbb{S}^3, \mathcal{L}_t)$  is not CR-embeddable. To get the spectrum of  $\square_b^t$ , we utilized spherical harmonic spaces such that the matrix representation of  $\square_b^t$  is tridiagonal on these finite-dimensional subspaces. We then exploited this fact to bound the smallest eigenvalue of  $\square_b^t$ , and find a sequence of eigenvalues that converge to 0.

#### 44. Zeros of polynomials generated by quadratic-factor denominator

**Abigayle Dirdak** California State University

**Daniel Hooker** CSU Fresno

**Advisor(s):** Khang Tran, California State University, Fresno

This project is a part of the research on the zeros of a sequence of polynomials  $\{H_m(z)\}_{m=0}^{\infty}$  satisfying a recurrence of order  $n$

$$\sum_{k=0}^n A_k(z)H_{m-k}(z) = 0,$$

where  $A_k(z)$ ,  $1 \leq k \leq n$ , are complex polynomials and  $A_0(z) = 1$ . We focus on the special generating function

$$\sum_{m=0}^{\infty} H_m(z)t^m = \frac{1}{(1 + b_1 B(z)t + a_1 A(z)t^2)(1 + b_2 B(z)t + a_2 A(z)t^2)},$$

where  $a_1, b_1, a_2, b_2 \in \mathbb{R}$  and  $A(z), B(z) \in \mathbb{C}[z]$ . If  $a_1 = a_2 \neq 0$  then the zeros of  $H_m(z)$  lie on the curve  $\mathcal{C}$  whose equation is given by

$$\operatorname{Im} \frac{B^2(z)}{A(z)} = 0 \quad \text{and} \quad a_1 \operatorname{Re} \frac{B^2(z)}{A(z)} \geq 0.$$

We also study other quadruples of real numbers  $(a_1, b_1, a_2, b_2)$  so that the zeros of  $H_m(z)$  lie on  $\mathcal{C}$ .

#### 45. $k$ th-Order Fibonacci-like Polynomials

**Alexandra Embry** Indiana University

**Katherine Arneson** St. Olaf College

**Advisor(s):** Aklilu Zeleke, Michigan State University

The well-studied Fibonacci polynomials are described by  $F_n(x) = xF_{n-1}(x) + F_{n-2}(x)$  with  $F_0 = 1$  and  $F_1 = x$ . We extend this recursive polynomial sequence to  $G_n^{(k)}(x) = xG_{n-1}^{(k)}(x) + G_{n-k}^{(k)}(x)$  and further to  $H_n^{(k)}(x) = xH_{n-1}^{(k)}(x) - H_{n-k}^{(k)}(x)$  with initial conditions  $G_0^{(k)} = G_1^{(k)} = \dots = G_{k-1}^{(k)} = 1$  and  $H_0^{(k)} = H_1^{(k)} = \dots = H_{k-1}^{(k)} = 1$ . When  $k = 2$ , the  $G_n^{(k)}$  polynomials are the Fibonacci polynomials with altered initial conditions. When  $k = 3$  and  $x = 1$ , the  $G_n^{(k)}$  polynomials describe Narayana's Cow Sequence. In this talk, we present a closed form for these polynomials from which we derive a class of significant integer sequences. For example, we find a sequence that describes a population that takes 5 months to mature and reproduces every 3 months, which has been submitted to the On-Line Encyclopedia of Integer Sequences. We also study some interesting analytic properties of these polynomials, especially concerning the behavior of their roots. We establish that the minimal real roots of  $G_n^{(k)}(x)$  converge uniformly to some number  $-3 < r^{(k)} < -1$ , and the maximal real roots of  $H_n^{(k)}(x)$  converge uniformly to 2. MATLAB simulations show that the roots of  $G_n^{(3)}$  are rational for only  $n = 3, 5, 6, 10$ , and 19, and those of  $H_n^{(3)}$  are rational for only  $n = 3$  and 12. Computer-assisted curve fitting using MATLAB suggests that the relative rates of convergence of the minimal real roots of  $G_n^{(k)}$  and the maximal real roots of  $H_n^{(k)}$  are of exponential order.

#### 46. The Necessary and Sufficient Conditions to Form a Hadamard Submatrix From a Fourier Matrix

**Robert Glickfield** Butler University

**Advisor(s):** John Herr, Butler University

A matrix is considered Hadamard if all of its entries are of unit modulus and the columns are mutually orthogonal. Fourier matrices, a subclass of the Hadamard matrices, contain entries of the form  $e^{\frac{2\pi i}{N}(j-1)(k-1)}$ . In this talk, we formulate a conjecture for the necessary and sufficient conditions for the existence of a Hadamard submatrix of a Fourier matrix for a particular dimensions. We prove the nonexistence of a Hadamard submatrix for many cases.

## 47. Simple Continued Fractions and Periodicity

**Vanessa Gomez** University of California, Los Angeles

**Jacob McCann** Lehigh University

**Advisor(s):** Shannon Talbott, Moravian College

Weinstraub and Anselm explored a generalization of simple continued fractions, in which they replace all numerators with a constant  $N$ . We further explore this type of continued fraction, beginning with the generalization of eventually periodic simple continued fractions, in which we replace all numerators with a constant  $N$  and evaluate the limit of such a continued fraction as  $N$  approaches infinity. We then look at simple continued fractions of generalized golden ratios divided by an integer and prove results about when such expansions are one or two-periodic and what the exact form of said expansions will be. Next, we discuss the particular difficulties that arise when evaluating the case of bounded aperiodic partial quotient sequences.

## 48. On the Roots of Lower Binomials

**Harold Jimenez Polo** University of California, Berkeley

**Carlos Osco Huaricapcha** San Francisco State University

**Esteban Madrigal** Harvard University

**Advisor(s):** Federico Ardila, San Francisco State University

Given a univariate trinomial  $f \in R[x]$ , we analyze the relation between the roots of  $f$  and the roots of the corresponding lower binomials. The roots of the lower binomials, conjecturally, provide high quality approximations of the roots of  $f$ . We implement Smale's  $\alpha$ -criterion to analyze whether our approximations converge quickly under Newton's method. We know that under certain conditions every root of a lower binomial is an approximate root of a trinomial, and we expect to determine when at least one root of a lower binomial is an approximate root in the sense of Smale. Moreover, we prove that  $f$  has the same number of positive roots as its lower binomials with high probability.

## 49. A Comprehensive Approach to the Toeplitz' Conjecture

**Kimberly Lopez-Zepeda** Fullerton College

**Advisor(s):** Dana Clahane, Fullerton College

The Toeplitz' conjecture states that every Jordan (in other words, simple and closed) path in the plane contains all four corners of some square. This question has been affirmatively answered for the cases when a simple closed path is convex or piece-wise smooth. We will give a rigorous development of the definition of a Jordan path that can be completely understood by a person with no higher mathematical background than intermediate algebra. We'll then describe attempts to answer the problem for randomly drawn Jordan paths and give more self-contained proofs of the conjecture in special cases. These special cases are bound, but not limited to, simple closed loops.

## 50. Computing measurable cross-sections with Mathematica

**Sophia Maniscalco** Bridgewater State University

**Advisor(s):** Vignon Oussa, Bridgewater State University

Let  $G$  be a matrix group acting linearly on a finite-dimensional vector space. We say the action of  $G$  is regular if there exists a measurable set meeting every orbit of the group at exactly one point. Such a set is called a measurable cross-section for the orbits of  $G$ . The present project is motivated by the existence of continuous wavelets arising from group actions in which the explicit construction of measurable cross-sections plays a central role. Under the assumption that  $G$  is commutative, we present a complete library of algorithms which is exploited to compute measurable cross-sections systematically.

## 51. Vanishing Dissipation Limits for the MHD- $\alpha$ Equation

**Danielle Pham** Creighton University

**Advisor(s):** Nathan Pennington, Creighton University

The Magnetohydrodynamic (MHD) system of equations governs kinematic fluids that are subjected to a magnetic field. The equation is a combination of the Navier-Stokes equation and Maxwell's equations. Due to the difficulty in solving the MHD system, it has become common to study approximating versions of the equation, including the MHD- $\alpha$  system, which regularizes the velocity field in exchange for the addition of non-linear terms. Both the kinematic and

magnetic parts of the MHD- $\alpha$  system have diffusive terms which dissipate the initial energy of the system. Setting those terms equal to zero returns the Ideal MHD- $\alpha$  system. The goal of this project is to show that solutions to the MHD- $\alpha$  system with diffusion will converge to the ideal MHD- $\alpha$  system as the diffusion parameters are sent to zero by adapting known results for the analogous problem of determining when solutions to the Navier-Stokes equation will converge to a solution of the Euler Equation for fluids in a disk and in a pipe.

## 52. Boundary Value Problems and Green's Functions on Magnetic Graphs

**Sawyer Robertson** University of Oklahoma

**Advisor(s):** Javier Alejandro Chavez-Dominguez, University of Oklahoma

Let  $G$  be a finite simple graph. We impose on  $G$  the additional structure of a signature, a function which maps edges into the set of complex numbers of modulus 1. This induces a second-order difference operator for complex-valued functions defined on the vertex set of  $G$  which is a discrete analogue of the classical Laplacian, and consequently discrete boundary value problems on proper and sufficiently connected subgraphs of  $G$ . We construct a solution to Poisson type problems and explore some applications, including the role of discrete Green's functions in constructions of solutions. These structures and problems arise in many physical models where discrete domains (namely, graphs) can more efficiently describe continuous regions; in particular, those of quantum mechanics, where a signature structure helps to describe atomic structures with the presence of magnetic potential.

## 53. Fractals And Iterated Function Systems

**Marko Saric** Benedictine University

**Advisor(s):** Manmohan Kaur, Benedictine University

Fractals are self-similar structures which can be defined by an iterative process. In this paper, we use a computer program and iterated function systems (IFSs) to study the role of matrix norms and its effect on the type of fractal generated. In particular, we propose a connection between the spectral norm of the matrices in the IFS and the type of attractor that is graphed.

## 54. Shift Operators on Directed Infinite Graphs

**Elyssa Sliheet** Southwestern University

**Advisor(s):** Ruben Martinez-Avendao, Universidad Autnoma Del Estado De Hidalgo

Let  $G$  be a directed graph which has an infinite number of vertices and for which each vertex belongs to only finitely many edges. By  $L^p(G)$  we refer to the Banach space of complex-valued functions defined on the vertices of  $G$  for which the sequence of function values is in  $\ell^p$ ,  $1 \leq p \leq \infty$ . We study linear operators on  $L^p(G)$  which we call shift operators. For a directed graph, the "backward" shift is represented by the graph's adjacency matrix, and the "forward" shift is represented by the transpose of the adjacency matrix. We give necessary and sufficient conditions for boundedness of these operators and present results regarding some other properties. We also prove the operators are bounded above by the maximum degree of the graph, show when this upper bound is achieved, and give a lower bound for their norms. We prove that for directed graphs, the "forward shift" operator is the adjoint of the "backward shift.". Finally, we describe the kernels of the operators and related properties regarding the range as well as give conditions for when the orbit of a function under the operators is dense in the function space, a property known as hypercyclicity.

## 55. Boundary Value Problems via Finite Difference Approximations

**Sara Stout** Lewis and Clark College

**Advisor(s):** Paul T. Allen, Lewis and Clark College

The standard approach to proving existence and uniqueness of solutions to boundary value problems requires graduate level theory from functional analysis. We present an elementary proof of existence and uniqueness of solutions to a simple class of boundary value problems using only tools from linear algebra and introductory analysis courses.

## 56. Linear Fractional Transformations on the Bidisc

**Sarah Strikwerda** Calvin College

**Amanda Cowell** University of Michigan-Dearborn

**Timothy Hollman** University of Michigan-Dearborn

**Advisor(s):** John Clifford, University of Michigan-Dearborn

We address linear fractional transformations on  $\mathbb{C}^2$  as they act on the bidisc. Let  $\mathbb{D}$  denote the open unit disc in the complex plane  $\mathbb{C}$  and  $\mathbb{D}^2$  the bidisc in  $\mathbb{C}^2$ . A linear fractional transformation (LFT) is a holomorphic function from  $\mathbb{C}^2$  to  $\mathbb{C}^2$  defined by

$$\varphi(z) = \frac{Az + B}{C^*z + d}$$

where  $A$  is a 2 by 2 complex matrix,  $B$  and  $C$  are vectors in  $\mathbb{C}^2$ , and  $d \in \mathbb{C}$ . Linear fractional transformations that are self-maps of the open unit ball in  $\mathbb{C}^n$  have been recently studied by Cowen and MacCluer. Through our investigation, we prove a necessary condition for an LFT to be a self-map of the bidisc. We also show forms of LFT's where the image of the bidisc is a disc cross a disc, that is  $\varphi(\mathbb{D}^2) = r_1\mathbb{D}(z_0) \times r_2\mathbb{D}(w_0)$ . In further exploration, we study the amount of fixed points LFTs have and determine a method for finding these fixed points. Finally, we use these ideas to construct LFT's with prescribed fixed points.

## 57. Morita Equivalence of Groupoids from Directed Graphs

**Vincent Villalobos** University of Texas at Tyler

**Advisor(s):** Scott LaLonde, University of Texas at Tyler

Groupoids are algebraic objects that generalize groups. This research studied a specific type of groupoid, called a *graph groupoid*. Graph groupoids arise from *directed graphs*. In particular, we considered moves on the graph level and investigated whether the resulting or starting groupoid was *Morita equivalent* to the other graph groupoid.

## 58. A Blast from the Past: Finding Chaotic Solutions to Differential Delay Equations

**Erik Wendt** Gettysburg College

**Advisor(s):** Benjamin Kennedy, Gettysburg College

We discuss the basic properties of equations of the form  $x'(t) = f(x(t - d(x_t)))$ , which are known as state-dependent differential delay equations. After reviewing the properties of chaos, we present some new results on finding chaotic solutions to such equations.

## 59. Modeling the Street Canyon Effect: A Study of Air Dispersion Models

**Zariluz Alvarado** University of California, Santa Barbara

**Advisor(s):** Chun Yin Dy, Hong Kong University of Science and Technology

The Hong Kong Environmental Protection Department currently uses CALINE4, a Gaussian plume air dispersion model developed by Caltrans, to predict air pollutant concentrations resulting from vehicle emissions. However, CALINE4 does not adequately predict air pollutant concentrations within street canyons, which are narrow urban roads lined with tall buildings on each side, a common occurrence in cities like Hong Kong. In a phenomenon known as the street canyon effect, roof level winds create a vortex in the street canyon that traps pollutants at the ground level. CALINE4 is only capable of modeling air pollutant concentrations within street canyons with roof level winds parallel to the street ( $\phi = 0$ ). In such a scenario, there is no street canyon effect. Thus, the goal of this research is to expand the capabilities of CALINE4 to model air pollutant, namely CO, concentrations throughout a street canyon where the wind angle relative to the street orientation is greater than thirty degrees ( $\phi \geq 30$ ). To accomplish this task, we incorporate the existing air dispersion models STREET and OSPM (Operational Street Pollution Model) into CALINE4. We use CALINE4 to calculate the total concentration of a pollutant within the cross-section of a street canyon and redistribute this quantity using a hybrid of OSPM, a plume and box model hybrid, and STREET, a box model. The results from this new model show a decrease in the total concentration of a pollutant within a street canyon for strong and near parallel winds, as expected. The accuracy of the new model has been evaluated against experimental data from the CODASC database.



## **60. Modeling the effects of crayfish invasion and drought on hypothetical crayfish population dynamics**

**Leah Bayer** Youngstown State University

**Advisor(s):** Daniel Magoulick, University of Arkansas

North American crayfish species face several environmental and ecological threats including natural ranges, invasive species, and intensified drought. Our objectives were to model the population dynamics of potential crayfish species with theoretical life histories and assess how these populations could be affected by the impacts of invasive species and drought. We used RAMAS-Metapop to construct stage-based demographic models using data obtained from various literature sources. We assessed population viability under various disturbance scenarios using estimates of terminal extinction risk, median time to quasi-extinction, and metapopulation occupancy. Models indicated that both r- and K-selected species appear to be highly susceptible to decline when under additive effects of reduced carrying capacity due to invasion and reduced vital rates due to drought. Better estimates of stage-specific survival and fecundity could reinforce our findings and more accurately predict species outcomes. By constructing models that explore a wide variety of life histories and disturbance scenarios, we hope to provide managers with tools to develop broadly applicable conservation strategies.

## **61. Combining Hydropower System Designs and Fish Life History Traits to Predict the Risk of Population-level Impacts as a Result of Turbine Passage Mortality**

**Zachary Becker** East Tennessee State University

**Advisor(s):** Ariel Cintron-arias, East Tennessee State University

The impact of hydropower facilities on fish populations is a common environmental concern. Past experimentation has shown that contact with these facilities can harm individual fish. Also, a well-documented link exists between certain species-specific life history traits and increased mortality from hydropower. However, the extent to which hydropower affects the trajectory of fish populations is still unknown. To address this, we developed a modeling framework that is based upon a combination of certain life history traits, specifically: fecundity, longevity, and age at first maturity. This framework relies on matrix models for age and stage structures. In the latter, the population is divided into four life stages: age-0; juveniles; young adults; mature adults. We applied the proposed modeling framework to five different fish species to cover a variety of life history traits. This illustrates the flexibility of the framework. Numerical explorations were completed to investigate which model parameters that the total adult population size would be most sensitive to. Latin hyper-cube sampling methods were implemented to this effect. Initial results show that the youngest stages are the most significant to the model.

## **62. Quantifying the Distribution of Urban Vegetation based on Google Street View Images**

**Jacob Beihoff** University of Wisconsin - Milwaukee

**Adam Honts** University of Wisconsin-Milwaukee

**Advisor(s):** Istvan Lauko, University of Wisconsin-Milwaukee

Measuring the amount of vegetation in a given area has long been accomplished using satellite and aerial imaging systems. However, satellite imagery is not very useful quantifying urban vegetation perceived at the street level. We measure the amount of vegetation beneath the canopy cover along street networks utilizing Google Street View (GSV) images, made accessible by the Google Street View Image API. We developed a multi-filter image processing technique to extract green vegetation pixels from GSV images, and computed a green index associated with each image location. Analyzing green vegetation through the use of GSV images provides a comprehensive representation of the amount of green vegetation found within an area, and facilitates an analysis rarely performed on a large scale at the street-level. We use this high-resolution GIS data to map the green index of Milwaukee County in Wisconsin. The results of statistical comparisons between urban greenery and known health and socioeconomic (e.g., mean income) outcomes will be presented.

### 63. New Stacked central configuration for planar 6-body problem.

**Gokul Bhusal** The University of Southern Mississippi

**Hamas Tahir** The University of Southern Mississippi

**Advisor(s):** Zhifu Xie, The University of Southern Mississippi

Newtonian  $N$ -body problem is one of the oldest problems of celestial mechanics and remains a challenge for many mathematicians and physicists. The problem has its own beauty in it as it leads to the study of the motion of celestial bodies such as earth, moon, sun and other planets. A Central Configuration (CC) is a special arrangement of masses in the  $N$ -body problem where the gravitational force on each body points toward the center of mass. A stacked CC is a CC that has a proper subset of the  $N$ -bodies also forming a CC. The stacked CC was first studied in the planar 5-body problem by Hampton in 2005. In our research, we construct a planar 6-body configuration with three bodies on the vertices of a fixed equilateral triangle and other three bodies on the vertices of another equilateral triangle. The later triangle can vary its size and move along an axis of symmetry of the fixed triangle. Such configuration can be parametrized by two variables. Many special cases were studied to determine the possibility of CC and to identify the region where it is possible to choose positive masses that will make the configuration central. The main results of our project are the existence of stacked CC which consists of two types of configurations. In the new type of stacked CC, two equilateral triangles do not have a common centroid. Mathematical software like Maple and MATLAB are used for numerical computation and analysis of the problem.

### 64. Mathematical Modeling of the Dynamic Process of Curiosity

**Claire Bodemann** Eckerd College

**Advisor(s):** David Kerr, Eckerd College

In this work, we initiate a dynamic model of curiosity process. This idea of dynamic modeling is based on the work of Berlyne. Curiosity process is composed of four sub-systems: novelty, uncertainty, conflict, and complexity. A large-scale dynamic model of curiosity process is described by large-scale systems of differential equations. A very simplified version of this dynamic model is illustrated.

### 65. Symmetric Rendezvous on the Unit Cube

**Jeffrey Braun** Johns Hopkins University

**Advisor(s):** John Wierman, Johns Hopkins University

A famous open problem in the field of Search Games and Rendezvous is that of the Astronaut Problem. In an effort to develop new strategies and tools to make progress on the Astronaut Problem, we postulated a new, similar scenario that is interesting in its own right. Given two indistinguishable agents randomly placed on the surface of a cube who move at unit speed, their location on the cube defines their vision in the following manner: If they are on a face they can see the entirety of the face (including the edges and vertices that comprise it) and if they are on an edge or a vertex, it is as if they are on the two or three faces simultaneously that join together to make the edge or vertex. Our goal is to either directly derive asymmetric and symmetric rendezvous values associated with this search space or provide good bounds for them. We have defined a strategy space which dominates among a large subset of all possible strategies. Within this strategy space, we closely examine two special cases, which provides us with excellent upper bounds for both rendezvous values. We also derived a common lower bound for both rendezvous values from some ad-hoc reasoning. In the process we also make some remarks on Search Games and Rendezvous problems in general that may be useful for further exploration of other problems.

### 66. An Application of Game Theory and Extreme Value Theory to Predict Baseball Game Outcomes

**Matthew Breen** University of Arkansas

**Advisor(s):** Chaim Goodman-Strauss, University of Arkansas

The game of baseball is made of a finite number of states. These at-bats are a zero-sum game that reward one team with an out or another with a base. We can use this property to create a model that can generate a probabilistic view of the pace of the game and the winner. The model will take as inputs: both lineups for the game, pitching statistics (handedness, K, WAR, RA9, FIP, IP and gmLI), batting statistics (SLG, OBP, PA, HR, BB, IBB, SO and WAR) and general statistics about the game, such as weather, location and how long each team has been traveling. Using these

measurements, we can generate payoff matrices for each at-bat and use Monte Carlo simulations to generate the most likely outcome for the game. The individual matrices will be generated using the given statistics, a time series analysis to update the pitcher's probabilities as he becomes fatigued and an application of extreme value theory to account for rare events like perfect games. Perfect games occur with measurable frequency over the course of a season, but are difficult to predict for any given game. This model is designed to generate a probable outcome for a single game, but if we iterate this method, we could use it to predict the outcome of a single season.

## 67. Comparing Songs Using Matrix Pattern Preservation

**Erin Bugbee** Brown University

**Claire Savard** University of Michigan, Ann Arbor

**Advisor(s):** Katherine Kinnaird, Brown University

The cover song task in music information retrieval involves correctly matching songs which are versions of the same piece of music. Many attempts have been made to accomplish this, including structure-based approaches that examine the repeated structures in the song from a given signal. Existing methods using these repeated structures are rigid and computationally expensive. Inspired by topological data analysis, we consider the times and lengths of these repeated structures, and then apply two new mappings, namely the Surface Pattern Preservation and the Matrix Pattern Preservation, to transform the data into matrix space and simplify the comparison of two songs. We then calculate distances between the matrices to determine if the dissimilarity is small enough to declare the songs to be covers of the same piece. Current testing shows promising results, with comparable precision and higher recall values than previous methods also analyzing repeated structures in songs.

## 68. Magnetic Resonance Recovery from Single-Shot Time Dependent Data

**Alyssa Burgueno** Arizona State University

**Advisor(s):** Rodrigo Platte, Arizona State University

Magnetic resonance imaging (MRI) captures the alignment of water molecules subject to a magnetic field, which are then computed with an inverse Fourier transform. Traditional MRI, using the standard image recovery model, traverses Fourier space to recover images accurately at the cost of scan time. This project employs a new reconstruction model; though it yields computational inefficiencies, the new model supports fast data acquisition, recovers additional information, and is more suitable to mitigate patient motion.

## 69. Automatic Conflict Detection in Police Body Worn Video

**Jelena Trišović** University of Belgrade

**Collin Cademartori** Brown University

**Advisor(s):** Jason Xu, UCLA Department of Biomathematics

Police body-worn cameras play a significant role in providing records of the interactions between police officers and the public. However, these procedures generate a massive quantity of data when deployed across police departments, making manual review intractable. Our research focuses on automatic conflict detection in audio recordings produced by these cameras. Classical approaches for speech detection and recognition fail due to the significant level of noise, and standard metrics for assessing conflict such as conversational turn-taking and overlap provide a poor proxy for conflict scenarios in public-police interactions. Using signal processing and machine learning techniques, we develop a pipeline catered to this relatively unexplored task that combines noise removal, detection of speech segments, and new measures of conflict. These metrics are based on detecting the quantity and volume of repeated phrases in an officer's speech, which in the majority of cases correspond to scenarios of non-compliance. We demonstrate the efficacy of our approach by applying our conflict detection algorithm to real body-worn data.

## 70. Evolution of microRNA diversity and regulation: Statistical Modeling

**Kangji Chen** Roger Williams University

**Advisor(s):** Yajni Warnapala, Roger Williams University

MicroRNAs (miRNAs) are small molecules found in all plants and animals. Despite their relatively small size and limited number, they regulate many cellular processes in healthy and disease states. In the c. 600-million-year history of animal evolution, the number of miRNA-encoding genes has grown from 8 in sponges, to over 1,400 in humans.

Despite this overall trend, the number of miRNA genes among species is highly variable, and has no obvious relationship to the age of the species. In our project, we are developing statistical models to evaluate the strength of the association between miRNA number and the following species-specific parameters besides evolutionary age: organism mass, surface area, tissue complexity, genome size and genome complexity. In order to investigate these relationships, we will consider both parametric and non-parametric statistical techniques (implemented using JMP software).

## 71. Analytical and Numerical Studies for Polytropes and Isothermal Spheres using Emden-Fowler Equations

**Sierra Chimene** Rochester Institute of Technology

**Saeed Akbar** Rochester Institute of Technology

**Advisor(s):** Fazal Abbas, Rochester Institute of Technology

We model the polytropes and isothermal sphere dynamics through Emden-Fowler type equations. We consider various non-linearity terms, shape factor values and the initial values to study the model equations in detail. We provide the convergent series solutions using Homotopy Analysis Method (HAM) and Adomian Decomposition Method (ADM) for the model equations. We present that the choice of Auxiliary parameter  $h$  used in HAM technique plays an essential role in achieving and improving the interval of convergence. Furthermore, the validity of the analytical solution is shown through Numerical solutions.

## 72. Study of Vortex Dynamics with Free Surface in a Shallow Water Regime

**Phoebe Coy** University of California, Santa Barbara

**Thomas Retzliff** Marquette University

**Hannah Barta** Central Washington University

**Alec Todd** University of California, Davis

**Advisor(s):** Christopher Curtis, San Diego State University

We examine the deformation of varying shapes of collections of point vortices beneath a free surface of an inviscid fluid with a flat bottom boundary. Initial simulations investigate the deformation of vortex sheets in a manner which resembles the Kelvin-Helmholtz instability. With large numbers of point vortices, the simulations we run show that these sheets tend to deform into elliptical patches. Several other shapes of point vortex arrays are simulated, and almost all deform into an elliptical shape. Upon deciding to simulate an ellipse as a starting shape, we observe that an initial elliptical shape deforms the least with time. To increase the realism of these simulations, we introduce a mollified kernel, which slows the speeds of vortices that are close together and yields ellipses that retain their shape with time. We introduce a metric to measure the deformation of these ellipses before and after mollification.

## 73. The Dynamics of Predator-prey Models in A Lake Environment

**Simon Deng** University of Nebraska-Lincoln

**Advisor(s):** Yu Jin, University of Nebraska-Lincoln

In this paper, we explore the dynamics of predator-prey models in the lake environment. We separate the lake into two depth levels for prey to live in and assume predator species live above the lake. In these models, we assume logistic growth for the prey in the surface level of the lake and no birth for the prey in the bottom level of the lake. Then we consider three different types of recruitment for predators and hence create three different predator-prey models. The purpose of this study is to examine how factors such as lake depth affect populations of both predator and prey. We discuss the stability and instability of the equilibrium solutions and provide numerical simulations to graphically demonstrate the population dynamics of the system.

## 74. Why do they not believe?: The network dynamics of opinion.

**Tucker Evans** Dartmouth College

**Advisor(s):** Feng Fu, Dartmouth College

We model opinion-holding individuals as nodes in a weighted graph such that the connections between nodes represent the strength of influence that nodes have on the opinions of neighbors. Pulling from cognitive dissonance theory, we allow these connections to evolve under the influence of a fitness function that penalizes incongruity between the opinions of graph neighbors while rewarding stronger connections and higher magnitudes of opinion. We show that

the relative emphases on each of these priorities can be used to characterize the resultant behavior of the network. We apply the model to a case study of the United States Congress and the growing phenomenon of partisanship in American politics.

## 75. Mathematics of Poker: Skill vs. Luck

**Xinying Fang** University of Illinois at Urbana-Champaign

**Advisor(s):** A.J. Hildebrand, University of Illinois at Urbana-Champaign

Is Poker a game mostly based on skill or is it a type of gambling that is prohibited by many states? Motivated by this long-lasting debate, we consider John von Neumann's classical poker model and its subsequent generalizations by others including noted economists and professional poker players. We classify players according to their skill levels and seek to quantify the relative effects and profit returns of skill and chance to determine which factor dominates in each poker model. Using game theoretic methods and Mathematica simulations and visualizations, we determine the long-term behavior of the profits for different levels of skill and different poker models.

## 76. Local and global well-posedness and long-term behavior for thermo-elastic beam models with degenerate damping

**Patrick Frankart** Otterbein University

**Advisor(s):** Pei Pei, Otterbein University

In this study, we will consider existence and uniqueness of solutions to, and energy decay of, thermo-elastic models based on Euler-Bernoulli beam equations. Our primary concern is the theoretical and numerical analysis for the beam equation with thermal effects and mixed damping terms. We investigate numerically the stability of one-dimensional beam equations and derive theoretical results that support these numerical approximations. The main results verify the global existence and uniqueness of weak and strong solutions. Based on the numerical approximation, we make a conjecture about the uniform energy decay rate.

## 77. Analyzing Fair Representation of Subpopulations in Multimember Versus Single-Member Districts

**Jasekani Furbert-Wade** Morehouse College

**Advisor(s):** Duane Cooper, Morehouse College

The mathematics community is heavily engaged at present on the issues of political re-districting and gerrymandering, analyzing compactness and the measures to compare suitability of district plans. However, district shapes are only one method of suitability. Appropriate representation of subpopulation is often another consideration, one that conflicts sometimes with district compactness. This project examines measures of fair representation of subpopulation and the abilities of single-member districts and multimember districts to attain appropriate representation.

## 78. Efficient Communication in Distributed Machine Learning

**Nina Galanter** Grinnell College

**Tiffany Jann** University of California, Berkeley

**Advisor(s):** Albert Ku, Hong Kong University of Science and Technology

To train large-scale machine learning models more quickly, researchers have adapted model optimization algorithms for distributed environments, where communication between machines can be costly. Microsoft Research Asia, our project sponsor, is interested in optimizing communication in distributed machine learning. Increasing communication frequency increases communication cost, but improves convergence rate. We investigate the effect of communication frequency on the convergence of distributed Stochastic Gradient Descent (SGD) through theoretical and experimental approaches. We conduct convergence analysis on convex models for synchronous and asynchronous distributed SGD. We perform synchronous and asynchronous distributed SGD experiments with the Logistic Regression, Convolutional Neural Network, and Long Short Term Memory Neural Network models. For each model, we vary communication frequencies and record the wall clock time and number of iterations SGD takes to converge.

## 79. Inverse Problem of Central Configurations in the Collinear 5-body Problem

**Scott Geyer** The University of Southern Mississippi

**Advisor(s):** Zhifu Xie, The University of Southern Mississippi

The  $n$ -body problem is as old as physics itself. It was Isaac Newton's fascination with calculating the motion of the Earth, sun, and moon that lead him to his discoveries in both physics and calculus. While Poincaré discovered the chaos of the three body problem, mathematicians have found ways to analyze the  $n$ -body problem by studying central configurations where the bodies can remain their shape in their orbits. The inverse problem of collinear central configurations has been studied in the collinear 3-body problem by Albouy and Moeckel in 2001 and in the collinear 4-body problem by Ouyang and Xie in 2004. In our research, we examine the collinear 5-body problem by solving for the masses with the positions as given variables. Through this process, we can determine possible positions for central configurations. In the symmetric case, it is surprising that any symmetric configuration could be a central configuration for some positive masses, which is different from the symmetric case for collinear 4-body central configurations. In general, we identify regions in the configuration space where it is possible to choose positive masses that will make the configuration central. Mathematical software Geogebra, Maple, and SageMath have been used to compute and plot the regions.

## 80. A Mathematical Model for Predicting Time of Florida Evacuation on Highways

**Haoming Guo** Shenzhen Middle School

**Chang Shu** Shenzhen Middle School

**Advisor(s):** Zhixiong Chen, New Jersey City University

In 2017, Floridians began a mass evacuation as the powerful category-5 storm, Irma, plowed towards the state. Thousands of cars headed north, causing traffic jams in major highways and long lines at gas stations. This was a significant hassle to both the evacuees and the governors. This research aims to model the relationship between evacuation population and total time in particular conditions. We modify the classical traffic flow and road capacity models with other factors that may prolong the time. The study then applies this model to Florida's evacuation to determine the relationship between evacuation population and total time under two ideal arrangements. Results provide useful reference for future evacuation arrangements given the amount of time available to evacuate.

## 81. Analyzing New Health Care Placement of Mercy Health Facilities

**Natalie Halavick** Youngstown State University

**Sara O'Kane** Youngstown State University

**Leah McConnell** Youngstown State University

**Advisor(s):** Thomas Wakefield, Youngstown State University

Mercy Health is a growing health care provider in the Northeast Ohio region. This poster employs an Operations Research Model to maximize demand covered by a given facility to suggest a possible new location for outpatient care. A MATLAB program was implemented to optimize the demand for each zip code. Three different cases were tested. The first case was placing one facility, the second placing two facilities, and the last placing three different facilities. The service lines were run through statistical analyses to determine which lines were in the highest demand for these locations. It was found in all cases, the same service lines should be offered.

## 82. Positivity and Slope Limiters for Locally-Implicit Lax-Wendroff Discontinuous Galerkin Methods

**Mariana Harris** Instituto Tecnológico Autónomo de México (ITAM)

**Camille Felton** University of Wisconsin- Plateville

**Ian Pelakh** University of Florida

**Stefan Nelson** Minnesota State University Moorhead

**Advisor(s):** James Rossmann, Iowa State University

Hyperbolic conservation laws model phenomena characterized by waves propagating at finite speeds. A feature of such equations is that smooth initial data can become discontinuous in finite time — such discontinuities are referred to as shock waves. In recent years, the discontinuous Galerkin (DG) method has become one of the standard approaches for obtaining numerical solutions to hyperbolic conservation laws. For sufficiently smooth solutions, DG methods can

be made arbitrarily high-order. However, when the solution becomes discontinuous (i.e., shock waves), DG methods can produce unphysical oscillations that result in large errors, numerical instabilities, and unphysical states such as negative densities. In this work we study the so-called Lax-Wendroff variant of DG. We formulate the method as a locally-implicit predictor step followed by an explicit corrector step. We develop a novel limiting strategy that works on both these steps and suppresses unphysical oscillations and maintains the positivity of key physical quantities. The resulting method is implemented in a Python code that we are making freely available and applied to several standard one-dimensional test problems.

### **83. Optimal Investment Strategies and Portfolio Analysis of Leveraged Exchange-Traded Funds (ETFs)**

**Jalen Harris** Fresno State University

**Melanie Abel** University of Maryland, College Park

**Daniel Maes** Williams College

**Jay Iyer** Washington University in St. Louis

**Advisor(s):** Tao Pang, North Carolina State University

An exchanged-traded fund (ETF) provides a convenient vehicle for an individual investor to seek return from a stock market index or sector. A leveraged ETF (LETF) with a leverage ratio of aims to replicate times the underlying index returns on a daily basis. Market size for ETFs and LETFs has been growing since their introduction in 2006. In 2015, the US ETF industry consisted of 1,600 funds with over \$2.1 trillion in assets under management. Despite their recent popularity, there is a scarcity of models available to predict and analyze the returns of these assets. In this project, we constructed a model for the expected return and risk of these securities by deriving a probability density function (PDF) in terms of the underlying ETF. Different distributions of the underlying ETF are tested in order to determine the most precise fit with historical data. Our model can show investors when it is optimal to invest in an LETF with positive or negative , and whether or not they should take a long or short position. We also created contour plots to help investors determine their investment strategy when considering LETFs. Utilizing mean-variance optimization, we derived the efficient frontiers and calculated the sharpe ratios to understand the effect that LETFs have on a portfolio. Lastly, we completed some risk analysis with LETFs.

### **84. Scaling and City Substructure**

**Cate Heine** Centre College

**Advisor(s):** Ellen Swanson, Centre College

As our world becomes more and more urbanized, understanding the intricacies of the human city becomes increasingly important. Existing research has identified nearly universal urban scaling laws: as cities grow, social outputs like income, creativity, crime, and disease grow even faster due to increases in number and diversity of social interactions. However, the underlying structure of that growth is less clear. This research explores the relationship between population growth and social outputs on a substructural level. We find that on average, substructural elements located in large cities perform better than elements of the same size located in small cities in terms of these social outputs. However, there is much variation in this data, and as much as 10 percent of the population of US cities may not be receiving any of the income benefit that comes with increased city population. Further, we identify certain heterogeneities in city connectivity patterns that may help to explain inequalities in income growth.

### **85. Using Differential Equations to Model a Zombie Apocalypse**

**Roberto Hernandez** California State University, Fullerton

**Advisor(s):** Roberto Soto, California State University, Fullerton

In our society infectious outbreaks are unpredictable as we recently saw with the Ebola and Zika viruses. These outbreaks can be worrying since we do not know how fatal they will be until the damage is done. Accurately predicting the resolution of these outbreaks can be vital to how we distribute our resources in the most appropriate place and time. Our goal is to model the fatality of an infectious outbreak given the correct parameters. We created a model that enabled us to predict a hypothetical zombie apocalypse which can also be used to model real infections/viruses. We used a Susceptible-Infected-Recovered (SIR) model which is a set of differential equations used to accurately describe the change in the population. This SIR model predicts what portion of the population is actively going from susceptible to infected, and from infected to recovered. We showed the two most extreme cases for our research, that is,

locations where you have extremely high chances of coming into contact with a zombie or extremely low. Our model predicts that an outbreak in a densely populated area can have devastating results as a small portion of the population remains susceptible after 30 days. Furthermore, in less densely populated areas, most of the population would remain uninfected after 30 days. In conclusion, our model can serve as an essential tool for the Centers for Disease Control and Prevention (CDC) to help in the appropriation of medical resources for the most impacted areas.

## 86. The Optimal Path through a Crowd

**Ethan Hobbs** Carthage College

**Advisor(s):** Haley Yaple, Carthage College

Have you ever arrived late at a concert and not been able to get a good view? Had the crowd already formed and there looked to be no good way to the front? Using collective motion models, we seek to figure out the optimal path to arrive at a better spot if you arrive late. We created a simulation of a crowd with two classes of agents (informed and naive) and directed the informed agent along two different paths (a direct and an indirect). We then analyzed the forces caused by the surrounding crowd to determine the best path. Through our analysis, we provide the basis for further investigation into the dynamics of noisy crowds.

## 87. Analyzing risk in resource allocation

**Rachel Hobbs** University of California Berkeley

**Advisor(s):** Stephen Hobbs, Space and Naval Warfare Systems Center

A mathematical model of funding natural disaster repair frequently involves a cost analysis and reallocation of funds after the damage expenses are already known—in other words, after the disaster has already occurred. We introduce a new model that deviates from the post-disaster approach by first illustrating the statistical risk involved in funding a pre-disaster resource pool, and then an abstraction of increased efficiency of funding allocation. Presented in MATLAB, this proposed algorithm maximizes collective-action resources while the variance of the loss to each party is calculable. We show *ceteris paribus* that resource reallocation post-disaster is inefficient, and present a mathematical way of conceptualizing pre-disaster cost analysis.

## 88. Math Meets Chemistry: Modeling Dicarboxylic Acids Spectra in the Infrared Region

**Cameron Hooper** California State University, Fullerton

**Advisor(s):** Laura Smith, California State University, Fullerton

Although there is much research linking climate change and greenhouse gases, there is less information on the effect of atmospheric aerosol particles. Aerosol particles can directly interact with incoming solar radiation and outgoing terrestrial radiation by either absorbing or scattering the radiation, which can either cause a warming or cooling effect at the Earth's surface. The amount of radiation transferred has multiple dependent variables ranging from chemical composition of the particle to its extinction spectrum. In previous studies, the optical properties of short chain C2 - C6  $\alpha$ ,  $\omega$ -dicarboxylic acids, and mixtures thereof, were examined in the infrared light region using Fourier transform infrared (FTIR) spectroscopy. Specifically, five acids of interest — oxalic, malonic, succinic, glutaric, and adipic acids and mixtures, were studied as a function of concentration and phase. In this study, the resulting spectra of the five individual acids and their mixtures were examined to quantitatively and qualitatively identify the components of their spectra through the use of cosine similarities, Fast Fourier Transforms, and linear combinations of spectra.

## 89. Withdrawn

## 90. Super Central Configurations of the Collinear 5-body Problem

**William Johnson** University of Southern Mississippi

**Advisor(s):** Zhifu Xie, University of Southern Mississippi

We consider the inverse problem of central configurations for the collinear five-body problem. The configuration  $q = (q_1, q_2, q_3, q_4, q_5)$  is given as the positions of celestial bodies with masses vector  $m = (m_1, m_2, m_3, m_4, m_5) \in (\mathbb{R}^+)^5$ . A central configuration is a special arrangement of masses in the  $N$ -body problem where the gravitational force on each body points toward the center of mass. Let  $S(q)$  be the set of masses such that it is a central configuration for a given  $q$ . Additionally, we define super central configurations as a configuration  $q$  such that  $q$  is a central configuration not only for mass vector  $m$ , but also mass vector  $m'$  which is a permutation of  $m$ . The existence and classification



of super central configurations were studied in the collinear 3-body and 4-body problems in 2010. We first prove some necessary conditions for existence of super central configurations. In the collinear five-body problem there are  $5! = 120$  cases to be investigated and we prove the nonexistence of super central configurations in 96 cases. We find the necessary conditions for the existence of super central configurations in the remaining 24 cases, as well as prove that there is a region in which super central configurations exist for each of those cases.

## 91. Slopes: A Graphical View of Differential Equations

**Frederick Joubert** Pepperdine University

**Frank Garcia** Pepperdine University

**Advisor(s):** Timothy Lucas, Pepperdine University

Slopes is a mobile application that allows students to immerse themselves in the world of differential equations. Designed with accessibility and intuitive interactions at its core, the app's five activities provide a platform for exploring the plots of solutions to several types of differential equations and systems. In the Slopefields and Phase Planes activities users can add solutions by simply tapping on an initial point in the vector field. Waves animates the solution of a mass-spring system. Systems plots the solutions to a system of up to nine equations. Methods constructs numerical approximations using Euler and Runge-Kutta methods. Unlike other mathematics software that produces static plots using obscure commands and syntax, the Slopes interface allows users to easily enter equations and produce dynamic plots that can be manipulated with gestures. Slopes was designed following the MVC paradigm. Users interact with the controller, the equations and parameters, to update the model, the numerical solver, in order to produce the view, the vector fields and solution graphs. Our most recent contributions to the project include an improved user interface and a release for iPhone with the same functionality as the original iPad version.

## 92. The Spectral Action on Multifractal Robertson-Walker Cosmologies and Apollonian Circle Packings

**Yeorgia Kafkoulis** California Institute of Technology

**Advisor(s):** Matilde Marcolli, California Institute of Technology

We study the spectral action for Apollonian Circle Packings for two different Dirac operators, to understand a more simple case of Apollonian Sphere Packing, which is the basis of the Swiss-Cheese Cosmology Model. We study the residual set of an Apollonian circle packing  $\mathcal{P}$  to determine the zeta function of the length spectrum of our packing, and take the multiplicities of the eigenvalues of our Dirac operator on the circle to be constant. Then, we find the zeta function of the Dirac operator on a single circle, and then of a packing of circles  $\mathcal{P}$ , which we find is the product of some constants, the Riemann zeta function, and a series of radii terms from our circle in  $\mathcal{P}$ . We study the series of radii in terms of a geometric zeta function, and try to find its poles and complex dimensions introduced by Lapidus. Moreover, we attempt to find a pattern in the spectral action coefficients for the Robertson-Walker metric via methods in linear algebra and pseudodifferential calculus, specifically a dependence between the coefficients and exponents of scaling factor terms in the spectral action.

## 93. A Household Model of *Blatella germanica* Infestations and their Effect on Atopic Asthma Symptoms

**Amandeep Kaur** California State University

**Advisor(s):** Anuj Mubayi, Arizona State University

Asthma is a chronic respiratory condition which affects 25 million people in the United States. According to the Center for Disease Control and Prevention (CDC) asthma is a significant health and economic concern, and also causes 270 deaths per month. Cockroaches are among the most common indoor pests, with its allergen being present in 63% of U.S. households. Improper extermination of cockroaches and the associated allergens has been shown to increase the occurrence of atopic asthma, a type of asthma triggered by the exposure to allergens. In this work, a household model was developed to study the dynamics of cockroach infestation in a neighborhood of houses with individuals sensitized to the cockroach allergen. The impact of extermination and removal of allergens in a household with recurrent atopic asthma was evaluated. Equilibria for the system with and without the influx or migration of cockroaches are determined, including equilibria in which both cockroaches and asthma are present. The average number of secondary infestations that can be produced by the presence of a single infestation in a population of households without cockroaches and asthma was calculated. Numerical simulations on the model and sensitivity analysis on the basic

reproductive number were used to examine the impact of key parameter values. Numerical backward bifurcations for a set of parameters were obtained and the results showed that it is more effective to prevent infestation of cockroaches, rather than attempt to remove cockroaches once they have infested a house. A mathematical study of cockroach infestations and their effect on atopic asthma has not been done. Therefore, this model will assist in educating the general public about the importance of pest control and its relationship with asthma.

#### **94. Dynamics of Red Coral Populations**

**Hye Kim** George Mason University

**Advisor(s):** Thomas Wanner, Department of Mathematical Sciences, George Mason University

Coral is an animal that plays an important role in the marine ecosystem. Consequently, a number of researchers have studied questions of conservation of the red coral population by introducing a data-based and high-dimensional discrete model. Numerical simulations of this model have given some first insights into the effects of changes in the mortality rate, as well as the effects of overfishing. We analyze their discrete time model for red coral populations to shed light on the long-term dynamics of the population. The model exhibits both fixed points and a Hopf bifurcation, as a function of the basic reproductive number. We demonstrate that after the Hopf bifurcation, neighborhoods of the fixed points converge to closed curves, which in turn approach extinction in certain parameter regimes. Furthermore, the numerical results have been verified through computer assisted proofs as a first step towards rigorous mathematical results.

#### **95. A Low Dispersion Numerical Scheme for Nonlinear Electromagnetic Propagation**

**Alex Kirvan** Arizona State University

**Advisor(s):** Eric Kostelich, Arizona State University

I will describe a low dispersion numerical scheme that approximates solutions to wave equations. The scheme employs spatial and temporal staggering. It is applied to the nonlinear Maxwell Equations. Simulations are performed using two electromagnetic waves propagating through linear and nonlinear media with different frequencies. The nonlinear effect causes secondary wave generation. Comparison of the spectral response of the generated wave with that expected from theoretical arguments demonstrates the accuracy of the scheme.

#### **96. Computational Analysis of Ventricular Oscillations Using the Van der Pol Equation**

**Kyle Kishimoto** Fairmont Preparatory Academy

**Advisor(s):** Anael Verdugo, California State University, Fullerton

In this study, we examined the Van der Pol (VDP) equation and its applications to biological oscillations. We used the VDP equation to model the left and right ventricle action potential duration (APD) and the action recovery intervals (ARI) of the heart from previously published experimental findings. The computational analysis was accomplished by examining both the linear and nonlinear cases of the VDP equation. Analyzing the linear case allowed us to predict the behavior of the solutions based off of different initial conditions and parameters. The nonlinear analysis was used to fit more realistic changes in the dynamics of the APD oscillation amplitude. We found that the APD and ARI ventricular oscillations were approximately modeled with the VDP equation.

#### **97. Predictive Modeling and Analysis of Golf Using the Massey Method**

**Marissa Koronkiewicz** Lewis University

**Advisor(s):** Amanda Harsy, Lewis University

Ranking sports teams and predicting future results from past games can be challenging. Golf is particularly difficult to predict because it incorporates individual scores into a total team score (with the highest individual stroke score dropped each day of the tournament). One linear algebra based system which can be used to help predict sport outcomes is the Massey Method. This method is relatively simple and basically involves setting up and solving a system of equations using least squares. We can possibly improve this method by incorporating weights into the system. In this poster, we will discuss the results of an undergraduate student project which tested the predictive power of using a weighted Massey Method to predict golf tournament results from the Great Lakes Valley Conference. In particular, we test whether or not incorporating the “dropped score” for each team is a better predictor for future tournaments.

## 98. Investigating Wealth Distributions of Econophysics Models

**James Larrison** Arizona State University

**Advisor(s):** Sebastien Motsch, Arizona State University

We investigate means of decreasing variance of wealth distributions utilizing different tax structures and a novel proximity-based exchange system where agents are given positions and the probability of exchange with a given agent is distance-dependent. The results of our numerical simulations show that economic systems with wealth-biased taxation and poverty-biased distribution decrease variance of the wealth distribution. Additionally, we show that variance can be decreased through a proximity-based exchange system by utilizing the inherent wealth-inequality.

## 99. Wavelet Based Intrusion Detection System Using Support Vector Machines

**Kevin Li** Western Connecticut State University

**Advisor(s):** Xiaodi Wang, Western Connecticut State University

Network security intrusion and attacks are becoming increasingly adaptive. Therefore, more accurate detection systems are necessary to protect public or private information. Existing Intrusion Detection Systems (IDSs) are only reliable to detect already known intrusions, as it is a self learning mechanism. Using discrete wavelet transform, we can capture both the frequency of attacks and the location on the server that has been attacked which allows us to categorize the intrusions. By using Support Vector Machines (SVMs) and feature analysis on the NSL-KDD Dataset, the efficiency of the generalization of the SVM can design better classifiers for more efficient IDSs and protect network information. By using a wavelet kernel function, the mechanism has a much more versatile learning curve to increase the chance of decreasing the false negative rate and increase the true positive rate. This method is compared to other models such as the Radial Basis Function (RBF) with SVMs to analyze the effectiveness of the methodology.

## 100. Computational Simulation of a Partial Differential Equation Based Model of Electrostatic Forces on Neuronal Electrodynamics

**Kaia Lindberg** Roger Williams University

**Advisor(s):** Edward Dougherty, Roger Williams University

Neurostimulation therapies demonstrate success as a medical intervention for individuals with neurodegenerative diseases. Despite promising results from these treatments, the influence of an electric current on ion concentrations and subsequent transmembrane voltage is unclear. This project focuses on developing a unique cellular-level mathematical model of neurostimulation to better understand its effects on neuronal electrodynamics. The Poisson-Nernst Planck system of PDEs is used to model electric potential, transmembrane voltage, and ion concentrations. This system is decoupled using the Gauss Siedal method and then the equations are solved using the finite element method on a biologically-inspired discretized domain. Using FEniCS we have conducted numerous numerical experiments on several two-dimensional neuronal geometries involving action potential generation and external current application. Preliminary results demonstrate the influence of applied external currents on membrane voltage. Future work will include extending these computational simulations to three-dimensional neuron domains and integrating an ODE based intracellular signaling pathway model. Hopefully this work will ultimately help elucidate the principles by which neurostimulation alleviates disease symptoms.

## 101. Modeling Biological Invasion with the Reaction-Diffusion Equation

**Jessica Linton** Benedictine College

**Advisor(s):** Shilpa Khatri, University of California, Merced

Every year, invasive species cause irreversible damage to economies and ecosystems worldwide. Preventing the spread of such species is an important step toward reducing impact on native flora and fauna, along with preserving local economies. A noteworthy example is Japanese knotweed, *Fallopia japonica*, a perennial native to Eastern Asia. It was introduced to the United States in the 1870s as an ornamental plant and has since displaced native vegetation and clogged rivers. Since fragments from the main plant can generate new sprouts, transport of such fragments by river networks may play a key role in its spread. To better understand the impact of a river on the spread of Japanese knotweed, we applied the Crank-Nicolson time splitting method to a reaction-diffusion model and compared our results with field data to assess its accuracy.

## 102. Stability Switches of Three by Three Matrices

**Yiyang Liu** College of William and Mary

**Advisor(s):** Junping Shi, College of William and Mary

We look into the stability of a one-parameter family of matrices  $A(t) = A - Pt$  where  $A$  and  $P$  are two independent  $n \times n$  real-valued matrices. We focus on  $3 \times 3$  cases and see how the stability of the system will change when parameter  $t$  changes from  $-\infty$  to  $\infty$ . The motivation of such problems comes from the famous Turing instability in the studies of biological or chemical pattern formation. In 1952, Alan Turing proposed that spatial patterns in embryonic morphogenesis were driven by diffusion-induced instability in a system of nonlinear reaction-diffusion equations (a type of partial differential equations). Previous studies have applied the Turing instability of a  $2 \times 2$  matrix in fields like biology and chemistry. However, more realistic models involve three or more variables. Our results in this research can lead to progress in some real-world Turing systems with three or more variables in biology or chemistry.

## 103. Numerical and Convergent Series Solution Analysis to Lane-Emden Type Equations

**Shoshanna Longo** Rochester Institute of Technology

**Advisor(s):** Fazal Abbas, Rochester Institute of Technology

In this Letter, analytical solutions for the Lane-Emden Initial value non-linear models are presented. Lane-Emden type equations describe the thermal behavior of a spherical cloud of gas acting on the mutual attraction of its molecules. Homotopy Analysis Method (HAM) is used to obtain the Convergent series solutions to the model equations. It is shown that the choice of linear operator and the particular value of the auxiliary parameter  $h$  improve the interval of convergence than in the case of the corresponding traditional series solutions. Furthermore, the validity of the analytical solution is shown through Numerical solutions using Runge-Kutta 4-5 technique.

## 104. Stochastic Differential Equation models of Ascaris (roundworm) infection

**Llasmín Lopez** California State University, San Bernardino

**Yaneth Reyes** California State University, Chico

**Advisor(s):** Ben Nolting, California State University, Chico

*Ascaris lumbricoides* is a parasite that infects 0.8–1.2 billion people globally. Its greatest impact is on children in developing countries. We developed mathematical models of how *Ascaris* is transmitted through populations, with the goal of improving the ability of epidemiologists to predict the spread of this neglected tropical disease. First, we developed a differential equation model that describes how *Ascaris* is transmitted between people. We extended this model to include pigs, which are an important reservoir species for *Ascaris*. We further extended our models to structure the infected human population by worm burden, because this determines the severity of symptoms and the probability of transmitting worms to others. We developed stochastic differential equation versions of our models that included both demographic and environmental stochasticity. We numerically simulated many realizations of these equations to approximate the dynamics of the means, variances, and probability distributions associated with each disease class. Our results showed that including pig populations and worm burden structure had important impacts on model dynamics, particularly in terms of the duration and severity of outbreaks. Funded by NSF 1559788.

## 105. Improved Website Fingerprinting Attacks and Defenses on Anonymous Networks

**David Lu** MIT PRIMES, Acton-Boxborough Regional High School

**Sanjit Bhat** MIT PRIMES, Acton-Boxborough Regional High School

**Advisor(s):** Albert Kwon, Massachusetts Institute of Technology

As the Internet becomes an increasingly central element of the modern world, digital anonymity is crucial to protecting an individual's freedom of expression and right to privacy. Although anonymous web browsers like Tor attempt to provide digital anonymity, these networks are susceptible to website fingerprinting (WF) attacks, which enable an adversary to infer which pages clients visit by observing their network traffic. In our research, we show that WF attacks are a far greater threat to privacy than previously thought. First, we use Gradient Boosting to develop the strongest WF attack to date, achieving 90.1% true positive rate and 0.3% false positive rate. Then, we develop a novel Convolutional Neural Network to *automatically extract features*. Our model shows, for the first time, that weak adversaries without in-depth knowledge of network protocols can compromise user privacy by circumventing the extensive manual feature extraction process. Given the severity of our attacks, we introduce a practical WF countermeasure with theoretical bounds on the accuracy of any attacker. Our defense defeats all attacks while increasing overheads in the network by

less than 80% (compared to 100–200% in past work). We conclude that a provably-secure WF defense is attainable without incurring high overheads or inhibiting ease of use.

## 106. Using Topological Data Analysis to Aid in the Effectiveness of Conservation Efforts

**Levi Lucy** North Dakota State University

**Advisor(s):** Robert Allen, University of Wisconsin - La Crosse

Large-scale conservation efforts require simplifying generalizations about focal systems for effective execution, but these generalizations often relate more to administrative zones than to critical habitats and associated species. Using data from the North American Breeding Bird Survey (BBS), we evaluate one such generalization as it relates to bird conservation: Bird Conservation Regions (BCRs). We use methods born from topological data analysis to detect community structures of birds in North America based solely on bird populations. We create a high-dimensional “profile space”, where each point in the space represents the profile of a BBS route, and use various metrics to quantify similarities between routes. We found that the Euclidean metric performs poorly, so we developed a profile metric which performs more effectively. We create a BBS-route network and split it into modules, where each module represents a region containing routes with highly similar profiles, and we compare this network to the existing BCRs. Our method produces a map of bird regions with appreciable overlap with BCRs; however, our method also revealed regions that were not well captured or described by the BCRs, suggesting that our method may provide improvements in defining these regions and represents a more efficient and biologically-relevant strategy for conservation efforts.

## 107. Convolutional Neural Network

**Kelby Madal-Hellmuth** California State University, East Bay

**Andrew Kim** California State University, East Bay

**Tibor Svraga** California State University, East Bay

**Benjamin Thomas** California State University, East Bay

**Advisor(s):** Ehsan Kamalinejad, California State University, East Bay

Convolutional Neural Networks have become the dominant technique for image classification tasks. We developed an application for the Microsoft HoloLens Mixed Reality Headset that can classify faces in the user’s field of view in real time. Potential applications of this technology include assistance for the visually impaired.

## 108. War-Gaming Applications for Achieving Optimum Acquisition of Future Space Systems

**Scott Mahan** Arizona State University

**Daniel Chertock** George Washington University

**Brittany Dyer** Ithaca College

**Advisor(s):** Hien Tran, North Carolina State University

This project contributes to The Aerospace Corporation’s development of a Unified Game-based Acquisition Framework - Advanced Game-based Mathematical Framework (UGAF-AGMF) and associated War-Gaming Engine (WGE) models by building a framework that solves for optimum acquisition strategies and contract parameters. The UGAF-AGMF combines game theory, probability and statistics, non-linear programming, and mathematical modeling to integrate Defense Acquisition Authority (DAA) and contractor perspectives into a mutually beneficial contract. During the 2016 NCSU Summer REU, the team developed a program that maps technology and market risk factors into Aerospace’s Program and Technical Baseline (PTB) and associated acquisition strategies. Our project enhances the mapping algorithm to produce strategies with low Total Ownership Cost (TOC), low risk, and innovative design while still meeting warfighter needs. Once the PTB model selects the optimum contractor and acquisition strategy, our contract models employ Monte Carlo simulations to solve for the optimum parameters for the given contract type. The models we designed will be implemented in the larger WGE models to procedurally optimize contracts based on warfighter needs and industry factors.

### 109. Modeling Chain Growth in Polymers

**Madison Martin** University of Central Arkansas

**Advisor(s):** Danny Arrigo, University of Central Arkansas

A polymer is a large molecule that is made up of repeating subunits, called monomers. Examples of polymers found in the human body such as proteins and DNA. Another example of common polymers is polyester along with all other plastics. There are two main types of polymers, step growth and chain growth. In this project we model a chain growth polymer. For a given a length of polymer, we create a set of nonlinear ordinary differential equations (ODEs) which model all the intermediates and final products. We are able to show: (i) the ODEs can be separated and (ii) that they then can be linearized. Several examples are considered for a particular class of rate constants.

### 110. Investigating the Mortality Rate in the United States through an Age-Structured Population Model

**Gabriel Martinez** California State University, Fullerton

**Freddy Nungaray** California State University, Fullerton

**Advisor(s):** Laura Smith, California State University, Fullerton

We analyze the mortality rate in the United States by modifying the Sinko-Streifer size-structured population model to incorporate immigration and emigration. We investigate how the mortality rates vary depending on age or other factors, such as education, gender, and race. The data from 2004-2014, divided into age groups, consist of births, deaths, population, and migration information. We estimate the mortality rate as a function of age by using the Hackney and Webb approach. Preliminary results on the estimates for the mortality rates are presented. We find that over this decade, the mortality rate for a given age group does not vary significantly. Further, we find that incorporating migration is important for finding these estimates.

### 111. Transportation Networks Optimized for Various Income Groups and Their Impact on the Spread of Airborne Disease

**Rachel Matheson** Vassar College

**Advisor(s):** Baojun Song, Montclair State University

With growing reliance on mass transit systems in American cities, the question of access becomes more important. This study aims to explore the spread of an infectious disease across a transportation network created to optimize access to most frequented destinations for distinct socioeconomic groups. First, we develop a theoretical model of a city, based on the Kohl model for urban growth which assumes distinct regions where income groups live and work. It is assumed that all income groups in this city are transit-dependent. In this framework, we maximize satisfaction, a measure of how easily the population of a neighborhood can travel to desirable destinations, through placement of bus routes. Within this framework we connect a single-outbreak multi-patch SIR model of Influenza A, incorporating the effects of attraction and travel time into the incidence rate. We track the populations' interactions through contact within their neighborhoods, within the transit network, and with other transit-connected neighborhoods. We observe how the basic reproductive number is affected by the layout of the optimized transportation network. Results show that use of public transportation largely does not affect the global epidemic but that more equal time spent in transit leads to less disparate patch-specific epidemic outcomes.

### 112. Optimizing Sparse Representations of Kinetic Distributions

**Eliza Matt** Williams College

**Julia Vasile** Philip de Castro

**Advisor(s):** Minh Pham, University of California- Los Angeles

The United States Air Force Research Laboratory uses kinetic simulations to reduce costs in their various research projects, including plasma simulations. When performing these simulations, probabilistic methods are employed to reduce the computational expense of estimating the physical entropy of the system. These techniques introduce an error term in the estimation, which we seek to reduce by developing a more efficient algorithm. We discuss the nature of kinetic simulations, relevant mathematical background, and methods for error analysis. We then present multiple algorithms to estimate the physical entropy from common sampling distributions. Some techniques explore the use

of Binary Trees and the roots of Legendre polynomials, as well as a combination of the two. Finally, we discuss the performance of these algorithms and provide suggestions for further research.

### **113. Predictive Modeling and Analysis of Softball Tournament Results Using Linear Algebra**

**Carley Maupin** Lewis University

**Advisor(s):** Amanda Harsy, Lewis University

Ranking sports teams can be a challenging task and using straight win percentage can be misleading at times. Among the many mathematically inspired sports ranking systems, the Colley and Massey Methods are among the most elegant and simple. Both involve setting up and solving a matrix system. While at their most basic level, these methods are useful for sports rankings, unfortunately, they are not particularly strong at predicting future outcomes of games. One way to improve these methods for ranking and predicting future outcomes is by introducing weights to these systems. In this talk, we will discuss the results of a summer research project in which we created and tested the predictive power of the weighted Colley Method using data from softball teams in the Great Lakes Valley Conference. Specifically we explored whether past and present seasonal results would transfer to the post-season conference tournament. We used cross validation to help determine the quality of our models.

### **114. A Fast-Slow Dynamical System Model of Addiction: Predicting Relapse Frequency**

**Monica McGrath** Saint Mary's College

**Advisor(s):** Jacob Duncan, Saint Mary's College

Substance use disorders are prevalent and endemic in modern society. A patient of a substance use disorder is likely to have periodic relapses and periods of recovery. We propose a fast-slow dynamical system model which relates the levels of addictive substance to the levels of dopamine in the patient's brain. High levels of dopamine (DA) are responsible for the rewarding effects of using drugs of abuse (DOA). A patient is likely to seek recovery, which is a slow process, but increases in DA lead the patient to recall the rewarding effects of using a DOA. This leads to a very fast relapse, which is characterized by a spike in DOA levels in the brain. Then, the body's liver works to remove the DOA from the system, which is a relatively slow process. Once the DOA is completely removed, the patient has re-entered the recovery phase, and the cycle repeats. We use our model to predict the DA level which triggers a relapse, called the relapse threshold. We also use the fast-slow dynamics (geometric singular perturbation theory) of the system to predict the period of the cycle, which can be interpreted as relapse frequency. Both predictions can indicate when a patient is at risk of relapsing.

### **115. Wavelet-based nonparametric algorithms for options pricing**

**Srihita Mediboina** Western Connecticut State University

**Advisor(s):** Xaiodi Wang, Western Connecticut State University

The purpose of this research is to explore wavelet and machine learning based nonparametric methodologies for pricing call options. We first apply wavelet transform to remove noise from raw data and then apply support vector regression as well as neural networks to predict call option prices. These methods, while being prominent in other fields of study, have not heavily been used for financial econometric applications. The accuracy of these methods are compared to the widely used Black Scholes Model. The empirical analysis has shown promising results for our nonparametric methodologies to further accuracy in accommodating with the stochastic volatility of financial markets.

### **116. The One-Dimensional Filter Equation and Bio-remediation of Heavy Metal Contaminated Water**

**Senayit Menasche** Borough of Manhattan Community College

**Abdulai Jalloh** Borough of Manhattan Community College

**Advisor(s):** Chris McCarthy, Borough of Manhattan Community College

Our research focused on developing kinetic models of adsorption and one dimensional models of adsorption based filters. These mathematical models have been developed in support of our college's interdisciplinary lab group (chemistry, engineering, mathematics). Our group conducts research into the bio-remediation of heavy metal contaminated

water via filtration. The filters are constructed out of biomass, such as spent tea leaves. The spent tea leaves are available in large quantities as a result of the industrial production of tea beverages. The heavy metals bond with the surfaces of the tea leaves (adsorption). The models we are developing involve differential equations, stochastic methods, and recursive functions. Our poster will explain our models and compare the models' predictions to data obtained from computer simulations and experimentally by our lab group.

### **117. Understanding Gambling Behavior Using Massive Online Casino Data Combined with Modeling of Risk Attitudes**

**Jonathan Meng** Dartmouth College

**Advisor(s):** Feng Fu, Dartmouth College

The classic problem of Gambler's ruin suggests that gambling has a large amount of intrinsic risk. Nevertheless, gambling at casinos and gambling on the Internet are both hugely popular activities. In recent years, both prospect theory and lab-controlled experiments have been used to improve our understanding of risk attitudes associated with gambling. Despite much progress, it remains a challenge to collect real-life gambling data, which is essential to validate theoretical predictions and experimental findings. To address this issue, we propose to take advantage of large amounts of publicly available online casino data that can be collected using a customized web crawler and parser. Next, we wish to analyze the dataset by closely integrating computational and mathematical modeling of risk attitudes. For this purpose, we have been collecting betting data from two online gambling websites, just-dice.com, and etheroll.com, which both instantly publish the outcome of every single bet (consisting of each bet's Date and Time, Wager, UserID, and Profit). Real-life data of this kind, owing to its sheer size and high quality, is indeed well suited for studying gambling strategies and the complex dynamic of risk attitudes involved in betting decisions.

### **118. Numerical Methods for Solving Monge-Ampere Equation**

**Ivan Mitevski** New Jersey Institute of Technology

**Matthew Illingworth** New Jersey Institute of Technology

**David Youssif** New Jersey Institute of Technology

**Advisor(s):** Brittany Hamfeldt, New Jersey Institute of Technology

The Monge-Ampere (MA) equation is a fully nonlinear degenerate elliptic partial differential equation (PDE) that arises in optimal mass transportation, beam shaping, image registration, seismology, etc. In the classical form this equation is given by  $\det(D^2\phi(x)) = f(x)$  where  $\phi$  is constrained to be convex. Previous work has produced solvers that are fast but can fail on realistic (non-smooth) data or robust but relatively slow. The purpose of this work is to build a more robust and time-efficient scheme for solving the MA equation. We express the Monge-Ampere operator as the product of the eigenvalues of the Hessian matrix. This allows for a globally elliptic discretization that is provably convergent. The method combines a nonlinear Gauss-Seidel (G-S) iterative method with a centered difference discretization on a variety of different coordinate systems, which is stable because the underlying scheme preserves monotonicity. In order to solve these systems efficiently, the V-cycle full approximation scheme (FAS) multigrid method is exploited with error correction within the recursive algorithm; this scheme is used to leverage the low cost of computation on the coarse grids to build up the finer grids. This work shows computational results that demonstrate the speed and robustness of the algorithm.

### **119. Within-host Mathematical Models for Orthohantavirus Infections**

**Quiyana Murphy** University of Kentucky

**Advisor(s):** Christina Edholm, University of Tennessee Knoxville

Orthohantaviruses, family Hantaviridae, are harbored by mice, rats, shrews, moles and bats. Intriguingly, only those viruses harbored by rodents cause disease in humans with up to 40% fatality for cases within the Americas. Infection of humans occurs through inhalation of rodent excreta into the lung. Orthohantaviruses target the endothelial cells of the lung or kidney but eventually spread throughout the body in lethal cases. Understanding the replication kinetics of these viruses in various cell types and how replication is abrogated by the host is critical to the development of effective therapeutics for treatment for which there are none. We formulated a series of mathematical models using ordinary differential equations to examine the viral kinetics of Orthohantavirus within cells. Our models were fit to experimental data of Black Creek Canal Virus infecting Vero E6 cells and then analyzed using Latin Hypercube Sampling. We also



determined the basic reproduction number,  $R_0$ , and the final size relation. Our models provide the basis for further research into mathematical models for.

## 120. A New Efficient Decoding Algorithm for Correcting Multiple Insertion or Deletion Errors in Helberg Codes

**Kaitlyn Myers** Rowan University

**Advisor(s):** Hieu Nguyen, Rowan University

In communications, data is represented as binary strings called codewords (e.g., 100101). Unfortunately, transmission or storage of the data is not always error free. For example, communication channels experience noise and therefore, errors can occur. Error correcting codes are used to fix these errors. A class of binary codes capable of correcting multiple insertion/deletion errors are Helberg codes. Recently, a decoding algorithm has been found to correct deletion errors. We present a new efficient binary search algorithm to decode multiple insertion (or deletion) errors in Helberg codes.

## 121. Candy Nim

**Rajiv Nelakanti** Fremont High School

**Alex Tholen** Proof School

**Advisor(s):** Simon Rubinstein-Salzedo, Euler Circle

CANDY NIM is a variant on the game of NIM in which both players aim to take the last candy in a game of NIM, with the added simultaneous secondary goal of taking as many candies as possible. We show an optimal strategy for the first player in the 3-pile game in a  $\mathcal{P}$  position. We also show how to construct a game with  $n$  candies such that the loser takes the largest possible number of candies.

## 122. Withdrawn

## 123. The One Dimensional Filter Equation (Recursive Solution)

**Ai Ngo** Borough of Manhattan Community College (CUNY)

**Advisor(s):** Chris McCarthy, Borough of Manhattan Community College (CUNY)

Our research focused on developing kinetic models of adsorption and one-dimensional models of adsorption based filters. These mathematical models have been developed in support of our college's interdisciplinary lab group (chemistry, engineering, mathematics). Our group conducts research into the bio-remediation of heavy metal contaminated water via filtration. The filters are constructed out of biomass, such as spent tea leaves. The spent tea leaves are available in large quantities as a result of the industrial production of tea beverages. The heavy metals bond with the surfaces of the tea leaves (adsorption). The models we are developing involve differential equations, stochastic methods, and recursive functions. Our poster will emphasize recursive methods and compare the models' predictions to data obtained from computer simulations and experimentally by our lab group.

## 124. Mathematical Model to Noninvasively Detect Dry-Eye Diseases

**Trini Nguyen** California State University, Fullerton

**Advisor(s):** Charles Lee, California State University, Fullerton

Noninvasive interferometry is a diagnostic technique for dry-eye diseases. The process requires shining light on the eye while measuring its reflectance at multiple wavelengths. The tear film layers can then be extracted implicitly. In a 2013 patent, a mathematical model for the reflectance measurement was presented; however, it did not characterize well for all data sets because it did not consider the nonlinear scattered light at the corneal surface. In this work, two models that describe the eye reflectance are derived. The models take into account electromagnetic properties, angles of the light, and the tear thicknesses, which are to be solved. In contrast to the patent's model, the scattered light terms of our models are of higher order, which results in higher accuracy. It should be noticed that the computer program described in the patent took up over ten pages of code and 45 minutes to produce the thicknesses. We have made effort in improving the computational program and our scheme runs within a few seconds, making it practical for clinical dry-eye disease diagnoses. Our future work will include comparing the models' results to validated results to confirm the models' performances.

## 125. M-Band Wavelet Based Machine Learning Algorithms for Financial Data

**Hieu Nguyen** Western Connecticut State University  
**Abdul Rahimyar** Western Connecticut State University  
**Advisor(s):** Xiaodi Wang, Western Connecticut State University

In the financial world analyzing and predicting stock trends is in high demand. Traditionally, this is often done through the use of a Moving Average and parametric regression. However this method often over-smoothens the data, leading to a loss in prediction accuracy. By implementing the Wavelet Transform instead, one can break down data signals to prevent over-smoothing. This transformation can be followed up by various non-parametric statistical methods, such as Principal Component Analysis (PCA), Support Vector Regression (SVR), Correlation and Regression Tree (CART), and Logistic Regression to predict future values. This can lead to better decision making when trading stocks.

## 126. Optimizing Water Flow in Non-Refilling Plants

**Sarah Oldfield** Sewanee: the University of the South  
**Matthew Capobianco** Marist College  
**Advisor(s):** Sergiy Koshkin, University of Houston - Downtown

With the impending effects of climate change, water conservation is becoming increasingly important, and therefore, watering plants optimally, without overwatering, will become more important. To pull the same amount of water under dry conditions plants have to create high negative pressures, which leads to some of their water conducting vessels getting filled with air (embolism) and unable to function, thus reducing future flows. We consider plants that are unable to refill their embolized vessels to make them function again, and use a leaf-stem model to determine the optimal regime for maximizing their water flow in the long term. It turns out that there are two different types of solutions: one where the conductance of the leaf limits the overall flow (bottlenecking), and the other, where the optimal flow is governed by characteristics of the stem alone. Existence of an optimal solution is proved, and an algorithm for finding it developed.

## 127. Internal Differential Equations

**Benjamin Oltsik** Hamilton College  
**Advisor(s):** Andrew Dykstra, Hamilton College

I will be presenting a poster based on some mathematical findings I have made and attending the conference and graduate school fair. Here is my abstract: This poster will focus on a characterization of solutions to an important class of differential equations. We are calling these internal differential equations because they have the form  $y' = y(f(x))$  (i.e.,  $y$  of  $f(x)$ ). We will describe a unique approach to solving and generalizing solutions to different types of internal differential equations and suggest some real world applications for them.

## 128. Withdrawn

## 129. The Dynamics of Math Anxiety as it is Transferred through Peer and Teacher Interactions

**Melissa Pulley** Campbell University  
**Meaghan Pearson** University of Missouri  
**Advisor(s):** Carlos Castillo-Garsow, Eastern Washington University

This research develops a simple dynamical system framework to study the role of social mechanisms on the prevalence of math anxiety in United States education systems. Math anxiety is the self-reported discomfort when attempting mathematical problems. This feeling prevents students from pursuing careers in science, technology, engineering, and mathematics in these fields. Female students are disproportionately affected by math anxiety, leading to poor representation. Previous studies have examined how teachers, with and without math anxiety, can “transmit” math anxiety to students. However, to our knowledge no mathematical models have been developed to thoroughly study long term intervention strategies for reducing transmission. In this paper, the effects of female teachers’ math anxiety are modeled as a contagion on female students who may become the next generation of teachers. The purpose of this research is to determine intervention strategies to effectively reduce students’ math anxiety. From our sensitivity analysis we conclude that, instead of focusing on professional development, math anxiety can be drastically reduced if teachers portray more positive attitudes towards math, and colleges focus on recruiting non-anxious math teachers.

### 130. Helping Students Make Wise, Data-Driven Academic Decisions

**Lexi Rager** Youngstown State University

**Marina Pavlichich** Youngstown State University

**Advisor(s):** Alicia Prieto Langarica, Youngstown State University

As technology has advanced, humans have progressed to the point where they have billions of pieces of information at their fingertips at all times. However, with that power often comes an overwhelming amount of data. To help people attend to only the most relevant items, information filtering systems sort through all known data and return only what is seen as useful or significant to an individual. One specific type of information filtering system is called a recommender system, and it tries to unearth implicit and explicit information about a user, combine all new information with any previous data about that user, and finally use the whole body of knowledge to predict the preferences of the user as accurately as possible. Recommender systems have become commonplace in companies trying to give their customers a personalized experience. For example, Amazon recommends items to an individual that are consistent with their past shopping habits, and that generates many more sales than if it just recommended items to their customer at random. Our research takes recommender systems and applies them to the field of higher education. After researching the different types of recommender systems, studying their strengths and weaknesses, and compiling a system of our own, we have created a program that can help students predict what classes and professors they would most like taking at our university. It is our hope that by using this program, students will have a convenient resource tailored to them to help make wise, data-driven academic decisions.

### 131. Observing the Behavior of Quasiperiodic Orbits

**Arsah Rahman** George Mason University

**Advisor(s):** Arsah Rahman, George Mason University

Quasiperiodicity is one of the three types of observed dynamical behaviors alongside chaos and periodicity and has large applications in Hamiltonian systems. Not only is it used in studying planetary motion, but researchers at NASA have observed this behavior to be a possible fuel-efficient path for a spacecraft to travel further distances as the Solar System is a Hamiltonian system containing quasiperiodic orbits, which are orbits that are close to being but are not periodic. Coding an algorithm based on the work of Das, Saiki, Sander, and Yorke and of Levnajic and Mezic, we observed the dynamics of the two dimensional standard map, that is, how the appearance of this map is influenced by different values of the parameter, radius being greater than or equal to zero in the system of equations used in the development of the map, and also by varying initial conditions and numbers of trajectory points, for an understanding of the behavior of quasiperiodic orbits. In addition, we are currently extending the techniques used for obtaining and understanding the standard map to study the four dimensional measure-preserving Froeschle map, and looking at new parameters, which we intend to apply to other equations that have not been studied yet.

### 132. Continuum Modeling of Traffic Systems with Autonomous Vehicles

**Brian Rhee** Phillips Exeter Academy

**Kaiying Hou** Phillips Academy

**Advisor(s):** Andrew Rzeznik, Massachusetts Institute of Technology

Describing the behavior of automobile traffic via mathematical modeling has been a field of study for mathematicians throughout the last century. In the past decade, the rise of autonomous vehicles (driven by software without human intervention) has presented a new problem for classical traffic modeling. Autonomous vehicles react differently from human-driven vehicles, resulting in modifications to the constitutive laws of the partial differential equation. In this project, we provide insight on constitutive laws of autonomous vehicles by using continuum modeling traffic systems with a mix of human and autonomous vehicles. We present a new model that includes both human drivers and autonomous vehicles in which each vehicle can only measure the total density of surrounding cars, regardless of type of car. By implementing the Lax-Friedrichs scheme, we test how different constitutive laws perform in our model and analyze the density curves. We also analytically derive and implement a Roe Solver for a class of coupled conservation equations, in which the velocities of the cars only depend on the total density of surrounding cars regardless of type. We present computational results showing how various constitutive laws for autonomous vehicles impact total traffic flow.

### 133. Searching for topological bubbles in parameter space for phononic honeycomb lattice

**Shuvom Sadhuka** Cambridge Rindge and Latin School

**Advisor(s):** Jenny Hoffman, Harvard University

Topological insulators are condensed matter systems where an electric pulse is conducted only along the boundary and not through the bulk as a result of eigenstate inversion about the chemical potential with respect to a vacuum. With inverted eigenstates, it is impossible to adiabatically map the eigenstates to those of the vacuum without crossing the chemical potential, resulting in topologically protected surface conduction. However, band inversions can be studied in any lattice system, and a recent problem has been to understand how band inversions in photonic and acoustic systems result in robust topological surface transport of light and sound waves respectively. Despite interest in topological systems, the relationship between the parameters which define a lattice and the ordering of energy eigenstates remains largely unexplored. Here we show by combining numeric and analytic calculations that band inversions occur in a phononic honeycomb lattice inside a closed manifold, or a bubble, defined by three parameters: two speeds of sound and a relative lattice. Band inversions in phononic systems can be mathematically mapped onto quantum mechanical analogs. Our results provide a simple understanding of band inversion in a phononic system which aids in the search for new topological quantum materials. These results could be generalized to other lattice geometries using our methods.

### 134. Recursively Constructed Keyboards for Xenharmonic Musical Scales

**Kecia Sako** San Jose State University

**Advisor(s):** Jordan Schettler, San Jose State University

Macro/microtonal scales such as the Bohlen-Pierce scale and the gamma scale of Wendy Carlos use recursive arrangements of black and white keys, which generalize the familiar layout on a standard piano keyboard. The number of notes in certain musical scales with roughly equal spacing is well-known to correspond with the denominator of a convergent in the continued fraction expansion of  $\log b(a)$ , where  $a$  and  $b$  are rational numbers. The 12 note-octave on a keyboard is the standard example derived from the continued fraction expansion of  $\log 2(3)$ , and this can be taken a step further to describe a procedure for constructing xenharmonic keyboard layouts using only the coefficients in the continued fraction. This framework is used to examine 41 equal temperament and help quantify why it is so under-used in music.

### 135. Using Kolmogorov's Spline Network to Assess Photolithographic Processes

**Jonathan Sepulveda** University of Central Florida

**Advisor(s):** Dr. Xin Li, University of Central Florida

The Kolmogorov Superposition Theorem (KST) prescribes the finite sum representation of univariate functions that can express any continuous function of  $n$  variables. In this study, the KST is extended to serve as the basis learning rule for a neural network architecture (NNA). Using spline approximating functions as its univariate functions, this NNA uses convolution and feature mapping to successfully partition a test set of photolithographic patterned samples into categories of "unexposed," "overexposed," and "adequately exposed" species. Back-propagation is utilized to make better error corrections during the supervised learning phase. Note that the general multivariate function is approximated by:

$$f(x_1, \dots, x_n) = \sum_{n=1}^{2d+1} g_n^s \circ \sum_{q=1}^N \phi_n(x_q) \lambda_q$$

The complexity of this NNA fits in line with that expected by the Kolmogorov spline network, first shown in the work of Igel'nik and Parikh, 2003.

### 136. Pokemon Go, a Case Study in Mathematical Modeling

**Devin Serna** Bakersfield College

**Patrick Chao** Bakersfield College

**James Plain** Bakersfield College

**Arthur Carrillo** Bakersfield College

**Robert Carrillo** Bakersfield College

**Advisor(s):** Brown Jonathan, Bakersfield College

Third-party models for pokemon and their move-sets in Pokemon Go, by Niantic, are well distributed but fundamentally flawed. Currently, the most accurate predictions are made by full simulations which disagree with the popular,

third-party models. We present more accurate mathematical models and heuristics for pokemon, move-sets, and predicting winners of match-ups. Our results find agreement with simulations, where other models have not. We demonstrate these results and rankings based on theorems from game theory, calculus, basic algebra, and linear algebra.

### **137. Predicting Separability from Partial Preference Matrices**

**David Shane** Grand Valley State University

**Advisor(s):** Jonathan Hodge, Grand Valley State University

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when a voter's preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. Preferences that are free from such interdependencies are said to be separable. Determining the extent to which a voter's preferences are separable often requires extensive information about their ranking of all of the possible election outcomes. In this poster, we explore what conclusions about separability can be made from partial voter preferences. Our work has potential applications to election sequencing, particularly when only incomplete information about voter preferences is available. This research was completed as part of the Summer Mathematics REU at Grand Valley State University.

### **138. Big Data and the Stock Market**

**William Shannon** Linfield College

**Advisor(s):** Xiaoyue Luo, Linfield College

In our modern competitive market, businesses are seeking efficient and innovative platforms to remain profitable and prepared, especially in the uncertain world of the financial stock market. One possible avenue for improving stock prices that companies can turn to is harnessing a substantial volume of information, known as big data. However, because of the nature of big data, distilling and analyzing the vast amount of information can require complex analytical methods. Using a keyword selection process based on word frequency, we were able to filter out the data amongst the noise and derive a sector specific keyword list. This list used in combination with a previously created trading method along with the implementation of a thresholding technique, allowed us to develop a more specific trading strategy focused on different market sectors. Our results show that the use of thresholding techniques in addition to the Google Trends strategy may improve returns in the stock market.

### **139. Estimating epidemic arrival times using linear spreading theory**

**Anne Shapiro** Carleton College

**Lawrence Chen** Kansas University

**Advisor(s):** Matt Holzer, George Mason University

We study the dynamics of a spatially structured model of worldwide epidemics and formulate predictions for arrival times of the disease at any city in the network. The model is comprised of a system of ordinary differential equations describing a meta-population SIR compartmental model defined on a network where each node represents a city and edges represent flight paths connecting cities. Making use of the linear determinacy of the system, we consider spreading speeds and arrival times in the system linearized about the unstable disease free state. Two predictions are presented. The first assumes that the dominant transmission pathway between any two cities can be approximated by a one dimensional lattice or homogeneous tree and gives a uniform prediction for arrival times independent of specific network features. The second method considers the full heat kernel solution of the linear system and produces an arrival time estimate that incorporates network features. We test these predictions on a real network describing worldwide airline traffic.

### **140. Constraints on the Oceanic Carbon Sink Using Atmospheric Oxygen Data**

**Julie Sherman** University of Minnesota

**Advisor(s):** Richard McGehee, University of Minnesota

Understanding the oxygen cycle and its relationship with the carbon cycle is an important tool in quantifying sources and sinks of carbon. Atmospheric measurements from the Scripps Oxygen Program have shown that while carbon dioxide levels are increasing, oxygen levels are decreasing, a trend consistent with their inverse relationship during photosynthesis and combustion. However, oxygen and carbon are decoupled with respect to oceanic exchanges, and

thus this flux can be determined with the proper equations. In this study we develop a simple model of the global carbon-oxygen budget in which we incorporate data from the Scripps Carbon Dioxide and Oxygen Programs. Our results are obtained from derivative free optimization techniques, and give minimum sources and sinks necessary to replicate atmospheric observations. We compare our results to large and complex global circulation models.

### 141. Economic Data Forecasting Using Wavelet Based Machine Learning Techniques

**Gabriel Soares** Western Connecticut State University

**Karim Naba** Western Connecticut State University

**Advisor(s):** Xiaodi Wang, Western Connecticut State University

In this research, we will be using data gathered on indicators of economic growth. These indicators are ones that have a high correlation to economic growth (such as per capita income, electricity consumption, etc.) We then apply a wavelet transform to the matrix created from the gathered data in order to remove noise. In order to determine the most significant indicators of economic activity, we apply Principal Component Analysis (PCA) to the previously obtained covariance matrix. We use the resulting principal components as feature vectors in applying a machine learning technique (such as support vector machine regression or neural networks) to be able to predict economic growth with greater accuracy.

### 142. Physiologically Based Pharmacokinetic (PBPK) Modeling for a Persistent Chlorinated Water Contaminant: 1,2,3-Trichloropropane

**Lee Spence** Kennesaw State University

**Jolie Even** Coastal Carolina University

**Frederick Law** University of California Berkeley

**Advisor(s):** Marina Evans, Environmental Protection Agency

The compound 1,2,3-Trichloropropane (TCP) has historically been used during the production of pesticides and polysulfide rubbers, and remnant traces of TCP can still be found in ground water. TCP shares structural similarities with toxic compounds, and it therefore is considered as possibly toxic. We developed a physiologically based pharmacokinetic (PBPK) model to determine internal doses of TCP in the body of F344 rats who were given an intravenous dose of TCP. Metabolism was modeled using Michaelis-Menton kinetics, and unknown metabolic parameters were optimized for using data from a 1984 intravenous study. Concentrations of TCP in each tissue with respect to time were predicted. We then altered the model to track internal doses in F344 rats that ingested TCP from contaminated drinking water, with drinking patterns modeled using a periodic intake function. Following our initial motivation, we scaled the drinking model to humans, with new parameters taken from literature or found through allometric scaling. We used this model to explore possible accumulation of TCP in human tissues after continued ingestion of contaminated water. Additionally, we performed sensitivity analysis on body fat percentage as it pertains to long-term accumulation.

### 143. Spectral Gluing Formulae of the Graph Laplacian and Quantum Mechanics

**Michael Toriyama** University of Illinois at Urbana-Champaign

**Advisor(s):** Ivan Contreras, University of Illinois at Urbana-Champaign

Spectral graph theory as a mathematical tool for modeling purposes has attracted much attention for a wide variety of practical applications, including those pertaining to networks and control theory. We propose a formalism for a discretized version of quantum mechanics by using the graph Laplacian matrix to describe the quantum evolution of a particle confined in a finite graph. Inspired by the locality principle of quantum mechanics, we prove a theorem that elucidates the morphology of the spectra of Laplacians after gluing two configuration spaces via a set of external edges. Such formulae give rise to explicit bounds for the algebraic connectivity of the glued graph, and it gives insights to the problem of understanding the electronic structure of matter with a large number of particles.

### 144. Stability of Periodic Fixed Points and Invariant Sets of the Modulated Logistic Map

**Khoa Tran** University of Illinois at Urbana-Champaign

**Advisor(s):** Eduard-Wilhelm Kirr, University of Illinois at Urbana-Champaign

In this talk, we are studying the stability of the 1- and 2-periodic fixed points and the invariant sets of the two-dimensional dynamical system proposed by Elhadj and Sprött in [1]. The map exemplifies a slave-master relationship

where the first, master component is defined by the logistic map and the second is also a logistic map regulated by the first component. We begin by finding the fixed points 2-periodic points of the map and determine their stability as a function of its three parameters  $a$ ,  $b$ , and  $c$ . In certain parametric regimes, we further analyze the basin of attraction of stable fixed points and 2-periodic points and fully describe the dynamics of arbitrary initial data. Lastly we want to numerically and analytically determine the invariant sets of the second component in the domain  $[0, 1]$  as it usually represents the validity region for the underlying models. We will detail our analysis using computer graphics and simulations to illustrate our results. [1] Elhadj, Zeraouia, and J. C. Sprott. "The effect of modulating a parameter in the logistic map." *Chaos: An Interdisciplinary Journal of Nonlinear Science*. 18.2 (2008): 023119.

### 145. Withdrawn

### 146. Applying fractional calculus in modeling cell viscoelasticity

**Anh Vo** Creighton University

**Advisor(s):** Nathan Pennington, Creighton university

Recent successful application of fractional calculus shows the effectiveness of the subject in understanding physical as well as biological models. In this research, we will apply the Mittag-Leffler function, a generalized exponential in fractional calculus to model cell viscoelasticity. Recent research shows that using fractional calculus can increase efficiency and fitness of models by decreasing parameters compared to using integer-order. Furthermore, fractional calculus describes differential equations which govern cell viscoelasticity and is more meaningful physically than traditional calculus. Given past experimental data by our research group, we will fit them to a non-integer order of a differential equation. Then we will draw a conclusion of correlation between the order of the differential equation and the state of the cell

### 147. Identifying Communities of Specialized Knowledge in a Tech Economy

**Sirui Wang** Cornell University

**Advisor(s):** Mei Wang, University of Chicago

In the past decade, New York City has seen tremendous growth in its technology sector, making it a prominent player in the world of technology today. With the rise of social media platforms, we are able to study the organization of specialized knowledge in this emerging community by building networks using data from a large online group focused on tech-based startups in the New York area. This study considers empirical estimates of edges using co-membership data where nodes in these networks represent agents of specialized knowledge while edges represent an extent of interaction between the specialties. With a given network, we cast the problem of community detection as a constrained integer linear program that optimizes network modularity, and approximate sub-communities of specialized knowledge that are present in the tech community.

### 148. Nonclassical Symmetries of a Generalized KdV Equation

**Andie Weaver** University of Central Arkansas

**Advisor(s):** Danny Arrigo, University of Central Arkansas

It is generally known the classical Lie symmetries of partial differential equations (PDEs) and equivalent systems can be different. Recently it has been shown for a general class of nonlinear Convection-Diffusion equation and nonlinear Wave equation, that their nonclassical symmetries are contained within the nonclassical symmetries of the equivalent system. It is natural to ask if this is true in general. Here, we consider a general class of KdV equation (i.e.,  $K(m, n)$  equation) and an equivalent system. We calculate the nonclassical symmetries of both and contrast similarities and differences to determine whether the conjecture true.

### 149. Population Movement in Epidemic Scenarios

**Azaryah Wilson** University of Central Arkansas

**Advisor(s):** Long Le, University of Central Arkansas

We present a study of population movement in an epidemic situation, including cross diffusion rates. A compartmental model is utilized, with susceptible, exposed, infected, and treated compartments to represent different groups in the population. Variable movement conditions through time, space, and relation to other populations are simulated using numerical methods.

## 150. Withdrawn

### 151. Mathematics of Poker - Adaptive Strategies

**Fan Wu** University of Illinois at Urbana-Champaign

**Advisor(s):** AJ Hildebrand, University of Illinois at Urbana-Champaign

Poker games are perfect natural experiments showing how people compete in a zero-sum game with imperfect information, gather information about their opponent's strategy, and adjust their own strategy accordingly. We investigate extensions and variations of classical poker models due to John von Neumann and others and seek to answer the question of how players should alter their strategies in a multi-round game, based on information from previous rounds of the game. Focusing on some specific extensions of the von Neumann model, we utilize minimax methods to determine the optimal strategy for the first player. We then explore how players adapt their strategies as they accumulate more information. Furthermore, we simulate how different strategies interact in real poker games based on our theoretical results, and visualize these simulations using Mathematica.

### 152. Robot Path Planning using Integer Linear Programming

**Jeffrey Yeh** California State Polytechnic University, Pomona

**Lori Lewis** California State Polytechnic University, Pomona

**Jacqueline Alvarez** California State Polytechnic University, Pomona

**Advisor(s):** Jillian Cannons, California State Polytechnic University, Pomona

The algorithm we present focuses on a multi-target search path planning problem from "An information theoretic based integer linear programming approach for the discrete search path planning problem" by J. Berger et al. In this problem, we focus on minimizing the target cell occupancy uncertainty, or entropy, within a given region. We formulate this search path planning problem as an integer linear programming problem and try to find the best path for the robots. Our method extends the existing algorithm by allowing the robots to rotate in place. This problem can become very large and complicated to solve analytically, so we implement the algorithm in MATLAB. The result is an optimized path for the robots in a specific region that we can map onto a grid.

### 153. A Random Graph Generator for Social Network Modeling

**James Zak** University of North Carolina, Chapel Hill

**Advisor(s):** Christopher Strickland, University of Tennessee - Knoxville

A method for the reliable generation of random networks that model known social networks is increasingly desirable as a tool in the study of how these networks are structured and change over time. This poster will present an algorithm capable of growing large directed networks that successfully model a range of observed social networks with regard to several key measures. Our algorithm is loosely based on the Barabasi-Albert algorithm for scale free graph generation. However, our model includes additional parameters that play key roles in social networks including a means of assigning trait attributes to individuals in the network at the formative stage, which allows for scenarios in which tolerance and diversity issues play a central role. In doing so, we have produced an algorithm that is not only intuitive in its implementation, but also extremely flexible and easily adaptable to a variety of situations.

### 154. A Markov Modulated Traffic Simulation To Increase Toll Plaza Efficiency

**Nathan Zavanelli** New Mexico Institute of Mining and Technology

**Advisor(s):** Aly El-Osrey, New Mexico Institute of Mining and Technology

A significant amount of traffic congestion occurs at toll plazas, where multi-lane divided limited-access toll highways use barrier tolls to collect tolls from motorists. The goal of this research is to improve upon the New Jersey Interstate System (NJIS) toll design by creating a Markov model for traffic in a toll plaza. In order to do so, several reasonable assumptions are made and the rationality of drivers is evaluated. The flow process through the toll plaza is divided into two stages: the toll collection and merging post collection. Each vehicle is modeled as a multidimensional vector within a grid, whose value at the next temporal step is a function of its current state and the properties of other vehicles within the grid. The grid state is updated using sequential cellular automata and the rate of traffic inflow is modeled with a Poisson distribution to match that of the NJIS. This model indicates that the current design of seven tolls merging into three lanes is the most effective method for toll collection given a high flow rate, but that significant inefficiency results from large vehicles and semitrailers. We propose a method for alleviating this issue by separating



these vehicles entirely from the general merging process through a separate toll plaza and note an average time wasted reduction from 27.4 to 20.3 seconds for a traffic flow of 500 vehicles per hour.

### 155. Optimal Vaccination Strategies for Combating *S. Typhi*

**Carmen Acosta Alonzo** Bennett College for Women

**Aaleah Lancaster** Bennett College for Women

**Advisor(s):** Hyunju Oh, Bennett College for Women

Typhoid Fever is a bacterial disease caused by *Salmonella enterica* serotype Typhi. The disease has been mainly eradicated in the developed world, but is still very much endemic in developing parts of the world. Transmission occurs through close contact, and unsanitary habits which can be combated through vaccination and education of the affected community. We developed an SVIR model to model the impact of the bacteria, in South Asia. We construct a game theoretic model to find the optimal individual vaccination and education strategies for the eradication of Typhoid Fever. We find that when the relative cost to vaccinate starts to increase individuals will not choose to vaccinate, however when education has a high relative cost, individuals will still use this strategy until max cost is reached. Due to this factor, a combination of both strategies is most effective when combating the disease.

### 156. An Automaton-Based Model of Uterine Contractions

**Lauren Alvarez** Loyola Marymount University

**Advisor(s):** Angela Gallegoa, Loyola Marymount University

Understanding how contractions begin in the uterus can lead to a better understanding of how uterine contractions regulate themselves. This in turn allows clinicians to properly monitor labor, and help predict preterm birth. Research in this area is important because 10% of births are preterm, and preterm births account for 85% of neonatal deaths. Based on previous work by Barclay, we have developed an organ-level model of the uterus in an automaton format that incorporates cell-to-cell communication. We have created the model using the computer software package called Netlogo, and used agent-based rules to describe how cells would be able to communicate. Allowing “cells” in the model to see their neighbor cells mimics the presence of gap junctions. Gap junctions are channels that connect one cell to its surrounding six neighbors, and allow ions to follow through the channel like messages. Gap junctions increase throughout pregnancy and a large number of them are considered essential for productive labor. Preliminary results indicate that when all cells can communicate, irregular signaling patterns are hard to achieve, supporting the importance of gap junctions. We explore the irregular communication patterns through the presence of scars. Future work includes restricting cell-to-cell communication through changing the rules about which cells can see their neighbors.

### 157. Modeling Fungal Community Dynamics within Chestnut Blight Cankers

**Jesica Bauer** Carroll College

**Advisor(s):** Anita Baines, University of Wisconsin-La Crosse

The American chestnut (*Castanea dentata*) has been infected with the introduced blight pathogen *Cryphonectria parasitica* (CP) since 1904. This fungal blight has decimated American chestnut populations because of its ability to spread and restrict nutrient transport within the tree by creating cankers. Hypovirulent strains of the blight are infected with an intracellular viral parasite which slows the growth of CP and can allow the tree to produce wound callus tissue to stop the blight. Using data collected from several stands of chestnut trees in Michigan, we analyzed the blight’s interactions with other fungal species on the tree. We also developed cellular automata to replicate the fungal communities on the American chestnut to further investigate the development of cankers. Simulations of various canker communities show that the development of the canker, and the resulting health of the tree, can depend heavily upon the interactions of these fungi. These results help to identify potential biological controls which can inhibit the growth of the blight pathogen and increase the survival of the American chestnut tree.

### 158. Modeling Giraffe Protein Absorption and Overall Health

**Kristina Benton** University of Central Oklahoma

**Advisor(s):** Brittany Bannish, University of Central Oklahoma

I use differential equations to model the protein absorbed by the endangered Rothschild giraffe during digestion. Protein absorption, the quantitative measure of giraffe health, is affected by consumed tannin from acacia tree leaves,

salivary proteins, and dietary protein. I build a model describing the interactions between these quantities and their movement from the stomach to the small intestine, where protein absorption occurs. I present results suggesting ways to improve the health of giraffes in conservatories.

### 159. Stochastic Modeling of Chemotherapy Treatment for Cancer with Rare Events

**Zachary Bezemek** Michigan State University

**Zachary Shay** University of Michigan - Dearborn

**Advisor(s):** Hyejin Kim, University of Michigan - Dearborn

In some very rare cases a cancer patient will spontaneously see complete eradication of their cancer without any treatment. Miracles like this make for uplifting stories, but what causes these extremely rare events to occur? In most cases, deterministic models can accurately represent the events that occur within a system, but they do not give rise to rare events that we may want to account for in predictive models. Stochastic modeling can help explain such events, thanks to its random characteristics. Therefore, we develop and investigate an equivalent stochastic differential equation (ESDE) model for an existing ordinary differential equation (ODE) model of tumor-immune system interactions with chemotherapy, with particular interest in how the stochastic component effects the original ODE system. We analytically calculate the probability of the rare event of tumor self-eradication after premature termination of adjuvant chemotherapy treatment by establishing the existence of a large deviation principle (LDP) in a simplified version of the ESDE model, as well as numerically estimate this probability. Additionally, we calculate the expected times for eradication of the tumor cells and for the patients' immune system health to meet critical levels, then analytically establish formulas for these results that agree with our numerical simulations. Beyond the rise of rare events, we discuss how the difference in expected times for these events from those of the system of ordinary differential equations yields changes when considering the comparable treatment methods to the optimal treatment plans established by quadratic and linear optimal controls for the deterministic model, even under standard conditions.

### 160. Comparison of Statistics in RNA Secondary Structure Matching Models

**Michael Breunig** University of Wisconsin-Eau Claire

**McKenzie Scanlan** University of Wisconsin-Eau Claire

**Kyle Goryl** University of Wisconsin-Milwaukee

**Advisor(s):** Manda Riehl, Rose-Hulman Institute of Technology

RNA forms bonds with itself, and this partially determines how the RNA functions. We investigate some models for this RNA secondary structure using a mathematical object called a matching. Some bonds are more likely than others based on how close two molecules are and which bonds are thermodynamically minimal. A number of different models have been created to approximate which matchings are likely to appear in RNA and which aren't. We will examine the crossing number, nesting number, and pseudoknot number statistics on L&P matchings, C&C matchings, and all perfect matchings.

### 161. Graph Connectivity Indices Applied to the Evolutionary Analysis of Chagas Disease Vectors

**Holly Caldes** Montclair State University

**Advisor(s):** Aihua Li, Montclair State University

*Trypanosoma cruzi*, otherwise known as Chagas disease is currently at epidemic levels in Central America. The disease is carried by the insect *Triatoma dimidiata*, which transmits the disease to the host. It commonly leads to heart and digestive problems. In order to understand the evolutionary relationships between species, DNA sequences can be analyzed using the pKa value of each of the bases and the Randic connectivity Index. By calculating the pKa connectivity indices as well as the ratio of the number of A and T to the number of G and C, a phylogenetic tree is constructed to help with the species analysis. For this analysis 120 different haplotypes with DNA sequences with a length of 489 base characters have been analyzed to create a phylogenetic tree for the ITS-2 sequence series.

## 162. A Model to Quantify the Importance of Habitats in a Migratory Network

**Alyssa Carson** Emmanuel College

**Shirley Qui** Emmanuel College

**Advisor(s):** Christine Sample, Emmanuel College

Understanding the influence of discrete habitats on the survival of a migratory species is an essential part of making successful conservation and management decisions. We model migratory populations using a weighted and directed network, where nodes and edges have sets of attributes that are time and density dependent. We develop and refine two measures to quantify the importance of each node in the network, which can be applied to populations with different migratory patterns, classes, seasons and nodes. These measures help us understand which nodes contribute most to the network's population and which nodes are most critical to the network's viability. Using data from real migratory populations we applied our model to calculate the two measures for various populations.

## 163. Modeling Vaccination Strategies to Control White-Nose Syndrome in Little Brown Bat Colonies

**Eva Cornwell** St. Olaf College

**David Elzinga** Wichita State University

**Shelby Stowe** Sterling College

**Advisor(s):** Alex Capaldi, Valparaiso University

Since 2006, the North American bat population has been in rapid decline due to white-nose syndrome (WNS), which is caused by an invasive fungus (*Pseudogymnoascus destructans*). The little brown bat (*Myotis lucifugus*) is the species most affected by this emerging disease. We consider how best to prevent local extinctions of this species using mathematical models. In 2017, development began on a new vaccine for WNS; we analyze the effects of implementing vaccination as a control measure. We create a Susceptible-Exposed-Infectious-Vaccinated hybrid ordinary differential equation and difference equation model informed by the phenology of little brown bats. We compare the effectiveness of annual, biennial, and one-time vaccination programs for multiple durations of immunity length. We also determine the optimal time to vaccinate, if vaccinating only once, as a function of average duration of immunity. Next, we perform a sensitivity analysis to determine the robustness of our results. Finally, we consider other possible control measures together with vaccination to determine the optimal control strategy. We find that if the vaccine offers lifelong immunity, then it will be the most effective control measure considered thus far.

## 164. Mathematically Modeling the Impact of Invasive Crayfish Removal on Endangered Steelhead and Rainbow Trout Persistence in the Santa Monica Mountains

**Madison Cox** University of Michigan

**Advisor(s):** Courtney Davis, Pepperdine University

Steelhead and rainbow trout comprise one species, *Oncorhynchus mykiss*. Steelhead trout are anadromous *O. mykiss* that migrate between the ocean and freshwater streams to spawn; rainbow trout remain in their natal freshwater stream for life. In addition, anadromous trout have much higher fecundity than resident rainbow trout. The populations of California's native *O. mykiss* that live in streams connected to the Pacific Ocean are endangered due to habitat loss, drought, and predation by invasive crayfish (*Procambarus clarkii*). The recent historic California drought has decreased the connectivity of the spawning streams to the ocean and has limited the anadromous members ability to reach spawning grounds. This is exacerbated by reduced reproductive success due to crayfish predation upon eggs and young trout. Recent conservation efforts focus in part on removing crayfish to boost *O. mykiss* survival. We create a discrete compartmental model of *O. mykiss* life history dynamics in Topanga Creek. We incorporate a model of invasive crayfish trapping and use numerical simulations and sensitivity analysis to investigate which crayfish trapping regimes most benefit *O. mykiss* persistence in Topanga Creek. The model is generalizable to endangered *O. mykiss* populations in other streams. Model results inform invasive crayfish removal efforts and aid *O. mykiss* conservation efforts.

### 165. A game theoretic model of mating strategies of territorial migratory songbirds

**Kimberly Dautel** Marist College

**Advisor(s):** Suzanne Lenhart, University of Tennessee Knoxville

It has been shown that territorial migratory songbirds exhibit both monogamous and polygynous mating strategies. In our work, we develop a game theoretic framework from which to model competing strategies among female and male members of the population. Analysis of the model shows that it is the mating preferences of female songbirds that most strongly influences the survivability and fitness of the entire songbird population.

### 166. Time-Independent Solution of the Poisson-Nerst-Planck System of Equations for Neurological Applications

**Andrew DelSanto** Roger Williams University

**Advisor(s):** Edward Dougherty, Roger Williams University

Electromagnetic brain stimulation has been shown to benefit medical patients with a variety of brain diseases. The molecular mechanisms by which these therapies operate are still largely unknown; nevertheless, these treatments are electromagnetic in nature, and so we hypothesize that they must have a direct influence on the distribution of ionic species adjacent to neural cells and their transfer between extracellular and intracellular spaces in neural tissue. We implement a finite element solution to the steady-state Poisson-Nerst-Planck system of partial differential equations to investigate these questions. To solve this system, we employ a variety of numerical methods including a Gauss-Siedal decoupling method and a weighted iterative Gummel scheme to facilitate numerical convergence. Implementation is performed using FEniCS on a biologically-inspired domain constructed in GMSH. We present our numerical simulation results which show the influence of an electric field on ionic distributions, and in addition, present numerical method convergence metrics which gauge simulation efficiency and accuracy.

### 167. A Diffusion Based Model of Kinesin Recycling in Neurons

**Ryan Fantasia** University of Massachusetts

**Katherine Borg** Cornell University

**Advisor(s):** Boris Slepchenko, University of Connecticut Health Center

The axon terminal of neurons has complex signaling and metabolic demands, requiring organelles, vesicles, and macromolecules, namely cargo, to be delivered from their site of synthesis. Motor proteins, such as kinesin, actively transport these cargo. What happens to kinesin after transport is less understood. Recent experimental work and modeling suggest that kinesin may be recycled through a diffusion-based mechanism, against the conventional hypothesis of degradation or return via dynein, the opposite directed motor. Continuing this line of exploration, we investigate a diffusion based bucket-brigade mechanism, in which cargo “changes hands” during transport, and the diffusion of unbound kinesin is restricted by free cargo jams. Diffusion and advection partial differential equations model three variables, cargo bound kinesin, unbound kinesin, and free cargo, related by reaction. Due to biological evidence we subject bound kinesin to spatial inhomogeneities that influence the dissociation of cargo. Disassociated cargo in turn decreases the diffusion rate of species by the excluded volume effect. With these assumptions we look to show that kinesin recycling via a diffusion based mechanism is feasible and competitive in long range neuronal transport.

### 168. The Spread of Cholera Through Water Networks

**Kendal Foster** Vassar College

**Advisor(s):** Benjamin Morin, Vassar College

Cholera is a waterborne disease caused by the bacteria *Vibrio cholerae* and is transmitted via the fecal-oral route. There are 1.3-4 million cases of cholera every year, and previous studies have extensively modeled this disease in a single human population. Instead of a single human population, our model considers the flow of bacteria between distinct human populations through water networks. Furthermore, we expand the previous compartmental models to include a class for individuals who are infected but not yet infectious and a class for an undetected population of asymptotically infectious individuals. We analyzed the long-term behavior of the disease and performed numerical simulations to determine the conditions for disease-free and endemic equilibria.

### 169. Modeling the Treatment of HIV in Children

**Kelley France** University of Central Oklahoma

**Advisor(s):** Brittany Bannish, University of Central Oklahoma

We build a differential equations model to study how the length of HIV treatment in young children affects the treatment success. The recent news of remission in some children who had been treated briefly for the virus early in life, shows that understanding the dynamics of the treatment at this stage of life is crucial to uncovering the potential for remission. Taking into account the uninfected T cell population, drug sensitive and drug resistant infected productive T cell populations, drug sensitive and drug resistant infected T cells in the eclipse phase, and drug sensitive and drug resistant free virus, we build a differential equations model of HIV treatment reflective of treatment in young children. We present our model and the results, identifying conditions under which the child would enter remission. We conclude by discussing how this information can be used to minimize the risks to other children/patients.

### 170. A Model of Iron Metabolism in the Human Body

**Mary Gockenbach** University of Texas at Arlington

**Tim Barry** University of Maryland, College Park

**Advisor(s):** Pedro Mendes, Center for Quantitative Medicine, UConn Health

Iron-related disorders are prevalent throughout the world. Anemia, which has iron deficiency as a major cause, affects nearly one quarter of the world's population. Hereditary hemochromatosis, a disease of iron overload, is the most common inherited disease of gene mutation in Caucasians. Understanding the mechanisms of iron metabolism in the human body will advance individualized treatments strategies for these and other conditions. A mathematical model using ordinary differential equations is developed to simulate the distribution of iron in the major organs of the body. The model is calibrated for a healthy person using experimental time course data obtained from literature. The inclusion of hormones in the model, such as erythropoietin and hepcidin, enable the investigation of common iron disorders and potential treatments. This model provides a foundation for the creation of a personalizable model in which the specifics of an individual's condition form the parameter set so that the outcomes of various treatments can be predicted.

### 171. An Interdisciplinary Approach to Computational Neurostimulation

**Madison Guitard** Roger Williams University

**Advisor(s):** Edward Dougherty, Roger Williams University

Mathematical models of electrotherapies are proving to be a valuable accompaniment to these medical treatments; in particular, modeling the transcranial direct current stimulation (tDCS) mode provides a predictive component to assess and quantify voltage and electrical current distributions in the head cavity, which in turn gives a direct measure of treatment efficacy. One fundamental drawback of these simulations, however, is the fact that precise material conductivity values are unknown for a particular patient, and in addition, are highly variable within a tissue type. As these values greatly impact simulation results, simulation utility is highly dependent on them. To address this issue, we have implemented a mathematical model of tDCS using a stochastic partial differential equation coupled with mixed boundary conditions. The finite element method is used to solve our governing system, and Monte Carlo experiments over scalp, skull, brain fluid, and brain tissue variabilities are performed. Numerical simulations are performed on an idealized two dimensional mesh, and then extended to an MRI-derived three dimensional head geometry. Our preliminary results showcase the importance of incorporating and quantifying this conductivity uncertainty within mathematical models of tDCS.

### 172. New Life Lineages

**Cristhian Gutierrez** University of California, Merced

**Advisor(s):** Arnold Kim, University of California, Merced

With the upcoming of metagenomics, many argue the credibility of the Tree of Life and our current method of taxonomic classification. Some outright argue that there is no universal tree. However, others say that the Tree allows us to analyze a physical representation of Evolution and ancient life and provides a means to classify and sort organisms. Placing organisms in hierarchical categories helps scientists understand unique physical features, biological function, and phylotypic genetic compositions. These taxonomic categories allow us to highlight similarities and differences amongst known organisms. By fully understanding the distinguishing factors between taxa, we hope to classify and

best organize unknown organisms. Here, we present a preliminary method that we hope will provide the framework in classification of novel life. Using supervised classifying methods we aim to filter known genomes from metagenomic datasets thus providing a list of candidate novel, unclassified genomes. This process will help you sort through large datasets and extract candidate genomes, which can be further tested and examined.

### **173. A Joint Model for Bivariate Longitudinal Data**

**Bridget Harris** College of the Holy Cross

**Advisor(s):** Shannon Stock, College of the Holy Cross

The development of viral resistance mutations among HIV-infected patients is the primary cause of antiretroviral treatment failure, which is observed clinically by a rebound in viral load and a decrease in CD4 cell counts. Here, we develop a tree-based method to determine the viral genetic mutation patterns associated with poor response to treatment. Such a model might prove extremely effective in guiding treatment decisions for physicians treating HIV-infected patients. Tree-based methods, such as CART, have been developed to model a univariate outcome measurement observed at a single time point. We extend the CART framework to accommodate a joint model for bivariate longitudinal data, which will allow us to simultaneously model patients' viral load and CD4 response trajectories. Application of this method to data from the ACTG 398 clinical study revealed distinct mutation patterns for different classes of drugs.

### **174. Name That Bird: Using Neural Networks to Classify Recorded Bird Songs**

**Russell Houpt** Hope College

**Advisor(s):** Darin Stephenson, Hope College

Can a computer learn to identify a bird by analyzing samples of its song? This research explores how neural networks can be used to identify different birds from recordings of their songs. We explore convolutions, wavelets, and neural networks, how they can work together, and what techniques were employed to teach the programs how to quickly and accurately identify birds. In earlier work, a research group at Hope College made progress on this question by using neural networks to classify bird songs on a somewhat limited scale. Our results extend this work by using similar techniques on larger data sets, improving the accuracy and speed of the analysis, and modifying the existing algorithms to take advantage of multiple core computers.

### **175. A Cellular Automaton Modeling Approach to Chestnut Blight Canker Development**

**Samuel Iselin** Valparaiso University

**Advisor(s):** Anita Baines, University of Wisconsin - La Crosse

The fungal pathogen *Cryphonectria parasitica* (CP) causes cankers on American chestnut trees (*Castanea dentata*). These cankers kill the branch or trunk if the canker girdles the infected branch. Treating cankers by introducing hypovirus-infected CP (HCP) as a biological control has been used to increase chestnut tree survival. Chestnut trees are able to survive infection if the tree can compartmentalize the CP and prevent it from girdling the tree. However, the mechanics of canker development remain poorly understood. We propose a cellular automaton as a model of canker development over time to theoretically test tree survival and the effectiveness of HCP treatment. The results of the cellular automaton predict the likelihood of a branch surviving a canker based on several factors, including branch size and fungal virulence. Ongoing work concerns the effect of HCP treatment on canker development and branch survival. Specifically, we investigate the effects of the treatment's timing and distance from fungal infection.

### **176. A Comprehensive Analysis of Predictor Variables for Change in Conners' Score Among Childhood Sleep Apnea Patients**

**Kingsley Iyawe** Morehouse College

**Advisor(s):** Renee Moore, Emory University

Obstructive sleep apnea (OSA) negatively impacts an individual's quality of sleep. Poor quality of sleep has been found to negatively alter one's quality of life. The adverse effects can include both cognitive and behavioral deficits. The most common treatment for children diagnosed with OSA is adenotonsillectomy (eAT), however eAT has not been thoroughly evaluated in its effectiveness for improving cognitive or behavioral functioning. The primary objective of this study is to determine if eAT is the most effective procedure when compared to watchful waiting with supportive care (WWSE).

### **177. An Analysis of the Global Population Genetics of Malaria Resistance.**

**Benjamin Juarez** University of California, Merced

**Advisor(s):** Suzanne Sindi, University of California, Merced

The World Health Organization reports that 3.2 billion people in 95 countries are at risk of being infected with malaria. With nearly 1 million deaths per year worldwide, malaria represents one of the deadliest infectious diseases. Because malaria first infected humans thousands of years ago, genetic mutations providing natural resistance to malaria have been under positive selection. Prior studies of genes promoting resistance to malaria analyzed only targeted populations experiencing malaria in the present. However, such studies may miss beneficial mutations carried by individuals who today live in areas where malaria infections are rare. We take a global-approach by studying a large collection of whole-genome sequencing data (the 1000 Genomes project) to look for past signals of malaria resistance. In this study, we focus on five gene locations linked to malaria resistance: HBB, ABO, ATP2B4, G6PD, CD40LG. We use Samtools and the human reference genome to report on the global nucleotide diversity of these target genes. In the future, we plan to extend our studies to the complete set of over 20 genes known to confer resistance to malaria. Understanding the complete genetic diversity of genes associated with malaria resistance will provide valuable information towards developing therapeutic targets to this important infectious disease.

### **178. The Phylogenetic Derivative: A Tool for Assessing Local Tree Reconstruction in the Presence of Recombination**

**Jacqueline Kane** Hobart and William Smith Colleges

**Advisor(s):** Joseph Rusinko, Hobart and William Smith Colleges

A limitation in many population genetics problems is the inaccuracy of inferred evolutionary histories of chromosomes sampled randomly from a population. This inferred evolutionary history differs among genomic locations as an artifact of recombination events along a chromosome. Thus, much recent attention has been focused on identifying these recombination points. However, many proposed methods either make simplifying, but unrealistic, assumptions about recombination along a chromosome, or are unable to scale to large genome-wide data like what has become commonplace in statistical genetics. Here, we introduce a phylogenetic derivative to describe the relatedness of neighboring trees along a chromosome. We use this tool to assess the accuracy of inferred local phylogenies in simulated data. This phylogenetic derivative is a computationally efficient, flexible metric that can be also be used to assess the prevalence of recombination across a chromosome. These proposed methods are tested and perform well in analyzing both simulated data and a real mouse data set.

### **179. Accurately Modeling the Healing Process of Chronic Wounds**

**Arjun Kanthawar** Western Kentucky University

**Nikhil Krishna** Western Kentucky University

**Advisor(s):** Richard Schugart, Western Kentucky University

In order to formulate a mathematical model that accurately represents the physiology of a wound, the model and its parameters must be identifiable when given actual data. Practical identifiability is a method used to determine whether parameters in a model can be uniquely determined given actual data. This work uses a differential equation model that describes the interactions among matrix metalloproteinases, their inhibitors, the extracellular matrix, and fibroblasts (Krishna et al., 2015). A singular value decomposition technique with a QR factorization combined with a correlation analysis is used to find an identifiable subset of parameters. Subsets are analyzed through model prediction intervals and parameter Markov chains and posterior densities. The goal of this work is to formulate a model that can accurately predict the healing process for individual patients.

### **180. Climate change and its impact on the migratory patterns of *Ficedula hypoleuca* and *Anser brachyrhynchus***

**Theoni Kasamias** Youngstown State University

**Advisor(s):** Alicia Prieto Langarica, Youngstown State University

There has been growing concern in regards to climate change and its impact on the organisms of our planet. Particularly of interest is the impact on migratory species which rely on environmental cues in order to travel in response to the seasons. In our study, we used an agent-based model in order to predict the changes in the migratory patterns of two

species of bird: *Ficedula hypoleuca*, the European pied flycatcher, which breeds mainly in Europe and western Asia but winters in Africa, and *Anser brachyrhynchus*, the pink-footed goose which breeds in Greenland, Iceland, and Svalbard but winters in northwest Europe.

### 181. Mathematical Models of the Mammalian Circadian Oscillator and Alcohol Dependency

**Grace Kelting** University of Central Oklahoma

**Advisor(s):** Brittany Bannish, University of Central Oklahoma

Circadian rhythms are the body's internal clocks that control brain wave activity, energy production, and other biological activities. When the body is subject to acute or chronic alcohol consumption, the regular circadian rhythm is disrupted and this disturbance affects mood regulation, sleep cycles, blood pressure, and other biological rhythms. This research presents a mathematical model to study the relationship between alcohol dependency and the mammalian circadian oscillator. We build and analyze a differential equations model to show how alcohol dependency disrupts circadian rhythms. We also compare our results to published data and suggest target genes for helping people with alcohol dependency recover their natural circadian rhythm.

### 182. Optimal Growth Allocation in *Sarracenia purpurea* in Varying Environments

**Brielle Kwarta** Houghton College

**Advisor(s):** Jason Bintz, Houghton College

*Sarracenia purpurea*, also known as the Northern Pitcher plant, is a perennial carnivorous plant found in nutrient poor bogs in the Eastern part of the United States, the Great Lakes region, and most of Canada. *S. purpurea* has modified pitcher-shaped leaves which collect prey and rainwater for nutrient consumption, with nitrogen being the most important nutrient, and rainwater being the primary means of obtaining it. These modified leaves are photosynthetically less efficient than other leaf structures (phyllodia) that are produced by the plant. *S. purpurea* is phenotypically plastic, and so the plant can allocate growth between carnivorous and non-carnivorous structures depending on environmental nitrogen conditions. Previous research has shown that an increase in phyllodia production is a direct and rapid response to nitrogen availability. Using optimal control theory, we created a model to show optimal growth allocation of *S. purpurea* in such a way that plant morphology represents a prediction of atmospheric nitrogen levels. Our model predicted a difference in carnivorous to non-carnivorous biomass ratios given different nitrogen uptake rates.

### 183. Optimal Vaccination Strategies for Combating *S. Typhi* Transmission in South Asia

**Aaleah Lancaster** Bennett College

**Belissa Acosta** Bennett College

**Advisor(s):** Hyunju Oh, Bennett College

Typhoid Fever is a bacterial disease caused by *Salmonella enterica serotype Typhi*. The disease has been mainly eradicated in the developed world, but is still very much endemic in developing parts of the world. Transmission occurs through close contact, and unsanitary habits which can be combated through vaccination and education of the affected community. We developed an SVIR model to model the impact of the bacteria, in South Asia. We construct a game theoretic model to find the optimal individual vaccination and education strategies for the eradication of Typhoid Fever. We find that when the relative cost to vaccinate starts to increase individuals will not choose to vaccinate, however when education has a high relative cost, individuals will still use this strategy until max cost is reached. Due to this factor, a combination of both strategies is most effective when combating the disease.

### 184. A simplified mathematical model to explore the output of a rhythmic neural network

**Madel Liquido** Saint Peter's University

**Nickolas Kintos** Saint Peter's University

**Advisor(s):** Nickolas Kintos, Saint Peter's University

The electrical output of rhythmically active neural networks is ubiquitous in the nervous systems of animals. Such networks are driven by a core circuit called a central pattern generator (CPG), and, due to their stereotyped activity, CPGs are tractable for mathematical analysis. We examine the gastric mill rhythm (GMR) of the crab, *Cancer borealis*, whose CPG activity is composed of the half-center oscillation between the lateral gastric (LG) neuron and interneuron 1 (Int1). The GMR is activated by synaptic input from the axon terminals of the projection neuron, modulatory



commissural neuron 1 (MCN1). Using the difference in fast-slow time scales that exist within the biological system, we construct a simplified, 2-dimensional ODE model of the GMR. Through phase plane analysis of our singularly perturbed system, we show that the MCN1-LG synaptic interactions drive the GMR oscillations in the model. Also, we show that the addition of a fast, synaptic input onto Int1 strongly influences the GMR frequency in the model. Our results agree with those of experiments, and our simplified model captures the underlying network dynamics of this CPG circuit.

### **185. Estimating Evolution from DNA Data**

**Jesse Maltese** Hobart and William Smith Colleges

**Advisor(s):** Joseph Rusinko, Hobart and William Smith Colleges

Traditionally, gene trees created from DNA sequence data are used to estimate species trees. However, constructing gene trees is computationally intensive. Using the coalescent model, we bypass this step and construct two taxa species trees directly from DNA data. We improve this estimate by accounting for the effective population size of a species. Finally, we use the neighbor joining algorithm to create larger species trees from refined two taxa estimates.

### **186. Modeling Resistance to Intermittent Androgen Suppression in Prostate Cancer Patients**

**Jose Martinez** Arizona State University

**Advisor(s):** Eric Kostelich, Arizona State University

Prostate cancer is one of the most common cancers in men. Because of the cancer's dependence on androgen, Androgen Deprivation Therapy is a standard treatment for advanced prostate cancer. This treatment is rendered ineffective once the development of androgen-independent tumor cells happens after several years. There is a need to predict the progress of cancer for physicians to decide on the best course of action. Mathematical models have been previously developed to model the progression of prostate cancer, and we propose a combinative model of two previous efforts. Using clinical data for patients under Intermittent Androgen Suppression Therapy, we show that the new model outperforms the previous models in both fitting and predicting power.

### **187. Modeling directed evolution with coupon-collecting and mixing problems**

**Devin Mattoon** Missouri Western State University

**Altan Tutar** Davidson College

**Advisor(s):** Laurie Heyer, Davidson College

Our interdisciplinary, interinstitutional team of undergraduate researchers sought to evolve a DNA sequence in a bacterial virus, using natural selection to alter the specificity of an RNA regulatory element called a riboswitch. We modeled random mutations in the virus, accounting for lethality due to damaged genes. We applied a generalized solution of the coupon collector's problem to calculate the expected value and a 90% confidence interval for the number of viral genomes required to observe all possible single, double and triple base substitutions in the DNA sequence, after removing from the population all viruses with one or more lethal mutations. We used our probability model to ensure that the spatial and temporal scale of the wet lab experiment was sufficient to explore the extremely large space of all possible DNA sequences of a given length. Further, we used differential equations to predict the time required for a small molecule to reach each of its target concentrations in a dynamic solution, and for particular virus genomes to dominate the population in each of three parallel compartments of a chemostat. Finally, we built web tools to make generalized versions of our probability and differential equation models available to biologists for the design of future experiments.

### **188. A Mathematical Model for Intracellular Calcium Release Through the IP3 Receptor in the Presence of Amyloid Beta Proteins**

**Joseph Minicucci** Norwich University

**Advisor(s):** Jocelyn Latulippe, Norwich University

Alzheimer's Disease (AD) is a devastating neurological disease affecting approximately 50 million people globally. Although the cause of this disease remains unknown, recent studies have focused on Amyloid beta ( $A\beta$ ) peptides as a possible factor in the pathogenesis of AD. The slow accumulation of  $A\beta$  has been shown to alter calcium signaling

mechanisms causing cytotoxic effects leading to a gradual decline in memory and cognitive skills, and neuronal death. Although  $A\beta$  alters normal calcium flux through the 1,4,5- Inositol- trisphosphate (IP3) receptor on the endoplasmic reticulum, the mechanism for how this occurs remains unknown. Our goal is to use mathematical modeling and analysis to study the effects of  $A\beta$  on calcium regulation by specifically tracking the contribution through the IP3 receptor. We aim to precisely identify the physiological processes in IP3 production that are directly affected by  $A\beta$ . We first develop a model for calcium dynamics in a closed cell environment and then test this model against experimentally observed data. Our results show that  $A\beta$  is increasing the rate of PLC mediated IP3 production and the rate at which G proteins are being activated.

### 189. Modeling Ebola Outbreaks: Analyzing the Effects of Contact Tracing

**Alexander Mitchell** Dixie State University

**Advisor(s):** Vinodh Kumar Chellamuthu, Dixie State University

Ebola is a disease transmitted by contact with the bodily fluids of those infected and can lead to internal bleeding, organ failure, and death. One method used to suppress the spread of Ebola is contact tracing, which consists of documenting and quarantining those who have come into contact with an infected individual. To understand the spread and containment of Ebola, we need to better understand the relationship between the suppression of the disease and the use of contact tracing. We developed a mathematical model utilizing a system of non-linear differential equations with the goal of investigating how variable contact tracing affects transmission dynamics and outbreak behavior, specifically as it relates to the 2014-2016 Ebola Zaire outbreak. We then validated our model with data from the World Health Organization and used sensitivity analysis to quantify the usefulness of the different aspects of contact tracing. Furthermore, we applied matrix theory to explore the dynamics of our model, and ran numerical simulations to verify the model's predictions and explore the use of multiple control strategies in effectively containing the Ebola virus. The results from this model can be used to help optimize the allocation of resources in future outbreaks.

### 190. Prostate Cancer Prediction Models

**Sierra Murphy** Arizona State University

**Advisor(s):** Eric Kostelich, Arizona State University

Though initially androgen deprivation therapy effectively treats prostate cancer, the tumor eventually becomes resistant. The hypersensitivity pathway of resistance bypasses therapy by producing androgen locally from precursors, like cholesterol. The correlation between high cholesterol and higher incidence of prostate cancer, the increase of ACTH during therapy, and the lower prostate cancer risk for patients on statin all support this mechanism of resistance. However, past mathematical models do not consider cholesterol. Thus, including cholesterol and related factors contributes to a new class of mathematical models that are more biologically relevant in determining when the tumor escapes treatment.

### 191. Biological Stability in Epidemiological Models of Zombie Outbreaks

**Matthew Naeger** Truman State University

**Advisor(s):** Robert Allen, University of Wisconsin-La Crosse

Applications of SIR models to the study of infectious diseases are well documented since the early 1900's. Recently, *Munz, et al.* (2009) applied certain SIR models to the spread of a zombie infection through a human population. Through studying perturbations of these models, *Allen, et al.* (2014) found that it is indeed possible to survive such an outbreak, and although the disease-free equilibria are not mathematically stable, they are biologically stable: over time the human population would decrease but remain positive, while the zombie population would decrease to 0. We extend the work of *Allen, et al.* (2014) and study the dynamics of three new SIR models applied to zombie infection: human immunity, zombie starvation, and competition. Through the dynamics, we determine if biological stability exists for disease-free equilibria, i.e., if humans can survive an outbreak of zombiism. In addition, we determine the conditions analytically for biological stability. These conditions are dependent on system parameters (infection rate, disposal rate, and initial populations of humans and zombies). Finally, we are able to verify these conditions through numerical simulation.

## 192. Cheating the Cheaters: Spatial Dynamics in the Evolutionary Stability of Antibiotic Resistance

**Joshua Nieve-Silva** Montclair State University

**Advisor(s):** Leon Arriola, University of Wisconsin-Whitewater

Infections caused by antibiotic-resistant bacteria are posed to be one of the most pressing health concerns of the twenty-first century. A common mechanism of resistance involves production of an antibiotic-degrading enzyme. In this case, neighboring, nonproducer bacteria can “cheat” by sharing the benefits of resistance while the metabolic cost of enzyme production falls solely on producer cells. The objective of this work is to explore how the spatial population dynamics of producers and nonproducers maintain the resistance found in biofilms. A three-dimensional spatial model was used to simulate growth of both producers and nonproducers under antibiotics with different characteristics. Standard antibiotics resulted in a heterogeneous populations with stable, homogeneous community structure. The population of resistant bacteria was most sensitive to altering the fitness cost of enzyme production. These results could suggest novel antibacterial treatments in order to create therapies less likely to favor the evolution of resistance.

## 193. Improving Automated Methods for Cell Identification in Calcium Images

**Cara Nix** University of Minnesota

**Meraiah Martinez** Benedictine College

**Advisor(s):** Mark Reimers, Michigan State University

New optical imaging technologies have the potential to revolutionize neuroscience, but are hindered by inaccurate automated cell identification. We are comparing results from current automated cell-sorting methods to careful human judgment. We are investigating the current algorithms which use singular value decomposition and non-negative matrix factorization. We are calculating single pixel statistics which are aiding our efforts to improve the automated methods in identifying neurons. We are exploring the correlations within the background noise of multiple neurons to augment our ability to identify, and then remove, the unnecessary background noise.

## 194. Modeling Chronic Vascular Responses Following a Major Arterial Occlusion

**Jordan Pellett** University of Wisconsin- La Crosse

**Advisor(s):** Julia Arciero, Indiana University- Purdue University Indianapolis

Peripheral arterial disease is a serious health concern characterized by a full or partial occlusion of a major artery in the systemic vasculature. Following an occlusion, blood supply to peripheral tissues is significantly reduced, causing patients to experience severe pain and reduced mobility. This study uses mathematical modeling to investigate the role of different vascular segments in restoring blood flow following a major occlusion. Vascular adaptations to collateral arteries and the microcirculation distal to the occlusion have been observed to occur on both acute and chronic time scales. Here two chronic vascular responses, arteriogenesis (increased diameter of existing vessels) and angiogenesis (new vessel formation), are investigated in a single vessel and a complex network. By coupling these chronic responses to acute responses, the model provides a framework for understanding the time frame and significance of vascular responses that help restore flow. Preliminary results suggest the number of collaterals increases following an occlusion while fewer vessels distal to the occlusion are required for optimal flow restoration. Ultimately, the model can be used to identify the most important vessels to target for future therapies.

## 195. A Stochastic Epidemiological Model of the Response of American Chestnut Populations to Fungal Blight

**Rebecca Rouleau** Saint Michael's College

**Kelsey Lieberman** Truman State University

**Advisor(s):** Anita Davelos Baines, University of Wisconsin - La Crosse

For the past 100 years, *Cryphonectria parasitica*, a fungal blight, has been infecting American chestnut trees through wounds on the tree, forming cankers. In an attempt to control the spread and negative effects of the blight, a hypovirus-infected form of the fungus has been introduced to the cankers. The hypovirus is an intracellular parasite of the fungus; it can only be transferred by direct contact between fungi. The hypovirus does not cure the cankers but rather slows the growth rate of the fungus and allows the tree to form “healing cankers” by producing wound callus tissue. We model the dynamics of various tree communities over time with the effects of virulent and hypovirulent forms of the blight

using an epidemiological model. Our model uses a stochastic approach to simulate tree health, growth, death, and reproduction, as well as the spread of blight. Initial conditions and parameters are generated based on biological data gathered from infected chestnut groves. We compare the average condition of our simulated groves at 10 year intervals up to 100 years under various treatment regimens—in which hypovirulence is introduced to various proportions of infected trees—to the results of leaving infections completely untreated.

## **196. Sparse Signal Recovery Methods for Variant Detection in Next-Generation Sequencing Data**

**Katharine Sanderson** Montana State University

**Jonathan Sahagun** California State University, Los Angeles

**Melissa Spence** University of California, Davis

**Advisor(s):** Roummel Marcia, University of California, Merced

Recent research suggests an overwhelming proportion of humans have structural variants (SVs): rearrangements of the genome such as inversions, insertions, deletions and duplications. The common methods of SV detection involve sequencing fragments from an unknown genome, mapping them to a reference genome, and analyzing the resulting configuration of fragments for evidence of rearrangements. While SVs occur relatively infrequently in the human genome, they are hard to identify due to the indirect nature of SV detection, resulting in high false-positive identification rates. Our approach aims to improve SV detection in two ways. First, we solve a constrained optimization equation consisting of a negative Poisson log-likelihood objective function with an attached penalty term that promotes sparsity. Second, we focus on detecting SVs in the analysis of multiple related individuals simultaneously. Therefore we are able to implement familial constraints, which limit a child's genome to express SVs if, and only if, one or more of its parents have first expressed a SV. This problem formulation is designed to improve SV detection despite a large amount of error attributed to both current DNA sequencing methods and noisy mapping processes from the test genome to the reference genome. Through careful modeling and leveraging related individuals to narrow the boundaries of study, we anticipate higher accuracy of SV predictions.

## **197. Evaluation of Circadian Time Series Analysis Methods Using Simulated Data**

**Alex Santos** Amherst College

**Advisor(s):** Tanya Leise, Amherst College

The study of biological oscillators often requires extraction of key characterizing aspects (e.g. period, phase, waveform) from experimental time series. A number of time series analysis procedures have been documented for the detection of gene expression periodicity, but the efficiency of these tests is frequently compromised by factors such as samples that are short in duration, series with varying rhythmicity, and noisy signals. We review and compare seven methods applied to circadian clock function analysis: Lomb-Scargle periodograms, sinusoidal curve fitting, Fisher's g-test, maximum entropy spectral analysis (MESA), discrete Fourier transform (DFT), autocorrelation and cross-correlation. Methods were assessed for their accuracy in period estimation, phase estimation and detection of rhythmicity, using simulations of circadian oscillations.

## **198. Analyzing the Spread of the Zika Infection in Puerto Rico through the Replicator-Mutator Model**

**Isabel Serrano** California State University, Fullerton

**Advisor(s):** Anael Verdugo, California State University, Fullerton

Within the last year, the Zika infection case count has risen in various states throughout America, with the largest number of infections in Puerto Rico. Although properties of the virus have been identified, the infection's subtle symptoms make the illness difficult to diagnose. Given that the infection was previously perceived as a mild illness, the disease's dynamics remain unknown. As simple models graze over the unique behaviors diseases exhibit, this work aims to create a robust model that analyzes disease transmission from an evolutionary perspective. We intertwine compartmentalizing techniques inspired by the Susceptible-Infected-Recovered (SIR) model, with the ordinary differential equations of the Replicator-Mutator (RM) model to describe the interactions between the three subgroups. Applying the RM model to the Zika virus, we analyze the model's dynamics and assess its ability to describe the Zika infection's geographic spread. Through MATLAB, we produce simulations of our equations to define our model's parameters.

Lastly, by using linear stability analysis we identify the model's fixed points to analyze the subpopulations' long-term behavior. Ultimately, by examining this model's dynamics we aim to give insight into the infection's outbreak cycle.

### **199. Curvature Profile of Crawling Worms**

**Faith Shaw** James Madison University

**Advisor(s):** Eva Strawbridge, James Madison University

*C. elegans*, a one-millimeter-long nematode worm, is being studied to determine if movement can be characterized and quantified. *C. elegans* are well studied organisms and are currently being used in research on a variety of diseases, such as Alzheimer's and Parkinson's. We study these worms by obtaining videos of them crawling on agar plates, using a microscope and worm tracker setup in the WORM Lab dark room at James Madison University. These videos are then processed by a program in the application MATLAB, which allows for their analysis by extracting information and creating figures to visualize the quantitative characteristics of locomotion. Visualization helps to determine a reliable method for distinguishing normal versus abnormal movement by quantifying the dynamic shape of a crawling worm. That is, we can see in these figures how the curvature along points of the worm changes from frame to frame throughout the video. When parameters of the centerline change as the worm crawls, we measure these changes to characterize what it means for a worm to move normally. Once we can say this, we will have a method of measuring abnormal locomotion, and its relationship to known genetic features. This research can eventually be applied to the study of diseases which involve problems with motor function such as Parkinson's Disease in humans.

### **200. Quantifying Electromagnetic Properties of Applied Electric Fields on Neural Tissues**

**Abigail Small** Roger Williams University

**Advisor(s):** Edward Dougherty, Roger Williams University

Electrical stimulation therapies continue to showcase great results as treatment options for individuals possessing a range of brain and nervous system related disorders. A fundamental unknown of these procedures is how stimulation source, location, waveform, and positioning, in conjunction with nerve cell geometry, affect the distribution of the electric current density around targeted neurons. In this work, we investigate these areas with mathematical modeling and computational simulation. In particular, we numerically solve the Poisson partial differential equation using the finite element method on biologically-inspired two- and three- dimensional neuronal geometries, all within an exhaustive series of scenarios that collectively examine how electric potential and electric current density distributions are influenced by stimulation parameters. To facilitate these *in silico* experiments, we are implementing a flexible modeling framework based on FEniCS to enable user-specified Dirichlet and Neumann boundary conditions, as well as domain and equation parameters. Our results reveal precise correlations among stimulation settings, cell structure, and the resulting electrical current distributions, which we compare with current heuristic arguments of the neurobiological community.

### **201. Land cover change shows little effect on migration patterns of Wood Storks and Great Egrets**

**Johanna Smith** Drake University

**Advisor(s):** Robert Allen, University of Wisconsin- LaCrosse

Wood storks (*Mycteria americana*) have been endangered in North Carolina, South Carolina, Georgia, and Florida since 1984. In 2014, the USGS changed the status of the wood stork from endangered to threatened in these areas. As an indicator species, they are one of the first species to react to changes in the environment. Evaluating the extent to which land-cover change effects wood storks, principally with respect to their movement, is of paramount importance. Conversely, great egrets (*Ardea alba*) rely on similar environments and are a more resilient species, less responsive to changes in their habitat than wood storks. We employ concepts from circuit theory to conduct a least-cost path analysis of the migration patterns of both species through the eastern United States. Least-cost path analysis applies differential weights to land-cover types corresponding to the expected relative ease of traversing each type, such that the resulting trajectory estimates the path of least cost through the landscape. We compared least-cost paths with GPS tracking data obtained from an online repository (Movebank.org). Estimated least-cost paths in 2001, 2006, and 2011 do not reveal substantive evidence to suggest that land-cover change is one of the motivating factors of variation in migratory dynamics in wading-nesters.

## 202. Assessing the Role of Temperature in Dengue Fever Outbreak Dynamics with *Wolbachia* Transinfection Control Methods.

**Colton Smith** Dixie State University

**Advisor(s):** Vinodh Kumar Chellamuthu, Dixie State University

Dengue fever is one of the most common mosquito-borne diseases worldwide; up to 500 million people are impacted each year. Several studies show that the *Wolbachia* bacteria transinfection can be used as a control strategy to mitigate the severity of Dengue epidemics. The bacteria cause decreased lifespan, as well as an inhibited ability to carry the Dengue virus. We developed a mathematical model with a system of nonlinear differential equations to investigate how *Wolbachia* transinfection could diminish the spread of the Dengue virus in human populations. We present numerical evidence from our model that shows a significant decrease in Dengue cases. Furthermore, we used our model to simulate the dynamics of the spread of the Dengue virus, and were able to illustrate the role of temperature in outbreak levels. Our model can also be adapted to test a wide variety of environmental factors to determine optimal release times of the *Wolbachia* infected mosquitoes into native populations. Overall, our model can be used as a tool to increase the efficiency of abatement centers in controlling Dengue outbreaks.

## 203. Allee Effects and Chaos in a Food Chain Model

**Wan Wen** SUNY Binghamton

**Advisor(s):** Zhifu Xie, The University of Southern Mississippi

The interaction between predators and their prey affects the population dynamics dramatically. In population dynamics, the spatial distribution of the population can be influenced by depressed growth rate at lower density, which is called the Allee effect. After researching the previous model, we notice that there are not many natural ecological systems for food chain model with an Allee effect in predator and supporting chaotic dynamical behavior. This research investigates a three species food chain model with an Allee effect in the second species which is a generalist predator. In this food chain model, the third species preys on the second one and simultaneously the second species preys on the first one. We first find all equilibrium points and conduct linear stability analysis. With different parameters, we prove that all the three species can survive. The third species can extinct and the prey and the middle generalist predator can survive. Using Matlab, we perform lots of numerical simulations for different parameters. This model numerically exhibits rich dynamical properties such as the existence of stable equilibrium, limit cycle, and chaos.

## 204. Evaluating Iodide Concentrations and its Effect on Neurological Development

**Heidi Whiteside** Winston-Salem State University

**Dominique Forbes** Coastal Carolina University

**Advisor(s):** Molly Fenn, North Carolina State University

In this study, a biologically based dose-response (BBDR) model representing iodide concentrations circulating through the lactating dam and nursing pup body compartments is modified to model a dam and fetus scenario. Iodide plays an essential role in developing thyroxine (T4) and triiodothyronine (T3) hormones. These hormones are secreted by the hypothalamic-pituitary-thyroid (HPT) axis and are suggested to impact fetal neurodevelopment. This model analyzes the dynamics of iodide interacting between a dam and fetus. Optimizing parameters and sensitivity analysis allow increased understanding of the factors that influence the development of T3 and T4 hormones due to iodide deficiencies. Further research will evaluate the impact of iodide deficiencies in fetal neurodevelopment.

## 205. Modeling epidemics on cliqued graphs

**Anlin Zhang** Canyon Crest Academy

**Advisor(s):** Laura Schaposnik, University of Illinois at Chicago

Social interactions have been shown to lead to symmetric clusters, and thus we propose here that symmetries play a key role in epidemic modeling. In 2016, a mathematical model on perfect d-ary tree graphs was shown to be particularly effective for simple networks by Claire Seibold and Hannah L. Callender. To account for symmetric relations, we extend their model to certain symmetric networks described by d-cliqued tree graphs, which we introduce by adding edges to the terminal nodes of a regular d-tree to form d-cliques. This setting gives a more realistic model for epidemic outbreaks originating, for instance, within a family or classroom which could reach a population by transmission via children in schools. Specifically, we quantify how an infection starting in a clique (e.g., family) can reach other cliques through the body of the graph (e.g., public places). In particular, we characterize the outbreaks and their dependence

on the structure of the network, including height, degree, and index case location. Finally, we examine the effects of different parameters on the outbreak duration, and propose the study of safe zones, subnetworks of the graph which have negligible probability of infection. Our new results provide a viable mathematical foundation for implementing symmetric constraints in epidemic modeling.

## 206. Restriction and induction in critical groups of group representations

**Ayush Agarwal** Dougherty Valley High School

**Advisor(s):** Christian Gaetz, Massachusetts Institute of Technology

In recent work, Benkart, Klivans, and Reiner defined the critical group of a faithful representation of a finite group, which is analogous to the critical group of a graph. In this paper we study maps between critical groups induced by injective group homomorphisms and in particular the map induced by restriction of the representation to a subgroup. We show that in the abelian group case the critical groups are isomorphic to the critical groups of a certain Cayley graph and that the restriction map corresponds to a graph covering map. We also show that when the group appears in a differential tower of groups, critical groups of certain representations are closely related to words of up-down maps in the associated differential poset. We use this to generalize an explicit formula for the critical group of the permutation representation of the symmetric group, and to give additional results about the number of factors in such critical groups.

## 207. Cluster Algebras and $k$ -positivity Tests

**Anna Brosowsky** Cornell University

**Advisor(s):** Pavlo Pylyavskyy, University of Minnesota, Minneapolis

A  $k$ -positive matrix is a matrix where all minors of order at most  $k$  are positive. Computing all such minors to test for  $k$ -positivity is inefficient, as there are up to  $2^n$  of them in an  $n \times n$  matrix. However, there are minimal  $k$ -positivity tests which only require testing  $n^2$  minors. These minimal tests can be related by exchanging the included minors according to certain rules, forming a family of cluster algebras. This family embeds into the total positivity test cluster algebra described by Fomin and Zelevinsky. We give a description of these sub-cluster algebras which give  $k$ -positivity tests, ways to jump between them using the structure of the larger cluster algebra, and an alternative combinatorial description of many of the tests.

## 208. Rubik's Cube: The Invisible Solve

**Allen Charest** Bridgewater State University

**Advisor(s):** Ward Heilman

The Rubik's Cube is one of the most popular and recognizable puzzles ever created. Even though it's been looked at and analyzed for over thirty years, there are still new things to learn about it. A  $3 \times 3 \times 3$  cube is solved when all six faces have the same colors on them. At first glance, there appears to be only one state in which the cube is solved. However, by labeling the center pieces of the cube (center cubies), one finds that there are different ways for the center cubies to be rotated. These different rotations of the center cubies create invisible solves for the cube that would only be noticeable if the center cubies were labeled. Once these invisible solves were discovered, the next step was to see how many there were and how each of them could be generated. After determining the number and finding a set of operations to generate all of the invisible solves from one, we discovered that the concept of invisible solves could be applied to many different sized cubes including a  $4 \times 4 \times 4$  cube. Each of which may have their own unique groups of invisible solves. In this research, we will be analyzing different sized cubes, their invisible solves and how to generate them.

## 209. The Combinatorics of The Zeckendorf Representation of the Natural Numbers

**Dean Dustin** Plymouth State University

**Paul Lagarde** Mississippi College

**Rachel Chaiser** University of Puget Sound

**Advisor(s):** Tom Edgar, Pacific Lutheran University

In this paper we acknowledge the idea that natural numbers exist regardless of their representation. We explore the properties of a representation that utilizes the Fibonacci sequence as the base, which is called the Zeckendorf representation. We examine the combinatorics arising from the arithmetic of these representations, with a particular emphasis

on understanding the “Zeckendorf tree” that encodes them. We introduce several new results related to the tree, allowing us to develop a partial analog to Kummer’s classical theorem.

## 210. Support Equalities Among Ribbon Schur Functions

**Will Hardt** Carleton College

**Marisa Gaetz** Massachusetts Institute of Technology

**Advisor(s):** Pavlo Pylyavskyy, University of Minnesota

We consider the question of when connected ribbons have the same Schur support. McNamara proved that two skew shapes can have the same Schur support only if they have the same number of  $k \times \ell$  rectangles as subdiagrams (McNamara, 2007). This implies that two connected ribbons can have the same Schur support only if one is obtained by permuting row lengths of the other. We present substantial progress towards classifying when a permutation  $\pi \in S_m$  of row lengths of a connected ribbon  $\alpha$  produces a ribbon  $\alpha_\pi$  with the same Schur support as  $\alpha$ ; when this occurs for all  $\pi \in S_m$ , we say that  $\alpha$  has *full equivalence class*. Our main results include a sufficient condition for a connected ribbon  $\alpha$  to have full equivalence class. Additionally, we prove a separate necessary condition, which we conjecture to be sufficient.

## 211. Set-Valued Young Tableaux and Product-Coproduct Prographs

**Maxwell Krueger** Muhlenberg College

**Ashley Skalsky** University of Minnesota

**Meghan Wren** SUNY Brockport

**Advisor(s):** Paul Drube, Valparaiso University

Standard set-valued Young tableaux are a generalization of standard Young tableaux where cells can contain (un-ordered) sets of integers, with the condition that every integer at position  $(i, j)$  must be smaller than every integer at both  $(i + 1, j)$  and  $(i, j + 1)$ . In this paper, we explore properties of standard set-valued Young tableaux with three-rows and various row-constant cell densities. We show that standard set-valued Young tableaux of rectangular shape and row-constant density  $\rho = (1, k - 1, 1)$  are in bijection with closed  $k$ -ary prographs: planar graphs where all internal vertices may be interpreted as either a  $k$ -ary product or  $k$ -ary coproduct when read from bottom-to-top. That bijection is extended to three-row set-valued Young tableaux of non-rectangular and skew shape, and it is shown that a set-valued analogue of the Schutzenberger involution on our tableaux corresponds to 180-degree rotation of the associated prographs. As a set-valued analogue of the hook-length formula is currently lacking, we also present direct enumerations of three-row set-valued tableaux for a variety of row-constant densities and a small number of columns. We then argue why the resulting integers for density  $\rho = (1, k - 1, 1)$  should be interpreted as a one-parameter generalization of the three-dimensional Catalan numbers that mirrors the generalization of the two-dimensional Catalan numbers provided by the  $k$ -Catalan (Fuss-Catalan) numbers.

## 212. Kirkman Packing Designs

**Mackenzie Maschka** Nebraska Wesleyan University

**Advisor(s):** Austin Mohr, Nebraska Wesleyan University

A Kirkman Packing Design  $KPD(v, n)$  is a collection of  $n$  partitions of the set  $\{1, \dots, v\}$  satisfying the condition that any pair of ground elements appear together in at most one block. In this poster we will discuss an upper bound  $n_{max}$  on the number of partitions for a given number of elements, along with discussing hill-climbing algorithms we developed to construct a  $KPD(v, n)$  for any  $v$  and any  $n \leq n_{max}$ . We will particularly focus on the  $KPD(19, 8)$  since 19 is the smallest  $v$  for which it is unknown if a design on  $n_{max} = 8$  partitions exists.

## 213. Pattern Avoidance and Statistics on Quasi-Stirling Permutations

**Bryan Pennington** University of Texas at Tyler

**Stephanie Slayden** University of Nebraska at Kearney

**Adam Gregory** Western Carolina University

**Advisor(s):** Kassie Archer, University of Texas at Tyler

Pattern avoidance was first developed by Donald Knuth in 1973, while studying stack-sortable permutations. He needed a way of distinguishing which permutations on the set  $S_n = \{1, 2, \dots, n\}$  could be sorted from least-to-greatest using a stack-sorting algorithm. During our research, which was done as part of the REU at the University of



Texas at Tyler, we studied pattern avoidance and statistics on quasi-Stirling permutations, which are permutations on the multi-set  $\{1, 1, 2, 2, \dots, n, n\}$ . Our focus was on avoiding combinations of patterns from  $\mathcal{S}_3$  and analyzing number of plateaus, ascents and descents.

## 214. Coefficients of Gaussian Polynomials Modulo N

**Dylan Pentland**

**Advisor(s):** Younhun Kim, Massachusetts Institute of Technology

Let  $\begin{bmatrix} n \\ k \end{bmatrix}_q$  denote the  $q$ -analogue of the binomial coefficient, given by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{[n]!}{[n-k]![k]!} \text{ where } [n]! = \prod_{i=1}^n (1 - q^i)/(1 - q).$$

Stanley conjectured that the function  $f_{k,R}(n) = \#\{i : [q^i] \begin{bmatrix} n \\ k \end{bmatrix}_q \equiv R \pmod{N}\}$  is quasipolynomial for  $N = 2$ . We generalize, showing that this is in fact true for any integer  $N \in \mathbb{N}$  and determine a quasiperiod  $\pi'_N(k)$  derived from the minimal period  $\pi_N(k)$  of partitions with at most  $k$  parts modulo  $N$ . We also determine the generating function  $F_{k,R}(x) = \sum_{n \geq k} f_{k,R}(n)x^n$  and study cases where it simplifies.

## 215. Counting integers with given number of generalized Fibonacci representations

**Mason Rogers** Stanford University

**Advisor(s):** Erik Bates, Stanford University

Every positive integer can be represented as the sum of distinct Fibonacci numbers. Let  $N(n, k)$  denote the number of integers with  $k$  such representations using the first  $n$  Fibonacci numbers. For any fixed  $k$ , this function is known to be linear in  $n$  for sufficiently large  $n$ . However, when a generalized Fibonacci sequence is used instead, the only known results are for  $k = 1$ . In this paper, we develop a generating function approach to compute  $N(n, k)$  for generalized Fibonacci representations. The method is tractable enough to provide recursive formulas for  $k = 2, 3, 4$  with tribonacci numbers. Our results show that the dependence on  $n$  is not consistent across various  $k$ , a behavior unlike the standard Fibonacci case or the generalized  $k = 1$  case.

## 216. Equivalence classes of binary strings under bit flips

**Anav Sood** Stanford University

**Arianna Serafini** Stanford University

**Advisor(s):** Erik Bates, Stanford University

Fix a binary substring  $R$  of length  $k$ , and define an equivalence relation on binary strings by allowing any instance of  $R$  to be replaced by the substring where each bit is flipped. How many equivalence classes of length  $n$  strings does this relation induce? We show that the answer is the sum of the first  $n + 1$  generalized  $k$ -th order Fibonacci numbers, regardless of  $R$ . When  $R$  equals 1 followed by  $k - 1$  zeros, this result follows from the well-known problem of enumerating strings avoiding  $k - 1$  consecutive zeros. For other  $R$ , the result appears to be new and holds even though the sizes of the equivalence classes vary dramatically with  $R$ . Graph-theoretic consequences and applications to Bernoulli convolutions are also given.

## 217. A Combinatorial Approach to RNA-Inspired Folds

**Stephanie Thrash** St. Edward's University

**Advisor(s):** Mitch Phillipson, St. Edward's University

RNA is a single-stranded molecule whose function depends on how it folds onto itself. Biologically, these foldings are determined by Watson-Crick Base Pairs. We consider a combinatorial abstraction of RNA where the bonding relations are given by a connection graph, instead of Watson-Crick base pairs, and foldings are represented by non-crossing matchings. Using this model, we were able to categorize graphs that guarantee a connected move graph, showing how foldings transform into one another.

## 218. Flow Polytopes of Partitions

**Zoe Wellner** Cornell University

**Connor Simpson** Cornell University

**Advisor(s):** Karola Meszaros, Cornell University

Recent progress on flow polytopes indicates that many interesting families have product formulas for their volumes. These product formulas are all proved using analytic techniques. Our work breaks from this pattern. We define a family of closely related flow polytopes  $F_{(\lambda,a)}$  for each partition shape  $\lambda$  and integral netflow vector  $a \in \mathbf{Z}_{\geq 0}^{n+1}$ . In each family, we prove that there is a “limiting polytope” that is a geometric product of simplices, allowing us to find volume formulas for the polytopes. We also show that all polytopes in each family  $F_{(\lambda,a)}$  have the same combinatorial structure. When  $\lambda = (n, n-1, \dots, 2, 1)$  and  $a = (1, 1, \dots, 1)$ , the results on volume specialize to a result of Morales, Rhoades, and Meszaros, which shows that the combinatorial type of the Tesler polytope is a product of simplices.

## 219. The Maximum Size of Subsets Forbidding Cyclic Arithmetic Progressions

**Kenneth Berglund** Brown University

**Advisor(s):** Peter Johnson, Auburn University

As a possible method of attacking the problem of computing van der Waerden numbers, we consider arithmetic progressions in cyclic groups. The quantity  $b(n, k)$  is defined as the maximum cardinality of a subset  $B$  of the cyclic group of order  $n$  that does not contain any cyclic arithmetic progression. We explore some conditions on  $b(n, k)$  related to the coprimality of  $k$  and possible common differences of cyclic arithmetic progressions in  $\mathbb{Z}_n$ . We end by giving formulas for  $b(n, k)$  for some values of  $n$  and  $k$ .

## 220. Equation of State for Matter at Extreme Conditions

**Robert Argus** George Mason University

**Brandon Sripimonwan** University of California of Northridge

**Olga Dorabiala** Pennsylvania State University

**Advisor(s):** Liam Stanton, Lawrence Livermore National Laboratories

We provide a new statistical model, the Saha-Hypernetted-Chain model, for computing certain state variables of a plasma, including pressure and charge moments. The model is designed to be less computationally expensive than molecular-level simulations but more accurate than existing statistical models. Such a model is of interest to the Lawrence Livermore National Laboratory because the large-scale fluid dynamics computations performed there rely on the calculation of plasma state variables (at every point in a discretization of space) as a fundamental subroutine. Our approach is to combine the Saha model for computing ionization level proportions with the hypernetted-chain (HNC) model for computing pair-correlation functions. This approach addresses the main draw-back of previous statistical methods that use the Saha model: a poor estimation for the shift in ionization energy due to many-body effects. This shift can be more accurately estimated using the pair-correlation functions given by the HNC model.

## 221. Graph Representations of Atomic Structures for Materials Data Analysis

**Montie Avery** University of New Mexico

**Ruth Lopez** California State University, Long Beach

**Miroslav Stankovic** University of Edinburgh

**Advisor(s):** Jonny Dadras, University of California, Los Angeles

Rapid growth of computing power has led to computer simulations that often produce more data than can be analyzed manually. As a result, there is a growing need for automated data analysis techniques. In the field of molecular dynamics (MD), molecular and material systems are simulated by calculating the interactions between tens of thousands of individual atoms. ParSplice is a parallel algorithm for MD developed by Los Alamos National Labs (LANL) which can greatly speed up these simulations. ParSplice relies on the ability to identify the “state” of the system of many atoms based on their configuration. We are exploring methods for interpreting the state of an atomic system by representing it as a mathematical graph, in which atoms are vertices and edges between atoms are drawn according to some nearest-neighbor algorithm. We have investigated both existing algorithms in literature and developed our own modified algorithms. In this poster, we present an overview of the challenges for choosing an algorithm to generate such graphs. We then present our results from testing several nearest-neighbor algorithms on systems generated by MD simulations.

## 222. Withdrawn

## 223. Root Finding with Chebyshev Polynomials in Two Dimensions

**Lucas Bouck** George Mason University

**Advisor(s):** Ian Bell, National Institute of Standards and Technology

Root finding is a commonly encountered problem in numerical analysis. The problem we specifically address is a global root finding problem in two dimensions, which is of interest to engineers for determining thermophysical properties of fluids (Kunick et. al. 2008). We utilize Chebyshev polynomials to approximate the component functions and then find the common roots of the polynomials. This first part of the algorithm is a replication of the root finding algorithm in the library `chebfun2`, which was developed by Alex Townsend (2014). This method is only for rectangular domains, and we also are currently developing and implementing ideas for extending the algorithm to nonrectangular domains.

## 224. Facial Verification

**Surabhi Desai** University of St. Andrews

**Prem Talwai** Cornell University

**Andrew Nguyen** University of California, San Diego

**Islam Faisal** American University in Cairo

**Advisor(s):** Shantanu Joshi, University of California, Los Angeles

GumGum is a company which specializes in computer vision and is currently developing a facial recognition engine to enhance the capabilities of their image recognition software. Our project focused on developing a system for facial verification, to augment the recognition capabilities of GumGum's software. The main aims of the project were to improve precision by using verification to omit high-ranking false positives returned by the recognition engine and thereby increase recall by considering lower ranked true positives. Our presentation outlines various results obtained from several different approaches to the face verification problem. We highlight the following contributions to the project; (i) the development and implementation of a novel metric learning algorithm that exploits the categorical covariance structure of the feature embeddings learned from the trained facial images using a deep convolutional network and (ii) the fusion of the recognition and verification performance by weighted combination of the recognition probabilities and verification scores.

## 225. Topology of Positive Zero Sets of $n$ -variate $(n+4)$ -nomials

**Sabrina Enriquez** University of Southern California

**Davina Boykin** Valparaiso University

**Advisor(s):** J. Maurice Rojas, Texas A&M University

Finding the real roots of sparse polynomials has many applications in algebraic geometry, computer science, and number theory. Recent work has shown that A-discriminants lead to faster algorithms for finding the topology of real zero sets than previous methods from computational algebra. A-discriminants are a recent tool used to keep track of polynomials having degenerate roots. We will use A-discriminants to create faster algorithms for finding real roots of certain polynomials. Specifically, we want to classify the possible topology of real zero sets of polynomials with  $n$  variables and  $(n+4)$  monomial terms. The complement of the resulting A-discriminant surface encodes the topological information we seek. Our work helps automate the process of calculating and understanding more complicated real zero sets.

## 226. An Algorithmic and Computational Approach to Optimizing Gerrymandering

**Kyle Gatesman** Thomas Jefferson High School for Science and Technology

**Advisor(s):** James Unwin, University of Illinois at Chicago

The need for fair and legal partitions of populations into voting districts raises several questions which are interesting from a mathematical perspective and relevant to modern society. Of particular interest are issues related to political *gerrymandering*, the act of constructing voting districts with a specific outcome in mind, typically the reelection of the incumbent party. Here we implement a lattice model of realistic voter distributions. We use quasi-Gaussian random population distributions and model the partisan bias via point sources and an inverse square law. This approach provides a flexible manner of modeling the voter distribution for two-party systems and is ideal for studying problems

of gerrymandering. Using this code, we show that highly aggressive gerrymandering strategies typically lead to disconnected districts which are legally prohibited (specifically, we apply this to the Friedman-Holden model (2008)). Subsequently, we construct an algorithm which aggressively gerrymanders arbitrary voter distributions and outputs equal-population, mostly connected districts. We use these outputs in statistical studies. Specifically, we quantify the impact of gerrymandering on electoral outcomes and argue that aggressively gerrymandered territories will generally fail isoperimetric quotient tests. Hence, our results provide quantitative support for implementing compactness tests in real-world political redistricting.

## 227. Deterministic weight initialization using SVD for MNIST handwritten digit recognition neural network

**Hyemin Gu** Ewha Womans University

**Advisor(s):** June-yub Lee, Department of Mathematics, Ewha Womans University

The objective of this study is to figure out a good initial weight for training a 2 layered neural network aimed at MNIST handwritten digit recognition. Conventionally, initial weights are set up with random values near zero. This made hard to understand where the weights converge. This article is going to introduce a deterministic weight initialization method based on the training dataset. By applying singular value decomposition on the training set, principal components are obtained. These principal components are going to be used for the first layer's weight. Then we find the relevant weight for the second layer. At the end, the performance of 2 neural networks constructed by random weight initialization and the new pc weight initialization is going to be compared.

## 228. Topological Data Analysis on Various Applications

**So Mang Han** St. Olaf College

**Xiaojun Zheng** St. Olaf College

**Advisor(s):** Matthew Wright, St. Olaf College

Topological data analysis (TDA) is a collection of the methods that finds the shape of data. TDA is especially useful to extract information of high dimensional and noisy data, which can be challenging for a geometric approach to analyze the structure of data. Persistent Homology is a main tool for TDA to bridge ideas between geometry and topology. Using RIVET, an interactive visualization software for two-parameter persistent homology, the structure of a network map of Game of Thrones was analyzed. RIVET detects structures of sub-graphs of the final network graph and provides detailed information that users can understand what is happening at each level. Throughout the filtration process, different numbers of structures(ex. connected components and holes) of the map were observed, and it gives a better understanding of the entire picture of data and the contribution of each character on the network map. We also analyzed the structure of a point cloud produced from a semantic analysis of Simple English Wikipedia articles. Specifically, the semantic algorithm converts each article to a 200-dimensional vector. We used the two-parameter persistent homology software RIVET to distinguish the Wikipedia point cloud from a point cloud of similar random vectors. We also compared the topological similarity of Wikipedia articles for major cities using semantic distance and geographic distance between the cities. In addition to analyzing the RIVET plots, we applied statistical tests to the topological differences between the data sets to confirm our conclusions.

## 229. Statistical Analysis of Blokus

**Michael Hanlon** College of the Holy Cross

**Advisor(s):** Eirc Ruggieri, College of the Holy Cross

Blokus is a two to four player competitive board game. The game is played on a 20 by 20 matrix board, and each player is provided with 21 pieces of different size and shape. The goal of the game is to place more of your pieces down than your opponents. The rule to placing pieces is that no pieces of the same player may touch side to side — they can only connect corner to corner. There are several strategies to Blokus. The two that are most prevalent are blocking and snaking. Blocking involves blocking other players from your controlled areas, while snaking involves spreading out across the board as much as possible. Through computer simulation, I plan to develop an optimal winning strategy for the game Blokus. I plan to test strategies with two, three, and four players. In order to do so, I will develop simple AI designed to mimic players with a blocking strategy, snaking strategy, or neither. By simulating hundreds of thousands of games I plan to calculate the winrate of the different strategies with different numbers of players. Additionally, I can calculate the effectiveness of a strategy against another specific strategy. This approach should allow me to develop a complete winning strategy for Blokus.

### 230. Modeling Insect Migration Patterns

**Alec Hutchings** Arizona State University

**Advisor(s):** Sebastian Motsch, Arizona State University

Simulations of migration patterns for insect swarms can be adapted to model the spread of the emergent viruses. We model such behavior using agent based models described as jump processes. Then study the long time behavior of the dynamics through numerical simulations estimating the characteristic speed of propagation of the swarm. Using the partial differential equation associated with the dynamics, we then investigate the dynamics with large number of individuals.

### 231. Modeling Monotone Boolean Functions

**Aubrey Laskowski** University of Illinois Urbana-Champaign

**Advisor(s):** Bertram Ludaescher, University of Illinois at Urbana-Champaign

Monotone Boolean functions are defined as Boolean functions which can be written without using the negation operation. Monotone Boolean functions, or MBFs, have many natural representations aside from the standard expression; acyclic directed graphs, colorings on the hypercube, and truth tables each provide properties of MBFs. The uses of these models have been investigated through research done at the National Center for Supercomputing Applications. I will be presenting a method for counting the number of MBFs of  $n$  variables, known as Dedekind's problem. Dedekind's problem is hard in general, with the number of MBFs growing exponentially. Using the truth table representation of MBFs, I am able to compute up to 7 variables in reasonable time, with  $n=8$  being the current upper bound. I will also give an alternative proof of Sperner's theorem, which states that for every Sperner family  $S$  whose union has a total of  $n$  elements, the size of  $S$  is bounded by  $\binom{n}{\lfloor n/2 \rfloor}$ . An analogue between MBFs and antichains gives rise to a graph-theoretic model of MBFs, which provides the tools to prove Sperner's theorem.

### 232. Comparative Analysis of Cryptographic System Strength

**Jennefer Maldonado** Adelphi University

**Mikaela Merolesi** Adelphi University

**Gerard Boniello** Adelphi University

**Advisor(s):** Salvatore Guinta, Adelphi University

We present a program that imports written works and encrypts and decrypts them using polygraphic substitution via matrix multiplication. This program has statistical capabilities to test the strength of these systems subject to frequency analysis. The strength of the system will be modeled as a function of the block size of the cipher, and the relative benefits of various block sizes will be discussed.

### 233. Three dimensional surface reconstruction from a single portrait based on supervised learning and optimization in Sobolev norm

**Yesom Park** Ewha Womans University

**Yoonkyeong Lee** Ewha Womans University

**Chohee Park** Ewha Womans University

**Seunghui Park** Ewha Womans University

**Advisor(s):** Chohong Min, Ewha Womans University

We introduce a new algorithm that produces a three-dimensional surface reconstruction only from a single frontal portrait. Our algorithm is mainly based on supervised learning and radial-basis-function, which seeks an optimal recovery with respect to Sobolev norm. Given prototypical images as a training set, the Adaboost algorithm is employed to automatically detect various facial expressions. Since the human face can be characterized by a few main features such as eyes and lips, our algorithm effectively reconstructs the surface reconstruction using the main features, the landmarks of a human face. Then, using the classified feature points and a reference surface, the smoothest surface in Sobolev norm is calculated via the polyharmonic radial-basis-function interpolation.

### 234. Studying Natural Calamities Through Mathematical Models

**Ian Russman** Hamline University

**Advisor(s):** Sayonita Ghosh Hajra, Hamline University

The world is in a constant state of turmoil; flowing, sinking, and growing. Predicting and modelling the behavior of this turmoil is a complex task. This paper extends work performed by Ghosh Hajra et. al. in 2017. The model is given by a

system of non-linear partial differential equations (PDEs) that accounts for viscous stress, generalized drag, and virtual mass. The method used for analysis is a common geometric tool; Lie Symmetry analysis. The method revolves around understanding the symmetries inherent in a system of PDEs, reducing the given system of PDEs into possibly a simpler system of PDEs or ODEs and mapping a known solution to a new solution with these inherent symmetries. Finding new solutions will aid in understanding flow dynamics of two phase mass flows in nature, thereby contributing to models that may help to predict and mitigate potential adverse flow dynamics seen in natural calamities (e.g. landslide, ice dam, dam burst). This research presents numerical solutions to a new set of P.D.E's and their physical interpretation. Ghosh Hajra, S., Kandel, S., and Pudasaini, S. (2017). Optimal systems of Lie subalgebras for a two-phase mass flow. *International Journal of Non-linear Mechanics*, 88, 109–121.

### 235. Withdrawn

### 236. Computations over Local Rings in Macaulay2

**Mahrud Sayrafi** University of California, Berkeley

**Advisor(s):** Michael Stillman, Mathematics Department, Cornell University

Local rings are ubiquitous in algebraic geometry and commutative algebra. Not only are they naturally meaningful in a geometric sense, but also they are extremely useful as many problems can be attacked by first reducing to the local case and taking advantage of their nice properties. We present a software package for performing computations over localizations of polynomial rings with respect to prime ideals. The main tools and procedures here involve homological properties, such as the flatness property of localization and the existence of minimal free resolutions for finitely generated modules over local rings, which follows from Nakayama's lemma. The procedures presented here are described as pseudocodes and implemented in Macaulay2, a computer algebra software specializing in algebraic geometry and commutative algebra. The main motivation for this work is enabling mathematicians to computationally study the local properties of algebraic varieties near irreducible components of higher dimension, such as the intersection multiplicity of higher dimensional varieties.

### 237. Nonsmooth Spectral Projected Gradient Methods on Convex Sets

**Vinayak Sharma** University of Washington Bothell

**Alexa McQuiston** University of Portland

**Rebecca Wood** Vanderbilt University

**Advisor(s):** Milagros Loreto, University of Washington Bothell

To solve nonsmooth minimization problems on a convex set, we combine the spectral step length with two sub differential schemes. Using the classical subgradient method and the simplex gradient method results in the Spectral Projected Subgradient (SPS) and the Projected Spectral Simplex Gradient (PSS). These methods use a nonmonotone globalization condition, and do not require estimates of the optimal solution to compute the step length. For the Spectral Projected Subgradient method, a subgradient has to be supplied. On the other hand, the Projected Spectral Simplex Gradient method is a direct search method that only requires function evaluations to build an approximation to the gradient direction. For both algorithms, we present numerical results on a set of nonsmooth test functions. The results indicate the spectral step length can improve the practical performance of both methods.

### 238. Exploration of Numerical Precision in Deep Neural Networks

**Yunkai Zhang** University of California, Santa Barbara

**Yu Ma** University of California, Berkeley

**Catalina vajiac** Saint Mary's College, Notre Dame

**Zhaoqi Li** Macalester College

**Advisor(s):** Susana Serna, Institute for Pure and Applied Mathematics

Reduced numerical precision is a common technique to lower computational cost in various Deep Neural Networks (DNNs). While it has been observed that DNNs are resilient to small errors and noise, there exists no general result capable of predicting the sensitivity to reduced precision for a given DNN system architecture. In this project, we emulate arbitrary bit-width using a specified floating-point representation and truncating the remainder after a certain number of bits. This truncation is applied to the neural network after every batch. We show results on two representative networks, MNIST and CIFAR-10. In these, we explore the impact of several model parameters and their impact on the network's training accuracy. We then present a preliminary theoretical investigation of the error scaling in both forward

and backward propagations. We end with a discussion of the implications of these results as well as the potential for generalization to other network architectures.

### **239. A Mathematical Model of Economic Growth of Two Geographical Regions**

**Xin Zou** College of William and Mary

**Advisor(s):** Junping Shi, College of William and Mary

A mathematical model of coupled differential equations is proposed to model economic growth of two geographical regions (cities, countries, continents) with flow of capital and labor between each other. It is based on two established mathematical models: the neoclassical economic growth model by Robert Solow, and the logistic population growth model. The capital flow, labor exchange and spatial heterogeneity are also incorporated in the system. The model is analyzed via equilibrium and stability analysis, and numerical simulations. It is shown that a strong attraction to the high capital region can lead to unbalanced economic growth even when the two geographical regions are similar. The model can help policy makers to decide whether the region should have an open economy or a more closed one. The results of the model can predict the trend of the trade between regions and provide a new insight into some hotly debated contemporary controversial topics.

### **240. An Intuitive Approach to Solving Ordinary Differential Equations: Utilizing Derivative Formulas**

**Huy Vuong** Pasadena City College

**Advisor(s):** Edward Riley, Pasadena City College

Solving explicitly First Order Ordinary Differential Equations (ODEs) is considered a relatively simple and straightforward task because various methods have been developed. However, most of the said methods require formula memorization, as well as the set-up and evaluation of integrals. These drawbacks not only lead one to expending much time and effort than necessary when attempting to solve a First Order ODEs but also take away the intuitive continuity when the solution to an ODE is reviewed by others. Therefore, one's process of learning to solve differential equations using the current methods can be hindered. In this research paper, the author proposed an alternative method, one based on more fundamental calculus formulas—derivatives of products and quotients, to observe and solve an exact first order ODE more directly, continuously, and intuitively. With some improvisation, this particular method can be used to solve Non-exact First Order ODEs (and any First Order ODEs). This paper also presented some specific applications of this method (certainly with slight variations) into solving higher order ODEs (such as Linear DE, Bernoulli's DE, Euler's DE, etc.).

### **241. Mathematical Billiards and the Search for a Finite Number of Shapes**

**Daniel Camacho** Simpson College

**Addison Grant** Simpson College

**Sara Lawson** Simpson College

**Advisor(s):** Robert Vance, Simpson College

We follow the methods of Henk Don to find a maximum of 39 polygons produced by a billiard flow on an L-shaped table. We use the result on the number of gaps produced by arc exchanges on the circle which generalizes the Three Gap Theorem for Rotations.

### **242. Tracking time-varying parameters in dynamical systems**

**Sergio Chavez** North Carolina State University

**Advisor(s):** Franz Hamilton, North Carolina State University

Mathematical models used in the analysis of physical and biological systems are often parameterized by numerous unknown parameters. Estimation of these parameters is key to making accurate model-based inference and prediction of system dynamics. While parameters are generally assumed to be constant, system non-stationarities often present themselves as drifting parameter values over time. The fundamental problem in tracking these changes is that there is usually no a priori knowledge of the parameter dynamics. Here, we investigate this problem of tracking time-varying parameters within a nonlinear Kalman filtering framework. Specifically, we compare the standard artificial noise evolution method with an approach that represents the parameters as a Fourier series expansion.

### 243. Continuity of Entropy for Piecewise Linear Lorenz Maps

**Zoe Cooperband** California Polytechnic State University, San Luis Obispo

**Matt West** California Polytechnic State University, San Luis Obispo

**Blaine Quackenbush** California Polytechnic State University, San Luis Obispo

**Jordan Rowley** California Polytechnic State University, San Luis Obispo

**Advisor(s):** Erin Pearse, California Polytechnic State University, San Luis Obispo

Dynamical systems are used to model and describe natural as well as technical and industrial processes which evolve over time. Gaining a profound understanding of dynamical systems, in particular of their complexity, helps to comprehend, characterize, and analyze these natural processes. Complexity can be measured in various ways, for instance via entropy, pressure or Lyapunov exponents. An interval map is a map from the unit interval to itself, which is piecewise continuous. Such maps arise naturally within the dynamics of Lorenz systems, which, for example, are used to model problems in hydrodynamics. Thus, there is much interest in understanding the dynamics of these maps; in particular the trajectory of an arbitrary point under repeated applications of the given map. In this project we want to study the entropy of such one dimensional systems and how it changes as one moves through the “Teichmüller space” of such transformations.

### 244. A Dynamical Systems Approach to Climate Modeling

**Cara Donovan** College of the Holy Cross

**Advisor(s):** Gareth Roberts, College of the Holy Cross

The goal of this project is to use low-dimensional mathematical models to better understand current climate, historical climate, and the drivers of climate change. In 1968, Mikhail Budyko developed one of the first Energy Balance Models, a differential equation that expresses the global average surface temperature of the Earth as a function of latitude. Budyko incorporated several relevant climate parameters such as albedo and the greenhouse gas effect. The latitude of interest in trying to model climate conditions of the past and present is that of the ice line. Using Budyko’s model with a new albedo function that incorporates land and the fact that glaciers form at  $-10$  degrees C, we can see how the ice line moves when varying certain parameters. Changes in the parameter values can cause qualitative changes in the equilibrium solutions also known as bifurcations. Climate scientists refer to these as tipping points because they often indicate major shifts in the climate system. So far, we have produced several bifurcation diagrams for different parameters to discern the possibility of extreme climate conditions in both the Neoproterozoic Era and the current climate state. Our model supports the theory of Snowball Earth and a subsequent ice-free state in the Neoproterozoic Era.

### 245. Ambiguous or Non-Generic Critical Portraits of Complex Polynomials

**Simon Harris** University of Alabama at Birmingham

**David George** University of Alabama at Birmingham

**Advisor(s):** John Mayer, University of Alabama at Birmingham

The dynamics of complex polynomials are complicated, and their study is made more manageable through studying less complicated, and more combinatorial, mathematical structures representing families of polynomials with similar dynamical behavior. Families of complex polynomials with similar dynamical behavior can be studied through dynamically equivalent Julia sets. Families of Julia sets can be studied through critical portraits. The penalty paid for compendious notation is that specificity is lost. We introduce a further generalization where families of critical portraits can be represented with bi-colored trees. The nodes of the tree are colored  $F$ , which correspond to regions of the critical portrait containing a fixed point, and  $P$ , which correspond to regions containing an invariant set with a non-zero rotation number. The coloring is weakly bi-colored, meaning no two  $P$  nodes can be adjacent. However,  $F$  nodes may be adjacent. We show how a weakly bi-colored tree represents multiple dynamically equivalent critical portraits up to rotation. We extend this to the case of a critical portrait containing an all-critical polygon. Weakly bi-colored trees correspond to generic critical portraits. Critical portraits with an all-critical polygon are nongeneric but possibly more fundamental. These latter types cannot be represented with weakly bi-colored trees. However, we can represent them as a “multi-colored” tree, where we still have two colors representing fixed and rotational regions, subject to the weakly bi-colored condition, and other “colors” representing the all-critical polygon(s). We will also explore possible counts for these critical portraits with differing numbers, sizes, and depths of all-critical polygons.



### 246. A Novel Algorithm to Compute the Cutting Sequence of Billiard Trajectories in Equilateral Triangles

**Sathwik Karnik** Boston University - PROMYS

**Steven Ma** Boston University - PROMYS

**Eugene Shao** Boston University - PROMYS

**Albert Xu** Boston University - PROMYS

**Advisor(s):** Max Hlavacek, Harvey Mudd College

A cutting sequence is an encoding of a billiard trajectory that corresponds with the sequence of sides hit by a billiard in a polygon. Computing cutting sequences is computationally intensive and time-consuming. In 2015, Davis provided an efficient algorithm that computed cutting sequences of billiard trajectories in the unit square. In this project, we present an efficient algorithm that computes cutting sequences of billiard trajectories in equilateral triangles. We also classify cutting sequences in equilateral triangles and reveal a bijection between cutting sequences in equilateral triangles and cutting sequences in the unit square. Moreover, we present a modified algorithm which generalizes Davis's algorithm to other integrable polygons.

### 247. Exploring Dynamical Systems: Number of Cycles and Cycle Lengths

**Christine Marcotte** Bridgewater State University

**Advisor(s):** Jacqueline Anderson, Bridgewater State University

In this project, we will observe the number of cycles and cycle lengths that are formed by the periodic points in dynamical systems  $f(x) = x^d + c$  over the finite field  $\mathbb{F}_p$ . Some data analysis has suggested which functions would create a cycle of length  $p$  and which functions would create an even or odd number of cycles. As a result, three major theorems have been created and proven to help describe the behavior of these dynamical systems. One theorem states any function of this form will create a cycle of length  $p$  when  $d = 1$  and  $c$  is nonzero. Another theorem describes the mapping of certain functions of this form as a ratio. The last theorem that there are only two possible nonisomorphic graphs of the dynamical system when the function is a bijection and  $d = \frac{p+1}{2}$ . In fact, these two distinct graphs correspond to whether or not the constant  $c$  is a quadratic residue modulo  $p$ .

### 248. Identifying Elliptic Islands in Angled Mushroom Billiards

**Emma Waters** Fairfield University

**Sally Franz** Fairfield University

**Advisor(s):** Mark Demers, Fairfield University

Mathematical billiards are important models of dynamical systems from statistical mechanics. This project concerns a class of billiard tables called mushroom billiards, which exhibit a wide variety of dynamical behavior. We study the dependence of features of the phase space, such as the existence of ergodic components and elliptic islands, on certain parameters which determine the geometry of the billiard table. We present a classification of the parameter space according to stable periodic orbits for an important class of angled mushroom billiards. This research was conducted at Fairfield University during the summer of 2017 with support from the National Science Foundation.

### 249. Sequences of Nested Tetrahedrons

**Rachel Andriunas** Sacred Heart University

**Advisor(s):** Andrew Lazowski, Sacred Heart University

In this thesis, we construct sequences of nested tetrahedrons from a given tetrahedron using an iterative procedure. We are interested in the limiting behavior such sequences. We briefly mention the relevant known results on sequences of nested triangles and generalize to a sequence of nested tetrahedrons.

### 250. Constant Vector Curvature of Skew-Adjoint and Self-Adjoint Canonical Algebraic Curvature Tensors

**James Beveridge** Lewis & Clark College

**Advisor(s):** Corey Dunn, California State University, San Bernardino

This research explored the property of constant vector curvature on model spaces with curvature tensors constructed from self-adjoint and skew-adjoint linear transformations. Constant vector curvature has been studied for general

curvature tensors in the past, but only in the 3-dimensional case. For this reason we look at the constant vector curvature of canonical algebraic curvature tensors constructed from skew-adjoint and self-adjoint linear transformation to try to generalize results in more than three dimensions. We determine the constant vector curvature for all canonical algebraic curvature tensors constructed from skew-adjoint linear transformations and some from self-adjoint linear transformations.

### 251. A Ladder of Curvatures in the Geometry of Surfaces

**Jasmine Camero** California State University, Fullerton

**Oscar Rocha** California State University, Fullerton

**Advisor(s):** Suceava, California State University, Fullerton

In the classical differential geometry of surfaces there are two curvature invariants: the Gaussian curvature (introduced by Gauss in 1827) and the mean curvature (introduced by Sophie Germain in 1831). At the end of the 19th century Casorati investigated another curvature invariant. In the present work we introduce a new curvature invariant, the tangential curvature, and we investigate its fundamental properties.

### 252. Withdrawn

### 253. Best Approximation on Metric Trees

**Ryan Chakmak** Claremont McKenna College

**Advisor(s):** Asuman Aksoy, Claremont McKenna College

When discussing best approximation, we are concerned with its existence and uniqueness. For instance on a compact set, we know that the best approximation always exists. The purpose of this project is to ask these questions in the setting of metric trees, instead of an arbitrary space. The literature has already established the conditions on some functions required for the best approximation to be unique. We plan to dive further into the topic, investigating the connections between Chebyshev sets, metric trees, and best approximation.

### 254. Analyzing Systems of Colliding Tangent Balls

**August Chen** PRIMES Massachusetts Institute of Technology

**Advisor(s):** Jayadev Athreya, University of Washington

In this paper, we analyze a system of colliding tangent balls. This is inspired by mathematical billiards and recent experimental approaches to model collisions of hard balls and disks, which are more realistic models for natural physical systems such as amorphous solid materials or colliding particles. First, we impose some simplifying assumptions in our model, most notably assuming each collision is instantaneous and occurs instantly after the previous one. From here, for the rectangular lattices in  $R^m$ , we find the number of collisions needed to reach the ending state where no more collisions are possible, the final configuration of velocities at the ending state, and the upper bound of the number of collision and its initial configuration of velocities. This is done using a combinatorial argument. Next, we proceed to analyze the hexagonal lattice configuration in  $R^2$  where the set of lines connecting tangent balls lie parallel to the sides of an equilateral triangle. Given an arbitrary initial configuration of velocities and an arbitrary, valid sequence of collisions, we find formulas for the configuration of velocities after these collisions using linear algebra and the theory of permutations.

### 255. Determining the Topologies of Real Zero Sets of Polynomials with Two Variables and Five Terms

**Ashley De Luna** California State Polytechnic University, Pomona

**Christian McRoberts** Morehouse College

**Malachi Alexander** California State University, Monterey Bay

**Advisor(s):** Maurice Rojas, Texas A&M University

A fundamental problem in many applications is determining the topology of the set of real zeros of a polynomial. The A-discriminant is a central tool derived by Gelfand, Kapranov and Zelevinsky that tells us when a polynomial has degenerate roots. Degenerate roots, in turn, determine when the topology of a zero set changes. We explore these topics by creating a MATLAB program that draws A-discriminant curves for polynomials in two variables with five terms. The program finds all discriminant chambers and automatically computes the topology of the zero set of any

bivariate pentanomials. This automatic topology computation for high degree polynomials will be quite useful for the real algebraic geometry community.

## 256. Extensions of Projective Association Schemes

**Russ Haight** Grinnell College

**Advisor(s):** Christopher French, Grinnell College

Hanaki and Miyamoto developed algorithms to find all association schemes of order 1–30, 32, and 34, and encoded the resulting schemes in arrays. These arrays, however, give little insight into the structure of the schemes which they represent. We present a more geometric interpretation for extensions of a certain projective scheme.

## 257. Relations Between Theta Functions in Genus One and Two from Geometry

**Thomas Hill** Utah State University

**Advisor(s):** Andreas Malmendier, Utah State University

Genus-two curves with special symmetries are related to pairs of genus-one curves by two and three-sheeted coverings. This classical work dates back to early 20th century and is known as Jacobi and Hermite reduction. In turn, Jacobians of genus-two curves can be used to construct complex two-dimensional algebraic manifolds known as Kummer surfaces. On the other hand, the defining coordinates and parameters of both elliptic curves and Kummer surfaces can also be understood as Theta functions of genus one and genus two, respectively, a result that goes back to seminal work of Mumford in the 1980s. We use the geometric relation between elliptic curves and Kummer surfaces to derive functional relations between Theta functions of genus one and genus two along Humbert varieties of low discriminant.

## 258. Most Planar Graphs do not Admit Faithful Tropicalizations in the Plane

**Sifan Jiang** Smith College

**Desmond Coles** Ohio State University

**Advisor(s):** Ralph Morrison, Williams College

Let  $k$  be a nonarchimedean field, such as the field of  $p$ -adic numbers. One of the most powerful tools for studying an algebraic curve  $X$  over  $k$  is its *minimal Berkovich skeleton*  $\Gamma(X)$ , a metric graph that encodes geometric information about  $X$ ; for instance, if  $X$  is a curve of genus  $g$ , then  $\Gamma(X)$  is a graph with at most  $g$  independent cycles. To study  $\Gamma(X)$ , we use *tropical geometry*, which transforms  $X$  into a piecewise linear subset of  $\mathbb{R}^n$ , denoted  $\text{Trop}(X)$ . This set contains some subgraph of  $\Gamma(X)$ , and if it contains the entirety of  $\Gamma(X)$ , we call  $\text{Trop}(X)$  *faithful*. In 2015, Baker and Rabinoff proved that every curve has a faithful tropicalization in  $\mathbb{R}^3$ . The next question is: which curves have a faithful tropicalization in  $\mathbb{R}^2$ ? In 2015, Brodsky, Joswig, Morrison and Sturmfels answered this question up to genus  $g = 5$  by determining which graphs with  $g$  loops appear in tropical plane curves. We extend their results to genus 6, including the important case of smooth plane quintic curves, and prove an important restriction on such graphs: as  $g$  goes to infinity, the percentage of trivalent planar graphs with  $g$  loops that appear in a planar faithful tropicalization goes to zero. Our proof uses discrete combinatorial methods, including estimates of the number of triangulations of lattice polygons, combined with previous graph theoretical work.

## 259. The Least-Area Tetrahedral Tile of Space

**Arjun Kakkar** Williams College

**Alejandro Diaz** University of Maryland

**Advisor(s):** Frank Morgan, Williams College

We prove the least-area, unit-volume, tetrahedral tile of Euclidean space, without the assumption of Gallagher et al. that the tiling uses only orientation-preserving images of the tile. The winner remains Sommerville's type 4v.

## 260. Geometries from Groups

**Casey Koch-LaRue** Grand Valley State University

**Advisor(s):** Steven Schlicker, Grand Valley State University

In this poster we present a new technique for geometrically representing the subgroup structure of a finite group. The approach is to create a finite geometry from the set of subgroups with the help of the Hausdorff metric and a notion of betweenness. The points are the subgroups of a finite group and the lines are sets of subgroups of the same finite group.

We have found a complete characterization of the lines in the geometries from finite cyclic groups using the standard generator. We display the statement characterization in the poster and aid the viewers with numerous examples. We also show one way in which the characteristics of families of these geometries are connected to the prime factorizations of the orders of the related groups. Geometries from groups is a research topic that allows for many avenues of further research, and we indicate one direction in which future students of geometries from groups can proceed. This research was conducted as part of the 2017 Student Summer Scholars Program at Grand Valley State University.

### 261. Constant Vector Curvature in 3 Dimensions: A Complete Description

**Andrew Lavengood-Ryan** California State University, San Bernardino

**Advisor(s):** Corey Dunn, California State University, San Bernardino

A relatively new area of interest in differential geometry involves determining if a model space has the properties of constant vector curvature or constant sectional curvature. The natural setting in which to begin studying these properties is in 3-dimensional space. This paper in particular examines these properties in the Lorentzian setting, where all Ricci operators take on one of four Jordan-Normal forms. We determine that three of these four model space families (Ricci operators) possess the property of constant vector curvature, and that under an orthonormal basis, only the diagonalizable family has constant sectional curvature, and that is only when the Ricci Operator has precisely one eigenvalue. By examining these families together, we draw some interesting and unifying conclusions that may be useful for exploring these properties in higher dimensions.

### 262. Line Arrangements: Combinatorics vs. Topology

**Baian Liu** Vassar College

**Advisor(s):** Moshe Cohen, Vassar College

A line arrangement is a finite collection of lines. Given the combinatorial data of a line arrangement, we can study its realizations in  $\mathbb{C}P^2$ . A Zariski pair of complex line arrangements is a pair of isomorphic line arrangements whose realizations in  $\mathbb{C}P^2$  have non-homeomorphic complements. There are no Zariski pairs of arrangements of at most 9 lines. A non-reductive line arrangement is line arrangement in which every line is incident to at least 3 intersection points of high multiplicity. In 1998, Rybnikov discovered the first Zariski pair. Rybnikov's example contains 13 lines and remains the smallest non-reductive Zariski pair. We identify smaller non-reductive line arrangements that are potentially Zariski pairs.

### 263. Heesch Numbers of Modified 3-Dimensional Solids

**Gabriel Lopez** California State University, San Bernardino

**Advisor(s):** Casey Mann, University of Washington

The Heesch number of a given tile,  $T$ , is the number of coronas that can be formed around a centrally placed copy of  $T$  without gaps or overlaps. Previously, there have been very few examples of 3D solids with nonzero finite Heesch number. We found the Heesch number of modified cubes and rhombic dodecahedra with all possible combinations of single interlocking protrusions and indentations on their faces through exhaustive computer search. This search revealed that while many had Heesch number zero or infinite Heesch number, several of these modified solids have finite Heesch number greater than zero. While this search is ongoing, this presentation will discuss the process of the search, and its results so far.

### 264. Lorentzian Geometry on Lie Algebras

**Nicole Marzolf** Longwood University

**Breana Figueroa** Longwood University

**Advisor(s):** Thomas Wears, Longwood University

On a general vector space, any scalar product can be diagonalized by choosing an appropriate basis for the underlying vector space structure. But what happens if one restricts the permissible changes of basis allowed on the vector space? Is it always possible to find a basis that diagonalizes the scalar product? Of course the answer is, "no", but can one still classify scalar products on a vector space using a restricted class of base changes? In our research, we study the classification of Lorentzian scalar products on Lie algebras up to a notion of equivalence determined by a change of basis corresponding to an automorphism of the Lie algebra. By making use of the automorphism group of the Lie

algebra and the geometry of Lorentzian scalar products, we are able to classify Lorentzian scalar products up to the indicated notion of equivalence on a number of finite dimensional Lie algebras.

### 265. Generalization of Inequalities to Non-Euclidean Geometry

**Jacob Naranjo** Kalamazoo College

**Advisor(s):** Ren Guo, Oregon State University

We aim to find analogues in hyperbolic and spherical space for geometric inequalities known in Euclidean space. First we prove a theorem which produces non-Euclidean analogues for inequalities involving a triangle's circumradius, inradius, and side lengths. This theorem is then applied to strengthenings of Euler's inequality  $R \geq 2r$ . We also prove  $\tan R \geq n \tan r$  for an  $n$ -dimensional spherical simplex, generalizing the inequality  $R \geq nr$  known for a Euclidean simplex.

### 266. The Orthocentric Distances

**Sushanth Sathish Kumar** Jefferey Trail Middle School

**Advisor(s):** Bogdan Suceava, California State University, Fullerton

In this poster we showcase a nearly 200 years old lemma and its application to Olympiad problems. The distances were originally discovered by Fagnano in 1770. In this article we will rederive these distances. Later, using power of a point and these distances, we will give an alternative proof of the 9 point circle theorem.

### 267. Degrees of freedom in constructing algebraic and tropical curves in the plane.

**Andrew Scharf** Williams College

**Advisor(s):** Ralph Morrison, Williams College

In algebraic geometry, one of the most important objects of study is  $M_g$ , the space of algebraic curves of genus  $g$ . In 2009, Castryck and Voigt introduced  $M_P$ , the locus inside of  $M_g$  of all curves with a given Newton polygon  $P$ , where  $P$  has  $g$  interior lattice points. This space has dimension at most  $2g + 1$ , although this upper bound is not achieved for all polygons. In 2015, Brodsky et al introduced the space  $\mathbb{M}_P$  as a tropical analog of  $M_P$ ; this polyhedral space parametrizes the metric graphs with  $g$  loops that appear in tropical curves with Newton polygon  $P$ . We prove that  $\dim(M_P) = \dim(\mathbb{M}_P)$  for maximal polygons, positively answering an open question by Brodsky et al. Our proof is constructive, and involves an explicit method of triangulating polygons to give rise to this equality. We also draw on previous work of Koelman (1991) on toric varieties to connect the algebraic and tropical dimensions. We apply our result to completely characterize which polygons  $P$  give the maximal  $2g + 1$  degrees of freedom when constructing a curve, answering another open question of Brodsky et al. The key players here are trigonal curves, curves of genus 6, and sextics with a decreasing number of nodal singularities.

### 268. Blocking Sets of Hyperplane Arrangements in PG(2,q)

**Bradley Scott** California State University, Fresno

**Advisor(s):** Oscar Vega, California State University, Fresno

A blocking set (of lines) in a finite projective plane  $P$  is a set of points that intersects all lines in  $P$ . The theory about blocking sets has been widely developed in the last few decades; these objects have been exhaustively studied. A possible generalization of a blocking set is that of a set of points that blocks arrangements of lines that satisfy a certain pre-determined condition. These new types of blocking sets have not been studied until now except, tangentially, by Settepanella (2009). Moreover, once the search for this new kind of blocking set is extended to higher-dimensional projective spaces, we encounter that the objects to be blocked are hyperplane arrangements (lines are hyperplanes in a plane), now found outside of their natural environment. In this poster we will explore these blocking sets of hyperplane arrangements by using combinatorial and geometric ideas only.

### 269. Double Bubbles on the Real Line with Log-convex Density

**Nat Sothanaphan** Massachusetts Institute of Technology

**Eliot Bongiovanni** Michigan State University

**Advisor(s):** Frank Morgan, Williams College

The classic double bubble theorem says that the least-perimeter way to enclose and separate two prescribed volumes in  $\mathbb{R}^N$  is the standard double bubble. We seek the optimal double bubble in  $\mathbb{R}^N$  with density, which we assume to

be strictly log-convex. For  $N = 1$  we show that the solution is sometimes two contiguous intervals and sometimes three contiguous intervals. In higher dimensions, we think that the solution is sometimes a standard double bubble and sometimes concentric spheres (e.g. for one volume small and the other large).

## 270. Dots on a Ball: What Do They Mean and Where Do They Com From?

**Mouhamad Fares Soufan** Carleton College

**Nupur Bindal** Carleton College

**Advisor(s):** Rob Thompson, Carleton College

The spherical Voronoi region associated to a generating point is the collection of all points on the sphere closer to that generating point than any other generating point. A spherical Voronoi diagram consists of the set of spherical Voronoi regions for a finite set of generating points on the sphere. The boundaries of three or more of these regions intersect at points, called Voronoi vertices. By taking these vertices as a new set of generating points we can generate a new Voronoi diagram, and iterate this process. This is called a Voronoi iteration. We analyze the dynamics of this iteration, count the number of vertices and find periodic patterns in the iteration.

## 271. Projectivity of some Bridgeland moduli spaces on Hirzebruch surfaces

**Talon Stark** University of California, Santa Barbara

**Advisor(s):** Cristian Martinez, University of California, Santa Barbara

There exists a class of Bridgeland stability conditions on the derived categories of Hirzebruch surfaces  $\mathbb{F}_d$  defined by choice of a general and an ample divisor. We prove that several of the moduli spaces arising from these stability conditions are projective. We follow a similar technique to [Arcara, Miles 2015] by finding suitable regions of stability conditions with hearts that are after “rotation” equivalent to the category of representations of a quiver.

## 272. Solving Liouville-type Problems in Calculus by Extending q-energy from Finite to Infinite

**Alice Wu** Borough of Manhattan Community College

**Advisor(s):** Lina Wu, Borough of Manhattan Community College

The study of Liouville-type problems in Differential Geometry is to discover constancy properties for maps between the domain and the target on Riemannian manifolds. Existence of constancy properties is determined by geometric structures on manifolds and energy growth for maps. In this project, we focus on solving Liouville-type problems in Calculus where manifolds are Euclidean spaces restricted with the real number systems and maps become functions. The original work in this project is to explore q-energy growth for functions from finite to infinite by the p-balanced technique. Calculus skills such as Holder Inequality and Tests for Convergence Series has been used to evaluate limits and integrations for functions. Computational methods and energy estimation techniques applied for functions in this project have been successfully generalized as an effective algorithm applied for maps. This successful algorithm has an in-depth impact on the research field of solving Liouville-type problems related with q-energy approaching to infinity.

## 273. Ordered Multiplicity Inverse Eigenvalue Problem on Six Vertices: Attainability

**Diane Christine Alar** San Francisco State University

**Audrey Goodnight** Agnes Scott College

**Cassandra Monroe** Princeton University

**Advisor(s):** Steve Butler, Iowa State University

Given a graph  $G$  with vertices  $v_1, \dots, v_n$ , a real symmetric matrix  $M$  is in the family of matrices  $\mathcal{S}(G)$  given it satisfies  $M_{i,j} = M_{j,i} = 0$  for  $i \neq j$  if and only if  $v_i v_j \notin E(G)$ . The Inverse Eigenvalue Problem of a Graph (IEPG) asks, given a graph  $G$  and a (multi-)set of numbers  $\mathbf{L} = \{\lambda_1, \dots, \lambda_n\}$ , does there exist an  $M \in \mathcal{S}(G)$  with eigenvalues  $\mathbf{L}$ ? We can also construct a relaxation of this question by considering the eigenvalues in increasing order and preserving this order when listing the multiplicity of each value. This creates an ordered multiplicity list, and so we ask similarly if there exists a matrix  $M \in \mathcal{S}(G)$  for a given graph  $G$  that achieves a specified multiplicity list. We solve the ordered multiplicity list problem for connected graphs on six vertices and the IEPG for most connected graphs on six vertices by demonstrating ways to determine if certain lists are attainable for a given graph.

## 274. Increasingly Grim Results

**Jagdeep Basi** California State University, Fresno

**Sam Barretto** California State University, Fresno

**Advisor(s):** Oscar Vega, California State University, Fresno

Grim is a deletion game played on the vertices of a graph where players take turns selecting a vertex to remove until all are gone. The winner of this game is the last player to make a legal move. This project is part of an on-going effort to find strategies for Grim using defining characteristics of different graphs; we are currently focused on complete multi-partite graphs having at least two partitions containing a single vertex. These types of graphs feature a reasonable pattern that is unexpectedly interrupted by special cases that become apparent only as the number of partitions in the graphs increases. This poster focuses on identifying this particular pattern, and analyzing the effects it has on our working notion of games played on complete multi-partite graphs. The techniques involved in this study include combinatorial game theory and graph theory in addition to an exhausting use of induction.

## 275. Zero-forcing in Generalized Petersen Graphs

**Marcus Cisneros** Texas State University

**Enrique Gomez-Leos** Texas State University

**Sarah Gibbons** Texas State University

**Nick Montana Taylor Baumgard**

**Advisor(s): Daniela Ferrero, Texas State University**

Zero-forcing is a graph process introduced independently in linear algebra, quantum physics and electrical engineering, between the years 2002 and 2008. Due to its broad range of applications, it has attracted the interest of mathematicians, computer scientists, theoretical physicists and engineers. A graph consists of a set of vertices together with some edges joining pairs of points. Assume the vertices of the graph are colored in blue and white. Iteratively apply the following color changing rule until it does not change the color of any vertex: if a blue vertex has exactly one white neighbor, its white neighbor turns blue. At the end, if all vertices are blue, the initial set of blue vertices is a zero-forcing set. The minimum number of vertices in a zero-forcing set is the zero-forcing number of the graph. Graphs are ubiquitous network models, and their zero forcing numbers provide valuable network information. However, determining the forcing number of an arbitrary graph results in an NP-complete problem, so there is no efficient algorithm to compute it. Therefore, it is important to find families of graphs for which there is a formula for their zero-forcing number. In this work, we construct zero-forcing sets in Generalized Petersen Graphs (GPG), prove they are minimal, and derive a formula for the zero-forcing number. In addition, we open up avenues for further research, since GPG are in the intersection of several important classes of graphs to which our results could be extended.

## 276. Network Security: Graphlet Counting and Degree Distribution

**Jian Cui** Gordon College

**Advisor(s):** Mike Veatch, Gordon College

Cybersecurity is very important to our daily life because networks are a huge part of our life. This project aims to further explore the network system and anomaly analysis. We mainly use two methods to detect the anomalies. The data we used is from Czech Technical University in Prague. The first method is graphlet counting: we divided the datasets into multiple time windows and counted the graphlets over each time window. After this, we use Kolmogorov-Smirnov test to detect anomalies. The second method is to analyze the distribution of the degree of nodes on the graph and use the Chi-Square test to detect anomalies. In the end, we compare the two methods and analyze the methods of detecting anomalies in general.

## 277. (Duplicate)

## 278. State Transfer on Graphs Through Magnetic Potentials

**Or Eisenberg** Harvard University

**Advisor(s):** Mark Kempton, Harvard University

Inspired by problems in quantum computational theory, the theory of perfect and near-perfect state transfer addresses questions regarding the transfer of information through quantum computational networks evolving through time. Specifically, they address the question of when a graph  $G$  with adjacency matrix  $A$  satisfies the condition that

$\langle u | e^{iTA} | v \rangle = 1$  for some vertices  $u$  and  $v$  and some time  $T \neq 0$ , or where there exists some sequence  $\{t_j\}$  of times such that  $\langle u | e^{it_j A} | v \rangle$  converges to 1. I hope to present on the area of state transfer in general but to focus on a new result which I and my coauthors established, namely, that there exist magnetic potentials (i.e., an assignment of a weighted real valued loop to each vertex) on any graph which has an equitable partition with a part of order 2 inducing state transfer between the element of said part. If space permits, I'd also hope to discuss some basic results we established on graph quotients (with a focus on "handle attachments") as they relate to state transfer and on the poset structure of the set of equitable partitions on a graph as they relate to state transfer.

### 279. Graphs with at most two trees in a forest building process

**Misa Hamanaka** Iowa State University

**Advisor(s):** Steve Butler, Iowa State University

Given a graph, we construct a spanning forest by taking all of the edges of the graph and temporarily pulling them out; sorting them in some fashion; and then going through them in order and only putting those edges back in which we see some vertex not previously seen by an edge. The resulting forest is dependent on the ordering of the edges, and so we can ask, for example, how likely is it for the process to produce a forest with  $k$  trees. We look at all graphs which can have at most two trees in this process and determine the probabilities of having either one or two trees. From this we construct infinitely many pairs of distinct graphs which have the same probabilities.

### 280. Withdrawn

### 281. The Hardness of Finding Hamiltonian Cycle in Grids Graphs of Semiregular Tessellations

**Kaiying Hou** Phillips Academy

**Advisor(s):** Jayson Lynch, Massachusetts Institute of Technology

The Hamiltonian cycle problem is an important problem in graph theory and computer science. It is a special case of the famous traveling salesman problem. A significant amount of research has been done on the special cases of finding Hamiltonian cycles in subgraphs of triangular, square and hexagonal grids. However, there is little work on more complicated grids. In this paper, we investigate the hardness of Hamiltonian cycle problem in grid graphs of semiregular tessellations, which are tessellations formed by two or more kinds of regular polygons. There are only eight semiregular tessellations, and we prove that the Hamiltonian cycle problem in all of them are NP-complete by reducing from NP-complete problems, such as the Hamiltonian cycle problem in max degree 3 bipartite planar graphs. Knowing the NP-completeness of Hamiltonian cycle problem in semiregular grids indicates that there will not be any polynomial time algorithm that solves the Hamiltonian path problem in these tessellations if NP does not equal to P, which helps show the limits of efficient motion planning algorithms and provides new information about what makes problems computationally difficult to solve.

### 282. Classification and Characterization of Networks

**Emma Ingram** The University of Alabama

**Adriana Ortiz Aquino** University of Puerto Rico, Rio Piedras

**James Canning** SUNY Geneseo

**Advisor(s):** Karl Schmitt, Valparaiso University

Networks are often labeled according to the underlying phenomena that they represent, such as re-tweets, protein interactions, or web page links. Our research seeks to use machine learning techniques to gain a better understanding of the categories of networks on the Network Repository ([www.networkrepository.com](http://www.networkrepository.com)) and then classify unlabeled networks into categories that make sense. It is generally believed that networks from different categories have inherently unique network characteristics. Our research provides conclusive evidence to validate this belief by presenting the results of global network clustering and classification into common categories using machine learning algorithms. The machine learning techniques of Decisions Trees, Random Forests, Linear Support Vector Classification and Gaussian Naive Bayes were applied to a 14-feature "identifying vector" for each graph. During cross-validation, the best technique, Gaussian Naive Bayes, achieved an accuracy of 92.8%. After training the machine learning algorithm it was applied to a collection of initially unlabeled graphs from the Network Repository. Results were then manually checked by determining (when possible) original sources for these graphs. Finally, we examined the accuracy of our results and discussed how future researchers can make use of this process.



### 283. Characterization of $(P, Q)$ connections in bump graphs

**Andrew Krenz** Purdue University

**Advisor(s):** Jim Morrow, University of Washington

The behavior of a resistor network is intuitively related to the network's connective properties. This relationship is generally studied by modeling resistor networks as weighted circular planar graphs where the edge weights represent conductance. In this setting, it is known that determinants of minors in the response matrix of the system completely determine the totality of disjoint paths one can find through the graph's vertices (i.e., the connective properties). We investigate the connective properties of circular planar graphs without the use of determinants by taking advantage of a construction arising from permutations. We found that by restricting the allowable inversions of permutations, the graphs subsequently constructed will behave predictably. Moreover, via interplay between the Catalan numbers and the permutations used, the connective properties of the circular planar graphs created may be realized in a geometric sense. In the future we hope to use this construction, or variations on the construction, in order to describe a more general set of graphs.

### 284. New Family of Edge-Isoperimetric Graphs

**Nikola Kuzmanovski** University of Wisconsin-Superior

**Pavle Bulatovic** University of Wisconsin-Superior

**Advisor(s):** Sergei Bezrukov, University of Wisconsin-Superior

In this talk, we present new infinite families of regular graphs for which all cartesian powers admit nested solutions in the edge-isoperimetric problem. These results can be applied to the bisection width or wirelength problems. For a given graph the problem is to specify a subgraph of a given order  $m$  that has max number  $I(m)$  of induced edges among all subgraphs of order  $m$ . Our results include as special cases most previously published results in this area. The graphs are specified by delta-sequences of the length given by the number of vertices in the graph. The  $m$ -th element of the sequence is  $d(m) = I(m) - I(m - 1)$ . It is known that  $d(m + 1)$  does not exceed  $d(m) + 1$ . We emphasize on delta-sequences that have several monotonically increasing segments of the same length, for example, 0, 1, 2, 2, 3, 4, 4, 5, 6 for a sequence with 3 segments of length 3 each. We show that by ordering the vertices of the  $n$ th cartesian power of our graphs lexicographically, the subgraph induced by any initial segment of this order spans the max number of edges. Previously such results were only known for graphs/sequences with just 2 monotonic segments. Based on a special representation of graphs as a union of disjoint cliques we introduce a new technique for extending a graph admitting nested solutions in the edge-isoperimetric problem.

### 285. Withdrawn

### 286. Solving 2-by-2 Scramble Squares Puzzles with Repetitions

**Maria Mota** St. Edward's University

**Advisor(s):** Jason Callahan, St. Edward's University

A Scramble Squares puzzle consists of nine square pieces with half of an image on each edge. To solve the puzzle the nine pieces are arranged in a 3-by-3 grid so that edges on adjacent pieces form a complete image. A repetition is a half-image that appears more than once on a puzzle piece. Previous research uses a graph-theoretical approach to establish necessary and sufficient conditions for solutions without repetitions to 2-by-2 Scramble Squares puzzles. We use a similar approach to establish necessary and sufficient conditions for solutions with repetitions to 2-by-2 Scramble Squares puzzles.

### 287. Graphs of Small Zero-Diagonal Minimum Rank

**Jake Mundo** Swarthmore College

**Advisor(s):** Cheryl Grood, Swarthmore College

The zero-diagonal minimum rank of a graph is defined as the minimum rank of a symmetric zero-diagonal matrix whose zero-nonzero pattern is given by the edges of the graph. We establish a method to characterize the graphs of zero-diagonal minimum rank less than or equal to a given positive integer. In particular, we provide an explicit characterization of the graphs with minimum rank less than or equal to 4 over the reals. We also characterize the complete graphs having minimum rank 4 and 5 over finite fields of odd order, and determine the minimum ranks of several graph families over the reals.

## 288. Radio Labeling of Square Cycle Graphs

**Joel Salazar** California State University, San Bernardino

**Advisor(s):** Min-Lin Lo, California State University, San Bernardino

Let  $G$  be a connected graph. The *distance* between two vertices  $u$  and  $v$  in  $G$  is defined by the length of the shortest path in  $G$  between  $u$  and  $v$ , which is denoted by  $d_G(u, v)$ . The *diameter* of  $G$ , denoted by  $\text{diam}(G)$ , is the maximum distance between two vertices in  $G$ . The *radio labeling* of  $G$  is a function  $f$  that assigns each vertex a non-negative integer such that  $|f(u) - f(v)| \geq \text{diam}(G) - d_G(u, v) + 1$  holds for any two distinct vertices  $u$  and  $v$  of  $G$ . The *span* of  $f$  is the difference of the largest and the smallest channels used. The *radio number* of  $G$ , denoted by  $\text{rn}(G)$ , is defined as the minimum span of all radio labelings of  $G$ .  $f$  is said to be an *optimal radio labeling* of  $G$  if  $\text{span } f = \text{rn}(G)$ . A *cycle* with  $n$  vertices, denoted by  $C_n$ , is a graph whose vertex set can be reordered as  $\{v_1, v_2, \dots, v_n\}$  such that  $E(C_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$ . The *square* of a graph  $G$  has the same vertex set as  $G$ , but the edge set is now  $E(G^2) = E(G) \cup \{uv : d_G(u, v) = 2\}$ . In this paper, we found an upper bound for  $\text{rn}(C_n^2)$ , where  $n = 4k + 3$  with  $k = 4m + 3$ , when  $m \equiv 1 \pmod{3}$ .

## 289. Intersection Graphs of Maximal Convex Sub-Polygons of $k$ -Lizards

**Kyle Salois** Willamette University

**Rebecca Robinson** University of Michigan

**Caroline Daugherty** Kenyon College

**Advisor(s):** Joshua Laison, Willamette University

A  $k$ -lizard is a simply connected polygon with sides parallel to a regular  $2k$ -gon. For a  $k$ -lizard  $P$ , let  $S$  be the set of all maximal sub-polygons contained in  $P$ . A graph  $G$  is a  $k$ -maximal sub-polygon graph (or  $k$ -MSP graph) if there exists a  $k$ -lizard  $P$  and a one-to-one correspondence between vertices of  $G$  and polygons in  $S$  such that two vertices are adjacent in  $G$  if and only if their corresponding polygons in  $S$  intersect. We find separating examples of graphs that are  $k$ -MSP graphs but not  $j$ -MSP graphs for  $j \neq k$ .

## 290. Reconfiguration graphs of prime labelings

**Robert Scholle** La Salle University

**Advisor(s):** Janet Fierson, La Salle University

This research applies the concept of reconfiguration graphs to prime labeling. In prime labeling, first introduced by Tout, Dabboucy and Howalla in 1982, vertices are labelled with distinct positive integers less than or equal to the number of vertices in the graph such that the labels of adjacent vertices are relatively prime. The reconfiguration graph,  $R(G)$ , of a prime graph,  $G$ , is constructed by creating a distinct vertex for each possible prime labeling of  $G$ , and placing an edge between two vertices of  $R(G)$  if the associated prime labelings of  $G$  differ in only 2 labels. We present results on which graphs may and may not appear as reconfiguration graphs, and we also consider subgraphs of reconfiguration graphs. In addition, we discuss findings on properties of reconfiguration graphs such as order, girth, and connectedness.

## 291. Withdrawn

## 292. Enumerating Diagonalizable Matrices over $\mathbb{Z}_p^k$ Using Complete Graphs

**William Sheppard** Bowdoin College

**Heewon Hah** Louisiana State University

**Advisor(s):** Brian Sittinger, California State University at Channel Islands

Matrix diagonalization is a well-studied topic in linear algebra when the entries of the matrices being considered are elements of a field (usually  $\mathbb{R}$  and  $\mathbb{C}$ ). Kaylor has explored matrix diagonalization over the field  $\mathbb{Z}_p$  for  $p$  prime, and Avni partially classified similarity classes of square matrices over local principal ideal rings. We generalize these results to prove that all diagonalizable matrices in  $M_n(\mathbb{Z}_p^k)$  have a unique diagonalization up to ordering of the diagonal entries. We also enumerate all diagonal matrices in  $M_n(\mathbb{Z}_p^k)$  for natural numbers  $n, k$ . For  $n > 3$ , we take a graph theoretic approach, which we demonstrate for the cases  $n = 4, 5$ .

### 293. Two-Terminal Reliability of Graphs with Terminal Distance 3 or 4

**Kassandra Short** Benedictine College

**Chloe Lewis** Northern Michigan University

**Allison Howard** Florida Southern College

**Advisor(s):** Christina Graves, University of Texas at Tyler

The two-terminal reliability polynomial of a graph is the probability that a source vertex  $s$  is connected to a destination vertex  $t$  when each edge is included with probability  $p$ . A graph on a fixed number  $n$  vertices and  $m$  edges is uniformly most reliable if it has a two-terminal reliability polynomial greater than that of all other graphs with the same  $n$  and  $m$  for all  $p \in [0, 1]$ . This poster identifies values of  $n$  and small  $m$  for which no uniformly most reliable graph exists when the distance from  $s$  to  $t$  is greater than or equal to 3. Additionally, the existence of a uniformly most reliable graph on specific values of  $n$  and large  $m$  is shown for graphs of distance 3.

### 294. Modeling DNA Self-Assembly Using Graph Theory

**Quinn Stratton** Lewis University

**Keller Dellinger** Lewis University

**Simon Merheb** Lewis University

**Advisor(s):** Amanda Harsy, Lewis University

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate level graph theory have resulted in significantly increased efficiency. One recent focus in DNA nanotechnology is the formation of nanotubes using lattice structures. These nanotubes are thought to have wide-ranging potential, such as containers for the transport and release of nano-cargos, templates for the controlled growth of nano-objects, and in drug-delivery methods. The rules governing the structure of these nanotubes are not yet well understood, and this naturally offers open problems in the realm of applied graph theory. In this poster, we present the results of a semester-long undergraduate research project applying graph theoretical techniques to nanotubes and other complexes which can be created from self-assembling DNA.

### 295. Anti-Ramsey Multiplicities

**Yunus Tuncbilek** Yale University

**Advisor(s):** Michael Young, Iowa State University

For a graph  $G$ ,  $M(G, n)$  is the minimum number of monochromatic copies of  $G$  in any coloring of  $K_n$  and is called *the Ramsey multiplicity*. A graph  $G$  with  $m$  vertices and  $e$  edges is called *common* if

$$\lim_{n \rightarrow \infty} \frac{M_r(G; n)}{\binom{n}{m} \frac{m!}{|Aut(G)|}} = 2^{1-e},$$

i.e., the proportion of monochromatic copies of  $G$  is asymptotically minimized by the random coloring. This paper discusses the *Anti-Ramsey Multiplicity*  $rb(G, n, r)$ , which is defined as the maximum number of *rainbow* copies of a graph  $G$  in any  $r$ -edge-coloring of  $K_n$ . A graph has *property*  $P_r$  if the proportion of rainbow copies of  $G$  in  $K_n$  with  $r$  colors is asymptotically maximized by the random coloring. We show that matchings, stars and disjoint stars have property  $P_r$  for all  $r$ . We prove that for any  $c \in \mathbb{R}$  such that  $c + (1 - c) \log(1 - c) \geq \frac{2}{m-1}$ , if  $e \geq c \binom{m}{2}$ , then  $G$  does not have  $P_{\binom{m}{2}}$ . We also show that  $C_4$  (cycle with 4 vertices) does not have  $P_4$ .

### 296. Counting Arithmetical Structures of Graphs

**Joshua Wagner** Gettysburg College

**Advisor(s):** Darrren Glass, Gettysburg College

Arithmetical structures are created by labeling the vertices of a graph so that each vertex is a divisor of the sum of its neighbors. The question is, given a specific graph, how many arithmetical structures does it have and what can you say about their structure. It was proven by Dino Lorenzini that any graph has a finite number of arithmetical structures, and there already exists an algorithm for computing all arithmetical structures for both paths (non-branching, non-cyclic graphs) and cycles (non-branching graphs). In this poster, we look at more complex graphs and find algorithms and

patterns for computing each graph's arithmetical structures. More complex graphs include those with branching and also those with multiple connections between the same points. In particular, graphs with a single vertex of degree three will be analyzed.

### 297. Packing Tree Degree Sequences

**Zhangyang Wei** Boston College

**Advisor(s):** Joshua Greene, Boston College

The research project looks at the edge-disjoint realization problem in graph theory. Erdős and Gallai gave a necessary and sufficient condition for a sequence to be the degree sequence of a simple graph in 1960. This motivated many graph theorists to aim at packing a finite number of degree sequences, that is, the existence of a simple graph with edge-disjoint subgraphs each realizing a given degree sequence. Among them, Kundu gave a sufficient condition for two tree degree sequences having an edge-disjoint realization. In this research project, we survey a new class of graphs called caterpillars and improve Kundu's conclusion by giving a necessary and sufficient condition for the same problem. We also give a sufficient condition for three tree degree sequences having an edge-disjoint realization, and we hope to generalize our conclusion to an arbitrary number of sequences.

### 298. Rendezvous Search on the Edges of Platonic Solids

**Elanor West** Johns Hopkins University

**Xiao Xie** Johns Hopkins University

**Hayden Helm** Johns Hopkins University

**Advisor(s):** John Wierman, Johns Hopkins University

The classic Rendezvous Search problem involves two players moving along the same line at random until they meet. We were inspired by the Astronaut Problem rendition in which two players are on a sphere moving until they meet. We have simplified the model to discrete units of time and to take place along the edges of platonic solids. We assume the search ends when the two players can see each other. We have compared the mean times to end on all five solids, and have altered assumptions and strategies in various versions to see how certain changes affect the mean time to end. Some variations we have tried are: simultaneous versus sequential versions, varying lines of sight, multi-step strategies, and additional edges and nodes. Most of these variations we have explored more closely on either the dodecahedron, icosahedron, or both, since they are closer approximations to a sphere than the octahedron, cube, or tetrahedron.

### 299. Metrics on Graphs: Refinement on Erdős Numbers

**Alexander Wittmond** California State University, Dominguez Hills

**Kayla Locke** California State University, Dominguez Hills

**Advisor(s):** Wai Pong, California State University, Dominguez Hills

The Erdős number of a person is the shortest path distance between the person and Paul Erdős in the collaboration graph where there is an edge between two people if they coauthored a paper. Shortest path distance is easy to compute, however, one may argue that it is an inadequate measure of "closeness" between two authors since, for example, it does not take into account the number of joint papers. In this project, we explore some known metrics (e.g., resistance distance) on weighted graphs and devise some new ones which leads to various versions of Erdős number that refines the measure of "closeness" between authors than the one classical one.

### 300. The MIGHTY-est Chicken: A Graphical Investigation of Chicken Pecking Orders

**Mikaela Wyatt** Grand Valley State University

**Ellen Grove** Grand Valley State University

**Morgan Oneka** Grand Valley State University

**Samantha Wyatt** Grand Valley State University

**Advisor(s):** Lauren Keough, Grand Valley State University

Stephen B. Maurer's essay The King Chicken Theorems is a prominent publication in graph theory that relies heavily on the use of directed graphs to describe the pecking interactions between chickens in a random coop. One of Maurer's principle endeavors was to derive a method to determine the most dominant chicken, or chickens, in a coop—regardless of its size. Our own exploration of this topic led us to define new classes of "chickens" such as the "Senator Chicken"

the “Boss Chicken” and the “Most Efficient Chicken” which were represented by vertices with given attributes in a directed graph. After studying the behavior of a variety of different graphs, we developed theorems about these different classes of “chicken” that revealed a number of interesting properties about directed complete graphs.

### 301. Even-Odd Laplacians and counting walks on graphs

**Chengzheng Yu** University of Illinois at Urbana-Champaign

**Advisor(s):** Ivan Contreras, University of Illinois at Urbana-Champaign

In graph theory, the adjacency matrix and its powers count the number of walks from vertex to vertex of a fixed length. When it comes to the Laplacian matrix and its powers, we define two new types of walks, denoted by “super-walks” and “edge super-walks”, which can be counted via two variations of the conventional Laplacian matrix. This construction is motivated by a discrete formulation of quantum mechanics and thermodynamics in the case of finite graphs, and it can be naturally extended to the case of higher dimensional objects, such as hypergraphs of topological nature (CW-hypergraphs).

### 302. Analyzing the Impact of Mastery-based Testing in Mathematics Courses

**Christina Carlson** Lewis University

**Lauren Klamerus** Lewis University

**Advisor(s):** Amanda Harsy, Lewis University

As educators, it is important to recognize that our assessment methods affect student attitudes. If we want students to learn from their mistakes and counteract a fixed-mindset of learning, perhaps we should look at what we incentivize in the classroom. One way that professors are attempting to counteract math anxiety, poor STEM retention, and a fixed-mindset of learning is through using and researching a new assessment model called “Mastery-based Testing” (MBT). In MBT, students are given problems in which they can only receive full credit for the problem after they demonstrate mastery of the concept being tested. Each test includes similar questions over the same concepts from previous tests which allows students who have not mastered an idea to retest and reevaluate old concepts. In order to help determine the effectiveness of Mastery-based Testing, we used statistical methods to analyze data collected from calculus and linear algebra MBT classes. Join us as we present the results of our findings on the impact of Mastery-based Testing in these math courses.

### 303. Volleyball: The Mathematical Sport?

**Lauren Hall** Saint Joseph’s University

**Advisor(s):** Tetyana Berezovski, Saint Joseph’s University

For this project, we created a mathematical model for defensive passing in volleyball. We analyzed this model for its applications and developed a sport-based mathematical curriculum that is appropriate to various high school subjects. Mathematical and physical tasks were designed in the context of the volleyball court, the volleyball, the bodily movements of the player, and the winning strategy of the match. The designed tasks were carefully aligned with the CCSS\_M and illustrate different levels of cognitive demand. These mathematical activities are appropriate for mathematics education at the secondary level. Sporthematics, a field where mathematics meets sports, sparks a greater curiosity from a broader range of students into understanding of mathematical content.

### 304. Using Lasers to Shine Light on Inverse Trigonometric Functions

**David Lary** California State University, Channel Islands

**Advisor(s):** Jennifer Clinkenbeard, California State University, Channel Islands

Students in high school mathematics in California are required to “Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context” (CCSS.MATH.CONTENT.HSF.TF.B.7). However, many high school students struggle with the concept of inverse trigonometric functions (and indeed, with the relationship between a function and its inverse in general). We present an innovative, kinesthetic approach to teaching this content using inexpensive laser pointers, boxes, and protractors. This approach is suitable for use in a typical high school classroom. We also present student feedback, which indicated enjoyment of the activity and retention of the content.

### 305. Examining Student Understanding of Domain and Range in Multivariable Calculus with a Visualization Tool

**Bridgette Scott** Robert Morris University

**Advisor(s):** Monica VanDieren, Robert Morris University

The purpose of this research is to look at students' understanding of domain and range of single and multivariable functions with the aid of the visualization tool CalcPlot3D (see [web.monroecc.edu/calcNSF/](http://web.monroecc.edu/calcNSF/)). This research extends Allison Dorko and Eric Weber's study on Generalizing Domain and Range from Single-Variable to Multivariable Functions (2014) by incorporating a graphical component. Through a series of task-based interviews, we examine how students think about domain and range in three different ways: inputs and outputs, independent and dependent quantities, and particular variables (Dorko & Weber, 2014). Additionally we investigate the generalizations that students create when transitioning from single variable calculus to multivariable calculus using Ellis' (2007) taxonomy and Lobato's (2003) actor-oriented transfer framework. We are interested to see how visualization with CalcPlot3D may impact the ways students generalize from two dimensions and how they understand domain and range in three dimensions. This project is funded in part by NSF-IUSE 1523786.

### 306. Sums of $k$ th Powers in $p$ -Adic Rings

**Lukas Anthony** McDaniel College

**Maia Hanlon** McDaniel College

**Advisor(s):** Spencer Hamblen, McDaniel College

Generalization of Waring's Problem—that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers—have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We will examine values of  $g(2)$  and  $g(3)$  for various  $p$ -adic rings and an  $ell$ -adic extension.

### 307. Generalizations of Waring's Problem in Quaternion Rings

**Timothy Banks** McDaniel College

**Advisor(s):** Spencer Hamblen, McDaniel College

Generalizations of Waring's Problem—that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers—have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We extend current results on  $g(2)$  for Quaternion rings with integer coefficients using a result of Watson on integral quadratic forms.

### 308. Withdrawn

### 309. On the Maximal Number of Roots of a Trinomial Over a Prime Field

**Jeshu Dastidar** Queens College, CUNY

**Viviana Peña Márquez** Konrad Lorenz Fundación Universitaria

**Ryan Pugh** California State University, Monterey Bay

**Advisor(s):** Federico Ardila, San Francisco State University

Canetti, Friedlander, et al. (2002) studied the randomness of powers over finite fields and along the way derived an analogue of Descartes' rule over the finite field  $\mathbb{F}_q$  with  $q$  elements: They showed that the number of roots of any univariate  $t$ -nomial, with exponents  $\{0, a_2, \dots, a_t\}$  and the differences  $a_i - a_j$  all relatively prime to  $q - 1$ , is  $O(q^{(t-2)/(t-1)})$ . The correct optimal bounds remain a mystery for prime fields, even in the case of polynomials with three terms. Following the work of Kelley (2016), we seek to prove the conjecture that the number of roots in  $\mathbb{F}_p$  of a trinomial with a linear middle term is always  $O(\log p)$ . We expand current evidence by using a supercomputer to determine the number of roots of these trinomials for  $139,571 < p \leq 191,491$ . We also prove that the search can be restricted to trinomials with a middle linear term when  $p - 1$  has less than three distinct prime factors.

### 310. Norm-Euclidean Ideal Classes in Galois Cubic Fields

**Kelly Emmrich** University of Wisconsin-La Crosse

**Clark Lyons** University of California-Berkeley

**Advisor(s):** Kevin McGown, California State University

Lenstra introduced the notion of a norm-Euclidean ideal class as a generalization of norm-Euclideanity of a number field. He classified all quadratic number fields possessing a norm-Euclidean ideal class. We investigate the Galois cubic case. We show that up to discriminant  $10^{11}$  at most two such number fields possess a nontrivial norm-Euclidean ideal class, and we conjecture no more exist. In an attempt to settle our conjecture, we prove explicit bounds on the first few non-residues of cubic characters under the generalized Riemann hypothesis.

### 311. Withdrawn

### 312. Infinite Trees of Primitive Pythagorean Quadruples

**Marcela Gutierrez** Northern Arizona University

**Advisor(s):** Jeff Rushall, Northern Arizona University

A primitive Pythagorean triple is a 3-tuple of natural numbers sharing no nontrivial common factors that satisfies the Pythagorean Theorem. Hall (1970) and Price (2008) found distinct perfect infinite ternary trees whose vertex sets are precisely all primitive Pythagorean triples. This talk will present progress towards the construction of an infinite tree whose vertex set consist of all primitive Pythagorean quadruples—i.e., 4-tuples  $(a, b, c, d)$  of natural numbers sharing no nontrivial common factors that satisfy  $a^2 + b^2 + c^2 = d^2$ .

### 313. Withdrawn

### 314. New Formulas with Euler's Totient Function

**Shida Jing** Grinnell College

**George Ge** Grinnell College

**Advisor(s):** Marc Chamberland, Grinnell College

The Euler Totient function is a elementary function used in Number Theory. Our advisor, Marc Chamberland, built a new basic identity involving the totient function and arbitrary sequences. From this, we were able to build new and old formulas that involve L-series, harmonic numbers, polylogarithms and Dirichlet characters. This is of interest because knowledge about the totient function illuminates our understanding of the positive integers, especially the primes.

### 315. Definition and first properties of Markov polynomials

**Rachael Kelly** Gettysburg College

**Samuel VanFossen** Gettysburg College

**Advisor(s):** Ricardo Conceicao, Gettysburg College

The sequence of Markov numbers (A002559 in the OEIS) 1, 2, 5, 13, 29, 34, 89, 169, . . . is generated from the integral solutions of Markov's equation  $x^2 + y^2 + z^2 = 3xyz$ . In our poster, we define a sequence of polynomials over a finite field  $\mathbb{F}_q$ , with  $q \equiv 1 \pmod{4}$ , which is analogous to the sequence of Markov numbers. We discuss some recently discovered and conjectured properties of these so-called Markov polynomials. For example, infinitely many Markov polynomials can be generated from one Markov polynomial, Odd-indexed Fibonacci and Pell polynomials are Markov polynomials, and more.

### 316. On Some Variations of Elliptic and Lucas Type Pseudoprimes

**Hyun Jong Kim** Massachusetts Institute of Technology

**Brad Bentz** Brown University

**Morad Hassan** Emory University

**Andr Hernandez-Espiet** University of Puerto Rico at Mayagez

**Advisor(s):** Liljana Babinkostova, Boise State University

Primality tests form the keystone of modern cryptography and computational number theory, so efficient implementations of these methods is essential. A pseudoprime is a type of composite number that acts like a prime number under some primality test. This research focuses on the existence of and relationships among pseudoprimes based on

elliptic curves and group structures derived from Lucas sequences. We further employ these notions for pseudoprimes to define their respective Carmichael numbers, which are pseudoprimes for all elements of a given group. We define equivalent conditions, akin to the Korselt Criterion, for an integer to be such Carmichael numbers. Moreover, we show that almost all elliptic curves which are elliptic Korselt numbers of type I for products of two distinct prime numbers are in fact anomalous for the two primes.

### 317. Explorations in Iterative Functions Over the Gaussian Integers

**Erik Knutsen** Humboldt State University

**Advisor(s):** Alejandra Alvarado, Eastern Illinois University

Let  $x$  be a positive integer and  $T(x)$  be the function defined by  $3x + 1$  if  $x$  is odd and  $x/2$  if  $x$  is even. The “ $3x + 1$  Problem” or “Collatz Conjecture” proposes that for any  $x$  there is a finite  $k$  such that  $T^k(x) = 1$ . We look at iterative functions similar to the Collatz function over the Gaussian integers. By partitioning the Gaussian integers into two sets modulo  $1 + i$  we obtain a definition of parity; a Gaussian integer  $z$  is even if  $z \equiv 0 \pmod{1 + i}$  and odd if  $z \equiv 1 \pmod{1 + i}$ . Then we can define the function  $\{C^1(z), C^2(z), \dots, C^k(z), \dots\}$  if  $z$  is even and  $mz + 1$  if  $z$  is odd, where  $m$  is an odd Gaussian integer. Our research explores patterns in the sequence  $\{C1(z), C2(z), \dots, Ck(z), \dots\}$  for arbitrary  $z$ , particularly the conditions for when an odd number precedes an even number within some such trajectory. We call such points coalescing points and determine a condition for their existence. From this result we are able to identify cyclic behavior in the sequence  $\{C^1(z), C^2(z), \dots, C^k(z), \dots\}$  if we characterize each  $C^i(z)$  as an ordered pair in  $\mathbb{Z}_{Nm} \times \mathbb{Z}_{Nm}$ .

### 318. On Factor Pair Latin Squares

**Bob Kuo** Colorado College

**Leah Miller** Gordon College

**Advisor(s):** James Hammer, Cedar Crest College

For a while, Latin Squares have been of interest to mathematicians. One variation of Latin squares is the popular game Sudoku, which we can be generalized to the concept of a factor pair Latin square. That is, we will label a  $n \times n$  square array with  $n$  characters. Then, we call this square a factor pair Latin square if for any  $a \times b = n$ , each  $a \times b$  subarray, tiled starting from the top left corner of the square, contains all  $n$  characters. This project is mainly concerned with when factor pair Latin squares cannot exist. In particular, we provide conditions such that if a number  $n$  satisfies the conditions, then a factor pair Latin square of order  $n$  cannot exist.

### 319. Magic Squares of Squares over Certain Integral Domains

**Giancarlo Labruna** Montclair State University

**Advisor(s):** Aihua Li, Montclair State University

A magic square  $M$  of order 3 over an integral domain  $R$  is a 3 by 3 matrix with entries from  $R$  such that the elements from each row, column, and main/minor diagonal add to the same sum. If all the entries in  $M$  are perfect squares in  $R$ , we call  $M$  a magic square of squares over  $R$ . The number of distinct entries in  $M$  is called the degree of  $M$ . An open question raised by Martin LaBar is: “Is there an order 3 magic square of squares over the set of integers which has all the nine entries distinct (degree 9)?” We approach to answering the same question over some integral domains other than the set of integers. We claim that over certain integral domains of finite characteristic, there exist order 3 magic squares of squares of degree 9. Furthermore, there are infinitely many prime numbers  $p$  such that there exist magic squares of squares of degree 9 over the set of integers modulo  $p$ . The proof is constructive.

### 320. Complexity of leading digit sequences

**Yuchen Li** University of Illinois at Urbana-Champaign

**Yunyi Zhang** University of Illinois at Urbana-Champaign

**Xinwei He** University of Illinois at Urbana-Champaign

**Advisor(s):** A J Hildebrand, University of Illinois at Urbana-Champaign

By a classical result of Diaconis, sequences such as the Fibonacci numbers and powers of 2 satisfy Benford’s Law; that is, leading digits in these sequences occur with frequencies given by  $P(d) = \log_{10}(1 + 1/d)$ ,  $d = 1, 2, \dots, 9$ . The resulting leading digit sequences are very far from being random sequences of digits generated with these frequencies.



In fact, they have a very low complexity and are, in some sense, very close to being periodic sequences. In our work we investigate the complexity of leading digit sequences of this type more closely, using several classical and new complexity measures, and we relate the observed behavior to properties of continued fractions and irrational rotations.

### 321. Hilbert Series and Square-Free Divisor Complexes of Numerical Semigroups

**Jessie Loucks** Sacramento State University

**Jackson Autry** San Diego State University

**Samuel Yih** Pomona College

**Paige Graves** University of La Verne

**Advisor(s):** Vadim Ponomarenko, San Diego State University

A numerical semigroup, or numerical monoid,  $S \subseteq \mathbb{Z}_{\geq 0}$  is a monoid under addition with finite complement in the nonnegative integers. For a numerical monoid  $S$ , we define the Hilbert series of  $S$  as the generating function  $\mathcal{H}(S; t) = \sum_{n \in S} t^n$ . There is a natural way of associating to each element  $n \in S$  a simplicial complex, known as the square-free divisor complex of  $n$ , that encodes information about how  $n$  factors into irreducibles within  $S$ . Studying these divisor complexes from a topological perspective affords us a closed rational form for  $\mathcal{H}(S; t)$ . We apply this approach to several classes of numerical monoids, as well as give a partial classification of which simplicial complexes can occur as square-free divisor complexes of a semigroup.

### 322. Rational and Irrational Distances

**Alexandro Luna** California State University, Fullerton

**Advisor(s):** Roberto Soto, California State University, Fullerton

Rational numbers are dense in the reals, and are easy to find on the real line. However, when measuring distances on a curve, they are not so easy to come by. Here we examine types of numbers, that when plotted as points on the function  $f(x) = x^2$ , will always have an irrational distance. From here, we are able to rule out families of numbers that will never work for finding rational distances between points. This brings us closer to the search for distances that are rational. We generalize and make similar conclusions about the distance between two points on the function  $f(x) = x^n$ , where  $n > 1$ .

### 323. Consecutive Products of Primes

**Eli Moore** California State University, Northridge

**Trevor Klar** California State University, Northridge

**Advisor(s):** Werner Horn, California State University, Northridge

Paul Erdős asked the following question about three distinct primes: If we construct a sequence of all the products of their powers, with the sequence arranged in increasing order, is it true infinitely often that consecutive terms in this sequence are both prime-powers? The way the question is stated indicates that the situation is known for two primes. However, we were not able to find a reference for this. We explore this question with two distinct primes, and give the proof that it does happen infinitely often. We also further explore the question with three distinct primes.

### 324. Counting Roots of Polynomials in $\mathbb{Z}/p^2\mathbb{Z}$

**Angela Patini** University of Pennsylvania

**Jeremy Johnson** Humboldt State University

**Trajan Hammonds** Carnegie Mellon University

**Advisor(s):** Maurice Rojas, Texas A&M University

Until recently, the only known method of finding the roots of univariate polynomials over finite rings and finite fields was brute force computation. In 2013, Bi, Cheng, and Rojas derived a deterministic algorithm that decides whether a polynomial with few terms has a root over a finite field. We will expand this idea to the case of finite rings of size  $p^2$ . Suppose  $p \in \mathbb{Z}$  is prime and  $f \in \mathbb{Z}/p^2\mathbb{Z}[x]$  is any univariate polynomial of degree  $d$ . We expect to prove a formula conjectured by Cheng, Gao, Rojas, and Wan that counts the number of roots of  $f$  efficiently. We will begin by discussing what modular reduction means in finite rings. The distinction between reduction in  $\mathbb{Z}/p^2\mathbb{Z}$  and reduction in finite fields is important because  $\mathbb{Z}/p^2\mathbb{Z}$  does not have unique factorization and division by  $p$  is undefined in  $\mathbb{Z}/p^2\mathbb{Z}$ . Using this reduction and modular arithmetic on ideals, we expect to get a new polynomial in a different polynomial ring that acts as a certificate of the number of roots in  $f$  in  $\mathbb{Z}/p^2\mathbb{Z}$ .

### 325. Prime Partitions

**Kyle Sutela** Michigan Technological University

**Advisor(s):** Jie Sun, Michigan Technological University

Prime partitions are partitions of integers into prime parts. In our paper, we first consider prime partitions with distinct parts. By using generating functions, we obtain some inductive formulas to calculate the number of prime partitions with distinct parts. Our formulas give two generalizations of the Euler's formula for the integer partition case. Then we consider general prime partitions with not necessarily distinct parts. By keeping track of the recurrence of primes in a partition and finding bijections between different prime partitions, we get some inductive formulas to calculate the number of general prime partitions. Finally by numerical experimentation we find an approximation of some analytical formulas for the number of general prime partitions.

### 326. Generalizations of the Collatz Conjecture.

**Noel Torrero** Los Angeles Mission College

**Nicholas Gomez** Los Angeles Mission College

**Advisor(s):** Werner Horn, California State University, Northridge

A collatz conjecture states that the sequence

$$C(n) = \begin{cases} \frac{3n+1}{2}, & n \equiv 1 \pmod{2} \\ \frac{n}{2}, & n \equiv 0 \pmod{2} \end{cases}$$

will converge to one starting with any natural number generator  $n$ . In our research, we explore the generalized version of this sequence

$$T_p(x) = \begin{cases} \frac{(p+1)x-r(x)}{p}, & x \equiv r \pmod{p} \\ \frac{x}{p}, & x \equiv 0 \pmod{p} \end{cases}$$

when  $p$  is prime and  $p > 2$ . The generalization of the conjecture is that for  $x > p$ ,  $T_p(x)$  converges to one of the numbers  $1, 2, \dots, p-1$ , or lapses into finitely many "exotic" limit cycles for each prime  $p$ . Our preliminary investigations have revealed that  $T_p(x)$  is subject to restrictions on stopping times given some conditions on the size of the generator. There also appears to be a connection between the existence of an "exotic sequence" and our list of "banned" stopping times.

### 327. Uniform Bounds for the Number of Rational Points on Symmetric Squares of Curves with Low Mordell-Weil Rank

**Sameera Vemulapalli** University of California - Berkeley

**Advisor(s):** David Zureick-Brown, Emory University

A central problem in Diophantine geometry is to uniformly bound the number of  $K$ -rational points on a smooth curve  $X/K$  in terms of  $K$  and its genus  $g$ . A recent paper by Stoll proved uniform bounds for the number of  $K$ -rational points on a hyperelliptic curve  $X$  provided that the rank of the Jacobian of  $X$  is at most  $g-3$ . Katz, Rabinoff and Zureick-Brown generalized his result to arbitrary curves satisfying the same rank condition. In this paper, we prove conditional uniform bounds on the number of rational points on the symmetric square of  $X$  outside its algebraic special set, provided that the rank of the Jacobian is at most  $g-4$ . We also find rank-favorable uniform bounds in the hyperelliptic case.

### 328. The Number of Primes for which a Polynomial is Eisenstein and Upper Bounds for the $k$ th Prime Non-Residue

**Mathias Wanner** Villanova University

**Devon Rhodes** California State University, Chico

**Shilin Ma** Carleton College

**Advisor(s):** Kevin McGown, California State University, Chico

The number of primes for which a polynomial is Eisenstein In a 2013 paper, Heyman and Shparlinski give an asymptotic formula with error term for the number of monic Eisenstein polynomials of fixed degree and bounded height. Let  $\psi(f)$  denote the number of primes for which a polynomial  $f$  is Eisenstein. We give the mean and variance of

the function  $\psi$  for each fixed degree, where the polynomials are ordered according to their height. Upper bounds on the  $k$ th smallest prime non-residue. In a 2017 paper, Pollack gives asymptotic upper bounds for the first several prime non-residues  $q_1$

### 329. Zeros of Eisenstein Series of $\Gamma_0(p)$

**Katharine Woo** Stanford University

**Patrick Revilla** Stanford University

**Advisor(s):** George Schaeffer, Stanford University

Modular forms are powerful tools used to study problems in number theory. We study the space of modular forms of  $\Gamma_0(p)$  for primes  $p$ , specifically locating the zeros of Eisenstein series relative to the fundamental domain. We derive a formula to find the number of zeros with multiplicity of a modular form of  $\Gamma_0(p)$ , bound the imaginary parts of the zeros of level  $p$  Eisenstein series and generalize Rankin and Swinnerton-Dyer's result for level 1 Eisenstein series to locate zeros along the arc  $\{(e^{i\theta} \pm 1)/p \mid \theta \in (0, 2\pi/3)\}$ . Additionally, we state several conjectures on the locations of the remaining zeros, based on quantitative evidence gathered.

### 330. On Cubic Residue Matrices

**Ryan Wood** Northern Arizona University

**Advisor(s):** Jeff Rushall, Northern Arizona University

In 2001, R. Chapman conjectured that a special infinite class of matrices, constructed using quadratic residue symbols, had constant determinant values. This conjecture, known as Chapman's Evil Determinant Problem was resolved by in 2014 M. Vsemirnov. We will present a generalization of Chapman's problem involving cubic residues.

### 331. Limiting Distributions in Generalized Zeckendorf Decompositions

**Jianing Yang** Colby College

**Shannon Sweitzer** University of California, Riverside

**Yujin Kim** Columbia University

**Eric Winsor** University of Michigan, Ann Arbor

**Advisor(s):** Steven Miller, Williams College

Zeckendorf's Theorem states that every positive integer can be uniquely decomposed into a sum of nonadjacent Fibonacci numbers. Interestingly this property can be used to define the Fibonacci numbers, and thus it is natural to consider constructing other sequences with decomposition laws. There has been previous work on sequences where bins have constant length  $b$ , and one can choose at most one element from a bin and cannot choose elements from two consecutive bins (the Fibonacci sequence is the case where  $b = 1$ ). In addition to unique decomposition, the distribution of number of summands in decompositions for these sequences is Gaussian, and the probability of a gap of length  $g$  has geometric decay. We extend these arguments to varying bin sizes  $b_n$ . Here the previous combinatorial techniques are no longer applicable, so we turn to the Lyapunov Central Limit Theorem to prove Gaussian behavior for the number of summands in a decomposition. The same behavior is also observed in generalized Zeckendorf decompositions with the legal number of summands chosen from the  $n$ th bin being a set  $A_n$ . We further generalize by examining sequences constructed with adjacency conditions on the bins, where we rely on some dependent CLT type results to study the limiting behavior.

### 332. An Exploration of Prime-Generating Polynomials

**Jeremy Collins** University of Arkansas

**Tyler Tracy** University of Arkansas

**Advisor(s):** Chaim Goodman-Strauss, University of Arkansas

The function  $n^2 + n + 41$  generates primes for all natural numbers such that  $n < 41$ . This fact carries interesting questions with it. If the values of  $n$  that yield primes are recorded, then patterns may emerge. What happens if these patterns are cracked and create a way to infinitely generate primes? If we set  $f(x) = n^2 + n + C$ , there are very few  $C$ 's that achieve this property. Are there infinite  $C$  values that generate primes from  $n = 0$  to  $n = C - 1$ ? Do these  $C$  values follow a pattern? If so, does one value of  $C$  make  $f(x)$  a more consistent prime generator after  $n > C$ ?

### 333. Variational Data Assimilation for Neuronal Network Models

**Jiajing Guan** George Mason University

**Advisor(s):** Tim Sauer, George Mason University

The large size of neuronal networks presents a challenge to understanding the behavior of the whole network. It is therefore of utmost importance to reconstruct the behavior of the network using the measurements of only a few neurons. In this research, we use coupled Fitzhugh-Nagumo models to simulate a chaotic neuronal network. Through the Levenberg-Marquardt method, we assimilate the behavior of a complete dynamical network given the data of a few observable nodes in the network, and assign each unobservable node a condition number describing the difficulty of reconstructing the node dynamics.

### 334. Improving the efficiency of Adaptive (Quasi-) Monte Carlo Methods using (Internal) Control Variates

**Yueyi Li** Macalester College

**Cu Hung** Biola University

**Advisor(s):** Fred Hickernell, Illinois Institute of Technology

(Quasi-)Monte Carlo ((Q)MC) simulations are used to compute the means of random variables whose distributions are too complex to admit analytic formulae. This work builds upon the automatic stopping criteria developed for (Q)MC simulations that use theoretically justified, data-based error bounds to determine when the error tolerance has been met. This research extends these automatic (Q)MC algorithms to include control variates and internal control variates. Control variates reduce variance and thus improve computational efficiency. Internal control variates could further deal with situations where multiples random variables share the same expected value. The stopping criteria must be modified, and the choice of control variate coefficients for QMC must be made differently than for simple MC. Numerical tests of our new algorithms and stopping criteria demonstrate that the error tolerance continues to be and that the computational effort required can be dramatically reduced.

### 335. Radiosity Equation Model for an Interior Space Illumination Design; Mars Project

**Hien Ngo** Roger Williams University

**Advisor(s):** Yajni Warnapala, Roger Williams University

This research project is focused on finding the true solution of the exterior Dirichlet problem for the Radiosity equation to determine the convergence of a Spherical Quatrefoil in three dimensions at its boundaries, using the Galerkin Method. A mathematical model, based on the Radiosity equation will be utilized to investigate the role of incoming light waves for different surfaces with different emissivity and reflectivity functions. Theoretical and computational details of the method will provide sufficient information for designing proper lighting of an interior space inside a spacecraft that can ultimately be used for future endeavors of Mars exploration.

### 336. Building Series-Parallel Voters

**Trung Nguyen** Macalester College

**Advisor(s):** Andrew Beveridge, Macalester College

When voting on different issues are carried out simultaneously, voters preferences on one may be dependent on results of others. In that case, the final outcome may not be sensible. We study this problem, namely the separability problem, using a formal mathematical framework. To be specific, we look at preferences as vectors in a vector space and use representation theory to find a convenient basis. This allows us to construct families of voters preferences with desired separability properties.

### 337. $n$ -enclosing Tiles that Admit Monohedral Tilings and Cavity Neighborhoods

**Jadie Adams** Westminster College

**Nhi Tran** University of Washington, Bothell

**Advisor(s):** Casey Mann, University of Washington, Bothell

H. Voderberg constructed a tile with the property that two copies of the tile can enclose one or two other copies. This tile's design can be extended to a general form which has the property that any number of copies can be enclosed within just two tiles. This property is known as the  $n$ -enclosing property, where  $n$  is the number of tiles enclosed.

This research uses the Voderberg tile to settle two previously open, tiling theory questions posed by Grunbaum and Shephard in *Tilings and Patterns*. The first is: Does there exist a tile with  $n$ -enclosing for each  $n > 2$  that admits a monohedral tiling? We settled this question by describing a general method of construction and pattern of tiling which demonstrates any  $n$ -enclosing Voderberg tile can be constructed so that it admits a periodic, as well as a nonperiodic, monohedral tiling. We also use one such tiling to disprove Grunbaum and Shephard's conjecture: the neighborhood of any given tile in a monohedral tiling will be equivalent to the patch generated by that tile. We demonstrate that double spiral tilings generated by Voderberg tiles with the  $m$ -enclosing property (where  $m > 2$ ) contain tiles with cavity neighborhoods, which are not simply connected; thus not equal to the patches they admit.

### 338. One-bit Sensing: Phase Transitions for the RIP Property

**Amadou Bah** Georgia State University Perimeter College

**Bryson Kagy** Georgia Institute of Technology

**Emily Smith** Agnes Scott College

**Advisor(s):** Michael Lacey, Georgia Institute of Technology

We study dimension reduction in the One-bit context. We take a finite collection of  $n$  high dimensional vectors and map them into an  $m$ -dimensional Hamming cube. Through this map, we are able to assess the effectiveness of One-bit sensing after concluding that it is possible to reduce the volume of data, while maintaining the structural integrity of the original set. We show that the probability of satisfying the Restricted Isometry Property is a phase transition within tight bounds for  $m$ . Finally, these bounds for One-bit sensing are essentially the same as those in Linear compressive sensing, even though the One-bit case only uses a few bit measurements.

### 339. On 2-Coloring the Integers to Forbid a Distance and Monochromatic Translates of a Triangle

**Kyle Binder** Baylor University

**Advisor(s):** Peter Johnson, Auburn University

An interesting coloring problem in the plane is as follows: given  $S, T \subseteq \mathbb{R}^2$ , with  $|S| = |T| = 3$ , how many colors are needed to color  $\mathbb{R}^2$  so that no translate of  $S$  or  $T$  is monochromatic? Previous results have shown only two colors are necessary except possibly for cases when  $S$  and  $T$  are each collinear and lie on parallel lines, and distances between points in  $S$  and distances between points in  $T$  are commensurable. This reduces the problem to looking at three-point sets in the integers. In this presentation, we consider a subservient problem: given  $S, T \subseteq \mathbb{Z}$ ,  $|S| = 2$ ,  $|T| = 3$ , how many colors are needed to color the integers so no translate of  $S$  or  $T$  is monochromatic? We prove that this problem can be further reduced and that "usually" only 2 colors are needed.

### 340. Algebraic Voting Theory

**Brian Camara** Bridgewater State University

**Advisor(s):** John Pike, Bridgewater State University

My research over the summer of 2017 through Bridgewater State University's Adrian Tinsley Program focused on the mathematical theory of voting, its connections with representation theory, and ways to explain some results in this area using linear algebra. Voting is of immense practical significance in our society, so understanding it from a mathematical point of view is of utmost importance. This poster will cover a ubiquitous class of voting systems and their connections with representation theory as detailed in a paper by Daugherty, Eustis, Minton, and Orrison. We will then sketch an alternate proof of the main theorem in that paper using less sophisticated mathematics. Similarly, we will also take a look at another theorem from Donald Sarri, and introduce our alternate proof for that as well.

### 341. Shifts on Substitution Alphabets

**Nathan Dalaklis** Carleton College

**Advisor(s):** Elizabeth Sattler, Carleton College

In symbolic dynamics, a substitution dynamical system results from a well-defined substitution map that replaces a letter with a word. In our research, we look at an alphabet of substitution mappings  $\{f, g\} = \mathcal{A}$ . Each substitution is defined on a common domain alphabet  $\mathcal{B}$ . In particular, we are interested in studying the properties of shift spaces generated by the one-sided full shift  $\mathcal{A}^{\mathbb{N}}$  and subshifts  $X \subseteq \mathcal{A}^{\mathbb{N}}$ . We investigate connections between these new spaces, different subshifts of finite type on  $\mathcal{A}^{\mathbb{N}}$ , and well-known substitution systems on  $\mathcal{B}$ .

### 342. Girth of Algebraically Defined Graphs

**Shannon Golden** Kutztown University

**Allison Ganger** Allegheny College

**Carter Lyons** Nebraska Wesleyan University

**Advisor(s):** Brian Kronenthal, Kutztown University

Projective planes and generalized quadrangles are structures in incidence geometry. One way to produce the point-line incidence graphs of such structures is to use algebraically defined graphs. An algebraically defined graph  $\Gamma_{\mathcal{R}}(f(x, y))$  is constructed using a specific ring  $\mathcal{R}$  and function  $f(x, y)$ . These graphs are bipartite with each partite set consisting of all coordinate pairs in  $\mathcal{R}^2$ . We denote the vertices of the first partite set by  $(a_1, a_2)$  and of the second by  $[x_1, x_2]$ . In order for two vertices  $(a_1, a_2)$  and  $[x_1, x_2]$  to be adjacent, their coordinates must satisfy the equation  $a_2 + x_2 = f(a_1, x_1)$ . In this poster, we will discuss the classification of  $\Gamma_{\mathcal{R}}(f(x, y))$  by girth, or length of a shortest cycle. We will motivate this classification with incidence geometry and then discuss theorems key to our research. Our main results derive from algebraically defined graphs where  $\mathcal{R} = \mathbb{R}$ .

### 343. The Ineseo Chumash Number System

**Annika Harder** Millersville University of Pennsylvania

**Advisor(s):** Ximena Catepillan, Millersville University of Pennsylvania

I am a descendant of the Ineseo Chumash indigenous people of the Santa Ynez Valley in California. Our indigenous band has been federally recognized since 1901, and, in the past 5 years, our language—Samala—has been slowly brought back into everyday life. I spend every summer with my family at Santa Ynez Reservation. My great-great-great-grandmother, Maria Solares, was the last full-blooded Chumash who spoke our language in the tribe; she is responsible for what we know today about our tribal language. The quaternary number system with rules to create compound numbers used by my tribe is presented in this poster with examples of the tribal use of this number system. I have had full support from my tribe to work on the project; they have provided me with oral history, documentation and literature. As a mathematics major and a descendant of the Chumash I am deeply committed to protecting and preserving my ancestors' culture and mathematical heritage.

### 344. Operator Quantization of Non-Regular Flips on Teichmüller Space

**Minhae Park** EWHA Womans University

**Advisor(s):** Hyun Kyu Kim, EWHA Womans University

The phase space of the  $2 + 1$ -dimensional pure gravity in physics can be described in terms of Teichmüller space with Weil-Petersson-Poisson structure. Quantization of Teichmüller space  $T(S)$  of punctured Riemann surface  $S$  has been established by Kashaev and by Chekhov-Fock, which draws the possibility of solving  $2 + 1$  quantum gravity. A proviso to that is a choice of an 'ideal triangulation' of  $S$ , which is a triangulation by homotopy class of edges running between punctures. Each edge yields a coordinate function on  $T(S)$ , which is replaced by a certain self-adjoint operator on a Hilbert space in quantization process. The key point is to build a unitary map between Hilbert spaces per each change relating these quantum coordinate operators. Such map is constructed for each elementary change of triangulations called flip along an edge, which alters only one edge, and the consistency relations of these operators is proved. However, in the literature, only nice class of flips are dealt with, and the flip involving triangles with two edges identified, are not fully treated. In the present work, we construct the unitary operators to such non-regular flips and prove the consistency relations, which fills a missing gap of quantum Teichmüller theory.

### 345. Playing Colorful Tic-tac-toe

**Troy Wiegand** Butler University

**Advisor(s):** William Johnston, Butler University

Analysis through Zermelo's Algorithm and other Game Theory techniques of a variant of the classic game Tic-tac-toe created by Dr. Jeremiah P. Farrell. This variant, Colorful Tic-tac-toe, introduces additional win conditions to the game besides going three in a row. The objective of this project is to map, via the game tree, every possible move in Colorful Tic-tac-toe, assuming rational play and mathematically prove that the first player has a strategy to win always.

### **346. Coxeter Nim: A variation on the classic mathematical game Nim to investigate the relationship between triangle Coxeter groups and Nim**

**Katherine Wu** Boston University - PROMYS

**Christianna Xu** Boston University - PROMYS

**Advisor(s):** Paul Gunnells, University of Massachusetts

Nim is a classic mathematical game thoroughly studied by game theorists. We studied the structure and properties of Coxeter Nim, a novel game which combines Nim with the geometry of triangle Coxeter groups. Coxeter Nim is played on a triangular Coxeter group where triangles are positions, one position is labelled “start”, and one position is labelled “final”. Players alternate turns, moving the playing piece from “start” to “end” through a series of intermediate positions. A player moves to “final” to win. There exist “cold” positions which ensure a win and “hot” positions which ensure a loss when players move to them, assuming optimal play. We focused on the (2, 4, 4), (3, 3, 3), and the (2, 3, 6) tilings of the Euclidean plane. We attempted to discover a winning strategy by examining patterns of “cold” and “hot” positions on these game boards. In particular, we analyzed how the orientation and distance of a given position from “final” affects whether it is “hot” or “cold”. With the knowledge of patterns described in this paper, a player can expect to find and move to cold” positions on any Euclidean game board. Our results may be useful on a hyperbolic or spherical game board. Further study could develop a generalized winning strategy for Coxeter Nim on Euclidean game boards.

### **347. Exploring the Iterated Prisoners’ Dilemma with Cards**

**Benjamin Adenbaum** Lafayette College

**Advisor(s):** Robert Root, Lafayette College

The Prisoners’ Dilemma and its iterated form are well studied models in Evolutionary Game Theory. We model the iterated game modified by imposing stochastic constraints on future play with a representation as a card game, the Iterated Prisoners’ Dilemma with Cards (IPDwC). In the unconstrained iterated game, the base Nash Equilibrium of always defecting enables the creation of Cooperative Equilibria promised by the Folk Theorem. In IPDwC, away from special cases, the strategy profile of defecting whenever possible ceases to be a Nash equilibrium, and by considering the IPDwC as a Markov process, we are able to identify new Nash equilibria in two distinct regimes. These new equilibria can play the analogous role in a Folk Theorem for the constrained game. Further, we demonstrate that the simplest mixed strategies all result in expected payoffs equivalent to random choice among available strategies. These results together demonstrate that stochastic constraints on future play can lead players to unforced cooperative play, suggesting that such constraints might serve as an evolutionary mechanism in the development of cooperative behavior.

### **348. Heart Disease Data Meets Mathematics**

**Jeenn Barreiro** College of the Holy Cross

**Advisor(s):** Eric Ruggieri, College of the Holy Cross

For decades, heart disease has been the leading cause of death in the United States for both, men and women. The oddity of this phenomenon is how heart disease is considered the most preventable disease around. Doctors and nutritionists have a lot of theories on what is causing this epidemic; theories from these professionals correlate poor diet and exercise for being the major factor into the development of heart disease. After learning how to code in the statistical analysis software, R, I analyzed a dataset from the Cleveland V.A. medical center that included 297 patients with heart disease. The dataset contains 14 different attributes of each patient, including each individual’s sex, age, cholesterol level, level of chest pain, etc., attributes which may or may have not been important in the development of heart disease for that specific individual. This summer, I created several models that allowed me to identify which attributes were the most influential in the development of heart disease for the dataset I used. In the frequentist setting, stepwise regression combined with the AIC criteria can be used to identify the ‘best’ model. On the other hand, a Bayesian approach to logistic regression allows one to quantify the uncertainty associated with the ‘optimal’ model. The Metropolis-Hastings algorithm was used to explore the set of potential models with the likelihood of each possible model determined by using Monte Carlo integration. The Metropolis-Hastings algorithm was run for 143,000 iterations with a burn-in of 10% and a lag of 3 to help reduce the correlation between successive observations. In the end, we identified both the most probable model and several viable alternatives. My most accurate model was 84% accretive, while the other two were 83% and 81% accurate. There were clear differences between Bayesian Logistic

Regression and Stepwise Regression, with Bayesian Logistic Regression providing a more accurate model. We would like to thank the National Science Foundation for their support on this project (DMS-1407670).

### **349. Winning Strategies in Coda**

**Jack Champagne** College of the Holy Cross

**Advisor(s):** Eric Ruggieri, College of the Holy Cross

Coda is a game of bluffing, hidden information, and deductive reasoning. Each player has a secret “code” that they build throughout the game, which other players are trying to decipher. The last player standing is the winner. We simulated Coda to test different strategies, including ones that act completely at random, ones that are willing to bluff and take chances, and the “ideal” strategy. We also considered the probability distribution of winning based on turn order to see if those who go first are more likely to win.

### **350. Spatial point analysis of segregated communities and greenhouse gas sources in New York**

**Rajita Chandak** Brown University

**Advisor(s):** Ben Nolting, California State University, Chico

In this research project, we hypothesized that sources of pollution are more likely to be built in highly segregated communities because segregated communities lack the social power to resist the establishment of these sources and disadvantaged racial minorities could be forced to live near existing sources. To test this, we designed a way to quantify segregation across a range of spatial scales using spatial pattern analysis tools: Ripley’s K function and the pair correlation function. We designed a coding framework to efficiently calculate these functions from 2010 census data. Previous studies typically only identify segregation at a single spatial scale, so our approach presents a novel way to understand this phenomenon. Using these results with EPA data on Greenhouse Gas emissions, we examined the relationship between segregation and the location of pollution sources. Preliminary results show that areas with higher levels of racial segregation were more likely to contain major GHG emission sources. Our computational design also allows us to quantify the relationship between racial groups and pollution sources across a continuum of spatial scales, and preliminary results show that disadvantaged racial groups are more likely to be clustered near pollution sources. NSF 1559788

### **351. Selective Hedging Against Regret: The Amnesiac Lookback Option**

**Ho-Chun Herbert Chang** Dartmouth College

**Advisor(s):** Seema Nanda, Dartmouth College

Everyone has had the experience and regret of not buying something at the right time. The Lookback Option, created in the late 20th century for volatile commodities like gold, is a financial instrument that hedges against this regret. The buyer pays a premium for the Lookback Call (Put) Option for the right to buy an underlying asset at its minimum (or sell at maximum) within a specified time period. This allows the holder to effectively buy or sell the underlying at its optimal price. However, as the condition of looking at all the previous prices is so strong, the premium of Lookback Options is very high and prevents the option from being more extensively traded. This talk presents the Amnesiac Lookback Option, which reduces the premium of a discrete Lookback Option by restricting the periods that can be looked back on. A fair price is shown using Monte Carlo simulations and compared with an adjusted binomial valuation. The Amnesiac Lookback’s use is then extended to cryptocurrencies, volatile commodities enabled by blockchain technology, which have recently been cleared for options and other financial derivatives trading. We conclude by showing that the region between 0% to 20% of the total periods provide regions for investors to precisely balance their risk and regret.

### **352. Probabilistic Models, Machine Learning, and the Future of Breast Cancer Risk Assessment**

**Alyssa Columbus** University of California, Irvine

**Advisor(s):** Argyrios Ziogas, University of California, Irvine

With 1 in 8 women in the United States at risk of being diagnosed with breast cancer at some point in her lifetime, the development and accuracy of breast cancer prediction models is pertinent to reducing the morbidity and mortality rates



associated with the disease. Three such prediction models are the Gail, BCSC (Breast Cancer Surveillance Consortium), and Tyrer-Cuzick models, each of which determine a woman's risk of breast cancer from risk factors including family history of cancer and mammographic density. However, these models have been shown to vary in accuracy among women with different ethnic backgrounds, which is why the Athena Breast Health Network, an extensive program at UCI that integrates clinical care and research to drive innovation in the prevention of breast cancer, is building and testing new breast cancer risk assessment models using machine learning. Experimenting with algorithms such as k-nearest neighbors (KNN), support vector machines (SVM), and decision trees (DT), Athena hopes to evaluate new breast cancer risk models and their predictive accuracies.

### 353. Propensity Score Matching: the Key to Comparative Analysis

**Anna Crosby** California Baptist University

**Linn Carothers** California Baptist University

**Advisor(s):** Linn Carothers, California Baptist University

In an environment where dynamic real-world data drive solutions and decisions in day to day industry life, the appeal of robust comparative analysis is increasingly apparent. In large-scale datasets containing multiple groups to be compared, propensity score matching (PSM) provides an efficient and reliable mode of comparison that extends beyond surface-level number summaries such as mean and standard deviation. Due to the increasing complexity and magnitude of data being collected, the need for more extensive analytics calls for statistical methods that can be easily automated and communicated. Propensity score matching involves the selection of covariates and outcomes to be analyzed from the data. Covariates are variables or factors that the analyst believes have an impact on the outcomes, and must be excluded from the covariates. The individual entries of data being compared from separate categories are then matched together based on their covariate values. The data entries are assigned a propensity score and then are matched so that only data with similar characteristics can be compared. The outcomes from the separate groups can then be fairly compared so that a simple p-value calculation can determine statistical significance.

### 354. Measuring Inter-Rater Reliability for Ordinal Data and Multiple Raters

**Gail Crunkhorn** University of Mary Washington

**Ekta Kapoor** University of Mary Washington

**Advisor(s):** Debra Hydorn, University of Mary Washington

Kappa statistics are used to measure Inter-Rater Reliability (IRR), which is important in both the corporate and scientific world for measuring validity of ratings made using a single criterion and rater comprehension of the criteria. We examined multiple measures of IRR and investigated how to modify a kappa statistic for ratings made on multiple criteria on the same set of subjects by the same set of raters. Comparisons of our proposed modifications were achieved using simulations written in R, as well as by applying the modifications to a set of real data where four raters rated 17 subjects on ten criteria. Simulations were conducted using a variety of different probability distributions for the possible ratings of 0-4 for each criterion. The simulations produced values for (1) the average Fleiss's Kappa across the multiple ratings, (2) a modified Fleiss's Kappa for the ratings, (3) the range between smallest and largest values for Fleiss's Kappa within the multiple ratings, (4) the average of the absolute values of Fleiss's Kappa, and (5) applying Fleiss's Kappa on the sum of raters' scores. The application of the modifications of Fleiss's Kappa to the example data set demonstrated that the data did in fact show a higher kappa than would be expected by chance. This indicates a certain level of shared understanding among the raters about the criteria, although perhaps not as high as those who developed the rating system might wish.

### 355. Monte Carlo Simulation of Drug Concentrations with Measurement and Process Noise

**Ty Danet** California State University, Channel Islands

**Cesar Manzo** California State University, Channel Islands

**Tyler Owen** California State University, Channel Islands

**Advisor(s):** Alona Kryshchenko, California State University, Channel Islands

Pharmacokinetic models describe how a drug behaves once inside the human body. We use these models to individualize patient therapies using measurable patient parameters, data about unmeasurable patient parameters obtained from clinical trials, and measurements taken from the patient during therapy. However, there are errors inherent to the therapeutic process or even errors in the models describing the drug's behavior. The two common sources of error

associated with drug treatment are measurement error and process error. It is especially important to account for these errors when dealing with high-risk drugs that have narrow margins of safety. Current widely used models only account for measurement noise, ignoring process errors such as dosage error, timing errors, and model misspecification. We show that all of these errors should be taken into account when designing dosage regimens. We run Monte-Carlo simulations on a system of linear stochastic differential equations modeling drug concentrations in the body. Measurements are taken at predetermined intervals to check the patient's concentration, and the dosage is updated through Kalman filtering. We utilize white noise to account for process error and will discuss running the simulation across different "error" scenarios where each of the relevant clinical factors is subjected to different magnitudes of error. The data given by each scenario will be analyzed using statistical inference methods and the results will be discussed.

### **356. Deterministic Constructions of Binary Edit Distance Tree Codes for Interactive Communication**

**Ronak Desai** Rowan University

**Advisor(s):** Hieu Nguyen, Rowan University

Error-correcting codes have numerous applications including the ability to recover data from lightly scratched CD's and damaged bar-codes. In communications, these codes are traditionally implemented in a one-way setting where each received message is immediately decoded; however, advances in distributed computing has motivated the need for an interactive communication setting where many short messages are alternately sent between two parties through an adversarially noisy channel before decoding is performed. The codes we consider are edit distance tree codes (EDTC), where encoded messages can be considered as paths on a tree with its symbols represented by edges. Since we allow for insertions and deletions of symbols (as opposed to just substitutions), this approach is more difficult compared to other tree encodings, and no explicit deterministic constructions of EDTC are known to date (although such codes have been proven to exist). We show an example of such a tree code construction that yields EDTC with good parameters and performs better on average than other natural and/or greedy approaches.

### **357. Detecting Bovine Lameness Using Three-Dimensional Limb Movement Variable Analysis to Achieve High Sensitivity and Specificity**

**William Dula** Morehouse College

**Advisor(s):** Nagaraj Neerchal, University of Maryland, Baltimore County

Bovine lameness is a common issue among commercial dairy farms, resulting in decreased productivity. In order to treat bovine lameness effectively, it is necessary that it is detected early. The main objective is to model the lameness status of a cow using three-dimensional limb movement measurements related to the cow's gait. Previously, a statistical model was generated using the software SAS<sup>®</sup> with its LOGISTIC and TRANSREG procedures. The model produces a binary classification: lame or sound. Current implementation requires running several SAS<sup>®</sup> procedures manually and therefore is not amenable to a large scale application. In this work, we implement regression algorithms in R to mirror the TRANSREG procedure and thus speed up exploration of a large number of candidate models to maximize goodness of fit criteria such as the area under the Receiver Operating Characteristic curve (AUC). The predictive models are also evaluated using quantities such as sensitivity (true positive rate) and specificity (true negative rate) which are quite important from the dairy industry's point of view. We also consider multinomial logistic models to divide the lame cows further into severely lame and mildly lame. These results can ultimately be used in the commercial dairy industry for early lameness detection.

### **358. Modeling Indiana Bat Migration**

**Emily Gentles** University of Arkansas

**Advisor(s):** Giovanni Petris, University of Arkansas

In the past decade over 5.5 million bats in North America have died of white-nose syndrome, a fungal disease active during hibernation. This, coupled with increasing urbanization, threatens current keystone bat populations and consequently impedes their ability to provide important ecosystem services, such as pollination and pest control. In order to provide support for these populations we must understand their movement through the environment. In this study we used telemetry data from 11 Indiana bat specimens to construct a continuous time model of bat migration. Utilizing Brownian motion and R, emphasis is placed on landscape analysis in an attempt to gain insight into roosting habits and wind turbine interaction.

### 359. Adjacent Levels in the Tridiagonal $\beta$ -Hermite Ensemble

**Gopal Goel** Krishna Homeschool

**Advisor(s):** Andrew Ahn, MIT

Given a matrix with real spectrum, one can identify it with a stepped function whose graph is a continuous function with slopes  $\pm 1$ . For a class of random matrices, known as Wigner matrices, it is known that the associated random stepped functions converge to the Logan-Shepp curve which arises in asymptotic representation theory. As a special case, we know that the Gaussian orthogonal ensemble (GOE) and Gaussian unitary ensembles (GUE) exhibit this limit. We show that for arbitrary Hermite  $\beta$ -ensembles the same limit shape appears. We also show that the second order fluctuations are Gaussian. Based on existing results on Jacobi matrix ensembles, we expect these Gaussian fluctuations to have an interpretation as some derivative of the Gaussian free field.

### 360. Fantasy Football Points Prediction Using Bayesian Statistics

**Scott Harbour** California State University, Fresno

**Advisor(s):** Steve Chung, California State University, Fresno

In this project, our main objective is to create a working model that predicts the fantasy points of NFL quarterbacks. According to the Forbes Magazine, 32 million Americans spend about \$11 billion to play Fantasy Football. It is estimated that the Fantasy Football market is \$70 billion. Therefore, this research will not only draw interest from academia but it will also have a high impact on the society. To predict the fantasy football points, we use the Bayesian statistics along with appropriate inflation methods due to various factors, such as weather and home team advantage. To the best of our knowledge, there is no published result on fantasy football modeling and prediction and hence, our work is novel in this area of research.

### 361. Demand Side Management Programs in Kentucky

**Marissa Hartsoe** Centre College

**Advisor(s):** Ellen Swanson, Centre College

Energy efficiency is a relatively new effort in Kentucky, and an increasing number of programs to promote energy efficiency are being developed and implemented. We examine the efficacy of a specific type of efficiency program, demand side management (DSM) programs. These programs are designed to decrease the amount of energy used during peak times and eventually reduce the overall amount of energy consumed. Through a regression analysis of six major utility companies in Kentucky, we determine the DSM program that is most effective in increasing energy savings for each utility. Additionally, we examine which utility best implements each program.

### 362. Effects of Early Standardized Mathematics Exams on Women in STEM

**Kaylee Henry** University of Arkansas

**Jameson Gillis** University of Arkansas

**Advisor(s):** Sarah McKenzie, University of Arkansas

According to a recent report by the American Action Forum, the U.S. will be short 1.1 million STEM workers by 2024. Additionally, women are currently underrepresented in STEM fields, so identifying factors that are positively or negatively affecting female enrollment in STEM courses in high school helps to increase their participation in STEM careers. Early standardized testing may be a factor that leads female students into a future including STEM. Students who score well on standardized mathematics exams may be more likely to take upper level math and science courses during their junior year of high school. Using student-level data from all public school students in a southern state over the years 2008-2015, we examine the STEM-based course enrollment in the junior year of high school for students who demonstrated high achievement on the 5th grade state assessment of mathematics. Creating a model applicable to all states, two Chi-Square tests will be performed for each cohort for students. The first test analyzes the ratio of STEM-based course enrollment in the junior year of high school by gender. The null is that there is not a significant difference in the male versus female student enrollment in STEM courses. The second test will examine each gender's junior year of high school STEM enrollment based on two factors: race and the student's free or reduced lunch status. For the second test, it is expected that the two factors should have equal ratios of students in STEM. Based on the results of the tests, policies and actions may be recommended to support students who demonstrate an early aptitude in mathematics to enroll in STEM courses later in their academic careers.

### 363. On the Speed of an Excited Asymmetric Random Walk

**Joseph Jackson** Swarthmore College

**Advisor(s):** Jonathon Peterson, Purdue University

An excited random walk is a non-Markovian extension of the simple random walk, in which the walk's behavior at time  $n$  is impacted by the path it has taken up to time  $n$ . The properties of an excited random walk are more difficult to investigate than those of a simple random walk. For example, the limiting speed of an excited random walk is either zero or unknown depending on its initial conditions. While its limiting speed is unknown in most cases, the qualitative behavior of an excited random walk is largely determined by a parameter  $\delta$  which can be computed explicitly. Despite this, it is known that the limiting speed cannot be written as a function of  $\delta$ . We offer a new proof of this fact, and use techniques from this proof to further investigate the relationship between  $\delta$  and limiting speed. We also generalize the standard excited random walk by introducing a "bias" to the right, and call this generalization an excited asymmetric random walk. Under certain initial conditions we are able to compute an explicit formula for the limiting speed of an excited asymmetric random walk.

### 364. Mathematical Modeling of Likelihood of Customers' Buying

**Kateryna Kaplun** Montclair State University

**Advisor(s):** Aihua Li, Montclair State University

This project works on a problem given by Staples. With collected customers' information from Staples, such as firmographics, transaction history, and online behavior, we try to determine which customers are most likely to respond to an email by making a purchase within seven days after receiving the email. This will be done in a binary case, determining whether or not the person will make a purchase within 7 days of receiving an email. The likelihood of customers buying is measured by a carefully designed scoring system. The software R is used to analyze the data and to build regression models. We hope the model can make reasonable predictions for the customers willingness of purchasing.

### 365. Kernel-Based Nonlinear Dimension Reduction for Face Analysis

**Katherine Kempfert** University of Florida

**Advisor(s):** Yishi Wang, University of North Carolina, Wilmington

Image data is often high-dimensional, which can hinder statistical and machine learning algorithms. Therefore, in this study the following nonlinear kernel-based dimensionality reduction techniques are investigated: kernel principal component analysis (KPCA), supervised kernel principal component analysis (SKPCA), and kernel Fisher's discriminant analysis (KFDA). As a preliminary, these techniques are studied on three simulated datasets. Then a novel machine learning pipeline is proposed for the longitudinal face aging database MORPH-II. First, images in MORPH-II are preprocessed, and several feature types are extracted from the preprocessed images. The KPCA, SKPCA, and KFDA techniques are used to transform then reduce the dimension of the extracted features. The transformed, reduced dimension data serve as input for a linear support vector machine (SVM) to classify gender of the pictured subjects. Finally, the dimension reduction techniques are compared in terms of gender classification results, which are promising.

### 366. Integrating Health Data Sources to Identify, Stratify, and Predict High Utilizers of Public Systems

**Benjamin Knisley** California Baptist University

**Advisor(s):** Linn Carothers, California Baptist University

The California Medi-Cal 2020 waiver includes a five-year pilot program known as Whole Person Care (WPC), which focuses on health interventions that coordinate physical health, behavioral health, and social service needs of beneficiaries who are high users of multiple county entities. San Bernardino County developed an analytic approach combining and matching health and social services data from multiple County departments utilizing disparate systems and identifiers. This required collaboration of data experts in multiple County departments to produce the most accurate matching approach. In many high utilizer projects, potential service recipients are identified through costs or basic utilization. However, both of these have limitations and may miss individuals who are not appropriately engaged in their care or their care is not appropriately coordinated. Through an iterative process, a scoring methodology was developed to stratify utilizers of County health services to identify individuals who are most likely to need care coordination and health engagement services. Additionally, in order to begin building and testing a predictive model, a

retrospective cohort was evaluated and scored and multiple logistic regression was implemented to demonstrate the factors and service utilization patterns that most contribute to high utilization. Over time, and with new incoming data, the model will be refined to better assess the combinations of factors, services, and score methodology that predict those who most need care coordination services to improve the quality of their care and access to outpatient services for better health outcomes.

### **367. A Probability Distribution Model for Bitterness Perception in Brassica oleracea**

**Natalie Kratts** Lenoir-Rhyne University

**Advisor(s):** Slavko Komarnytsky, North Carolina State University

Bitter taste receptors have been associated with glucose homeostasis, and their function affects the perception of bitterness in humans. Glucosinolates are compounds found in the Brassica oleracea species known to give these cruciferous vegetables their bitter flavor. The goal of this study was to investigate the link between bitter flavor perception, taste receptors, and glucosinolate compounds. In this investigation, a scoring metric was developed based on glucosinolate content in the vegetables and the probability of taste receptors binding with the compounds and their breakdown products. These scores were used to predict bitter taste perception and were compared with data from a taste panel to validate them.

### **368. Applying the Kelly Criterion to College Football Betting**

**Jamie Kunzmann** Stony Brook University

**Keri D'Angelo** The College of New Jersey

**Joshua Radack** Lafayette College

**Quyen Do** Millersville University

**Advisor(s):** Trent Gaugler, Lafayette College

The Kelly criterion is a mechanism used for properly sizing a portion of one's capital for betting and investment purposes. In our research project, we explore a realistic application of the Kelly criterion. Our objective is to maximize a bettor's return while simultaneously minimizing the risk of ruin in the context of betting on FBS college football games. We build a selection of logistic regression models using scraped college football data of the 2008–2016 seasons, simultaneously comparing our models to the well-known PFR model designed by pro-football-reference.com. We then apply the predicted probabilities to our expansion of Kelly criterion to more than one betting event. Simulations are then run with varying approaches—differing number of bets, game filtering methods, and fractional Kelly sizes are all considered to come up with the most profitable betting strategy for an FBS season.

### **369. A Monte Carlo measurement of Gerrymandering in Pennsylvania through random tessellation**

**Benjamin Lieberman** Muhlenberg College

**Advisor(s):** James Russell, Muhlenberg College

We examine the prevalence of gerrymandering in Pennsylvania through Monte Carlo simulation. To generate the simulations, the area of Pennsylvania is divided using Centroidal Voronoi tessellation with centroids sampled from inhomogeneous Poisson point processes. To ensure that each of the districts constitutes an approximately equal share of the population, a large number of tiles are generated and assigned to a particular district using an acceptance-rejection sampling algorithm. The vote of each individual is simulated using voter registration information and the results from the simulated election are compared to the actual distribution of congressional seats.

### **370. American Roulette: How Boldly Can You Play?**

**Amira Mahler** St. Edward's University

**Advisor(s):** Jason Callahan, St. Edward's University

American Roulette is a casino game in which players place various types of bets on subsets of 38 numbers. A wheel is then spun to determine the winning number; players who bet on this number win their bet amount plus a payout multiplier specific to each type of bet while players who did not bet on this number lose their bet amount. Previous research uses Markov chains to compute probabilities of players accumulating a desired amount of money ("success") or losing all their money ("ruin") for each type of bet. We use Markov chains to compute the probabilities of success

and ruin for new betting strategies based on bold gambling and find that these strategies yield higher probabilities of success.

### 371. Applications of Multiplicative LLN and CLT for Random Matrices

**Rajeshwari Majumdar** University of Connecticut

**Anthony Sisti** University of Connecticut

**Advisor(s):** Maria Gordina, University of Connecticut

The Lyapunov exponent is the exponential growth rate of the operator norm of the partial products of a sequence of independent and identically distributed random matrices. It usually cannot be computed explicitly from the distribution of the matrices. Furstenberg and Kesten (1960) and Le Page (1982) found analogues to the Law of Large Numbers and Central Limit Theorem, respectively, for the norm of the partial products of a sequence of such random matrices. We use these analogues to efficiently compute the Lyapunov exponent for several random matrix models and numerically estimate the corresponding variances. For random matrices of order 2, with independent components distributed as  $\xi \text{Bernoulli}(\frac{1}{2})$ , where  $\xi \in \mathbb{R}$ , we obtain analytic estimates for the Lyapunov exponent in terms of a limit involving Fibonacci-like sequences.

### 372. Statistical Tests for Convergence of Some Random Walks to Perturbed Brownian Motion

**Jacob Menix** Western Kentucky University

**Advisor(s):** Jonathon Peterson, Purdue University

A perturbed Brownian motion is a stochastic process that is like a Brownian motion with added effect each time a new extrema is achieved. While Brownian motion is the scaling limit of simple random walks, it is known that perturbed Brownian motion can arise as the scaling limit of some self-interacting random walks called excited random walks. Excited random walks are a type of self-interacting random walk where the transition probabilities, instead of being the same for every step, are dependent on the number of prior visits to the current location. We consider some examples of other self-interacting random walks where it is conjectured, but not yet proved, that the scaling limit is a perturbed Brownian motion. Using computer simulations, we perform statistical tests for properties of perturbed Brownian motion in rescaled Markovian Cookie Random Walks and Have Your Cookie and Eat It Random Walks to look for evidence of convergence to perturbed Brownian motion.

### 373. Withdrawn

### 374. Bounds for the bond and site percolation thresholds of a 2-uniform lattice

**Gabrielle Moss** Johns Hopkins University

**Advisor(s):** John Wierman, Johns Hopkins University

A percolation model is an infinite graph, from which edges are deleted independently with probability  $p$ . The percolation threshold of an infinite graph is the critical probability  $p_c$  above which there exists a connected, infinite component. Most research so far has focused on calculating exact values and rigorous bounds for the percolation threshold of one-uniform tilings, known as Archimedean lattices. We will investigate how to calculate the percolation threshold of a two-uniform tiling. In a two-uniform tiling, each vertex of the tiling has one of two sequences of faces surrounding it. We use these vertex configurations to name the tiling. This talk will be focusing on the  $(3, 4, 3, 12)$ ,  $(3, 12^2)$  lattice. Each vertex of this graph has one of two face sequences: triangle-square-triangle-dodecagon, or triangle-dodecagon-dodecagon. We will derive bounds for the site and bond percolation thresholds and discuss the methods used for calculating these bounds.

### 375. Transformation of categorical variables to numeric variables to increase statistical power

**Louis Mutter** California State University, Monterey Bay

**Advisor(s):** Steven Kim, California State University, Monterey Bay

In statistical hypothesis testing, assuming the alternative hypothesis is true, the probability of rejecting the null hypothesis is called statistical power. Researchers prefer high statistical power, and we generally need large sample sizes for high power, but obtaining large samples presents a practical and logistical cost to research. Our study aims to increase

power in the Chi-Square Goodness-of-Fit (G.O.F.) Test by transforming categorical variables into numeric variables. A numeric optimization algorithm is implemented to maximize power under numeric transformation, and the power is compared to the G.O.F. Test and other hypothesis tests for categorical variables. The power under the numeric transformation relies on the specified null and alternative hypotheses, and the truth. Therefore, multiple scenarios were designed to investigate the sensitivity of the specified alternative hypotheses for a fixed truth and a uniform null hypothesis. The simulation results show that an optimal numeric transformation allows researchers to gain power without increasing sample size for a correctly or closely specified alternative hypothesis to the truth. However, an alternative hypothesis specified in the wrong direction results in a power loss. Researchers confident in their prior knowledge relating to their study may use this method to increase power, or to decrease their sample size without a loss of power.

### **376. Ensemble method recommender system**

**Eshita Nandini** University of California, Merced

**Advisor(s):** Arnold Kim, University of California, Merced

A recommender system was constructed; originally to help Harvard University's College of Arts & Sciences organize the large amount of courses offered by rating similar courses based off of course description and syllabi data. The Python-based Dash system employs common Natural Language Processing (NLP) statistical methods, including Term Frequency-Inverse Document Frequency (TF-IDF) and Latent Semantic Analysis (LSA) (topic modeling) in order to make recommendations. The system allows the user to upload textual data, select methods for cleaning and modeling the data, and then outputs a ranked list of similar documents. The ensemble method is a way to give off recommendations based off overall ranking of all the method outputs instead of focusing on how one NLP method performed. This is in order to increase accuracy of predictions since there are very few ways to quantify accuracy in purely qualitative data.

### **377. Irrelevant Information Probability Theory**

**Lucas Neville** University of Arkansas

**Advisor(s):** Chaim Goodman-s Strauss, University of Arkansas

The main topic of this project is an extension on the "son born on Tuesday" riddle. The riddle first asks, "If a man with two children has at least one boy, what is the probability that the other child is a boy?". The answer to this is  $1/3$ . The riddle then asks, "If a man with two children has at least one boy who is born on a Tuesday, what is the probability that the other child is a boy?". Surprisingly, the answer is not  $1/3$ , but  $13/27$ . This project will be analyzing what happens as varying amounts of seemingly irrelevant information is added to a system and what conclusions may be drawn based on this application of conditional probability. To accomplish this, a few systems similar to the ones in the riddle will be posed and analyzed using conditional probability to form a hypothesis. Then, a computer program made to model the situation will test the hypothesis. Using these hypotheses, more general conclusions will be drawn about irrelevant information's effects on a system.

### **378. Comparing Several Models and Model Averaging for Testing Hormesis**

**Nathan Sanders** California State University, Monterey Bay

**Advisor(s):** Steven Skim, California State University, Monterey Bay

In cancer studies, hormesis is a phenomenon where low doses of a carcinogen reduces the risk of cancer while high doses increase the risk. There are several models to test for hormesis, however some are not flexible enough to detect hormesis. Our research objective was to compare five individual models as well as the method of model averaging (MA). The MA method utilizes multiple models for the hypothesis testing, and a simpler model with better fit has greater contribution in the procedure. We hypothesized that the MA method would avoid the worst individual result and perform closely to the best individual result. We designed sixteen simulation scenarios to emulate various cases occurred in real life experiments. Five monotonic scenarios were designed to test a significance level ( $\alpha = .05$ ), and eleven non-monotonic (hormetic) scenarios were designed to test statistical power. In the results, when the null hypothesis was true there was an instance when an individual model violated  $\alpha = .05$  and the MA method relieved the violation. When the alternative hypothesis was true the MA method relieved significantly low power. However when the truth was one of the models in MA, the MA method performed worse than we expected.

### 379. Monopoly Probability Simulation

**Hilary Shea** College of the Holy Cross

**Advisor(s):** Eric Ruggieri, College of the Holy Cross

Monopoly is a game where players take turns rolling a pair of dice to moving around the board, purchase properties, and build houses. You earn money by passing 'Go' and when other players land on your property. The game ends when you run out of money and go. The project will simulate the different outcomes of the game and computationally show what decisions make you more or less likely to win. The choices that may make you more likely to win include the different decisions regarding purchasing spaces and houses. Additionally, this project would attempt to simulate how long the game would take to play. Monopoly games are often not complete, which is why this research will simulate the amount of time it takes to complete a monopoly game. By simulating the different amounts of players allows you to analyze what makes a game longer or shorter, in addition to what makes a player more likely to win. The decisions in the program will be simulated by different probabilistic distributions in an attempt to discover the outcomes that are most likely to occur and the amount of time it would take to complete a game. In addition to researching how the different time lengths are affected by group, other points of interest are what spaces are landed on the most and what spaces prove to be most profitable.

### 380. Black Scholes Using the Central Limit Theorem

**Anthony Sisti** University of Connecticut

**Rajeshwari Majumdar** University of Connecticut

**Advisor(s):** Maria Gordina, University of Connecticut

The standard method of deriving the Black-Scholes European call option pricing formula involves stochastic differential equations. We provide an alternate derivation using the Lindeberg-Feller central limit theorem under some technical assumptions. This method allows us obtain the Black-Scholes formula using undergraduate probability. Theoretical results are supplemented with market simulations.

### 381. Noise-Induced Stabilization of Hamiltonian Systems

**Daniel Weithers** Carleton College

**Sarah Sparks** Frostburg State University

**Anthony Coniglio** Indiana University, Bloomington

**Advisor(s):** Tiffany Kolba, Valparaiso University

Noise-induced stabilization is the phenomenon in which the addition of randomness to an unstable deterministic system of ordinary differential equations (ODEs) results in a stable system of stochastic differential equations (SDEs). A Hamiltonian system is a two-dimensional system of ODEs defined by a Hamiltonian function, which is constant along each solution curve. With stability defined as global stochastic boundedness, Hamiltonian systems cannot be stabilized by the addition of noise that is constant in space. Therefore we seek to deterministically perturb the Hamiltonian systems in such a way that the qualitative behavior of solutions is preserved, but noise-induced stabilization becomes possible. Our goal is to provide a systematic framework for methods of perturbing the systems and proving noise-induced stabilization.

### 382. Investigation of Generalized Linear Mixed Models with Application to Salary Prediction

**Abigale Wynn** Butler University

**Advisor(s):** Rasitha Jayasekare, Butler University

In linear models when the observations are not independent due to different group structures or hierarchies that exist in data, the applicability of Generalized Linear Models (GLM) is violated. In such cases Generalized Linear Mixed Models (GLMM) extend GLM to allow data that are not independent, by capturing both random and fixed effects of observations. The goal of the project is to apply GLMM to develop a model that would predict salary as a binary response by identifying random effect variables within a data set of diverse individuals. The variables about individuals in the data set include information such as gender, race, sex, native country, occupation, and more. The validity of the model and accuracy of predictions are also discussed.



### **383. Investigating the Role of Random Forest in Classifying Cognitive Domain Functionality**

**John Yannotty** Slippery Rock University of Pennsylvania

**Parker Servello** Slippery Rock University of Pennsylvania

**Warren Geither** Slippery Rock University

**Advisor(s):** Dil Singhabahu, Slippery Rock University

To analyze the effects of ginkgo biloba on cognitive decline, a multi-center study produced neuropsychological (NP) test scores and magnetic resonance imaging (MRI) data for participants aged 75 or older. Various studies have shown correlation between the exercise of a cognitive domain and increased levels of activity in corresponding regions of the brain. Also, current research strongly suggests the existence of a link between the relative size of these regions and the functionality of the corresponding cognitive domain. The main focus of this study is to apply machine learning techniques to the NP scores and MRI measurements to investigate the relationship between the relative size of brain regions and cognitive domain functionality in the context of the progression of Alzheimer's disease. Currently, random forest is the primary method used for discrete classification. In this case, the algorithm generates a set amount of decision trees from a bootstrapped sample in effort to construct a forest that is capable of classifying a patient's cognitive ability. Each node in the tree focuses on a particular prediction variable. Those variables with the highest entropy are selected for the initial levels, as they present the most randomness in the bootstrapped sample. Once completed, variables that indicate cognitive impairment can be identified along with the overall classification performance through the means of a confusion matrix and an out of bag error estimate. In order to understand machine learning and its appropriateness to investigate this data, machine learning was examined through underlying probability and statistical theory.

### **384. Pricing Mortality-Linked Securities: An investigation into the Swiss Re Mortality Bond**

**Kee Shen Yim** University of Wisconsin - Eau Claire

**Yee Wen Heah** University of Wisconsin - Eau Claire

**Advisor(s):** Marie-Claire L Koissi-Kouassi, Actuarial sciences program, University of Wisconsin - Eau Claire

The original Swiss Re mortality bond, issued by the reinsurance agency Swiss Re, is a bond in which the principal is not paid out in case of an extremely high value of a mortality index based on mortality figures of the following five selected countries: the United Kingdom, United States, Germany, Japan, and Canada. Studies have outlined this selection as a possible weakness of the original bond. In this study, we investigated several modifications of the original bond index: (1) first we modified the number of countries selected and diversified their locations; (2) secondly, we incorporated an economics variable into the model. We made a comparative study using R and Excel.

### **385. An Analysis of Sequence Bias Introduced by Two Different DNA Fragmentation Methods**

**Weiqiu You** Gordon College

**Advisor(s):** Jonathan Senning, Gordon College

DNA fragmentation plays an important role in DNA sequencing. However, different fragmentation techniques have different advantages and disadvantages. Acoustic shearing (AS) is known to damage DNA and is not very scalable since DNA must be transferred to special reaction tubes or plates. Enzymatic Fragmentation (EF) typically does not cause damage and fragments faster, but enzymes usually react more quickly on some DNA sequences than others. This can result in a start site bias such that certain sequences are more likely than others to begin some reads. To analyze the best way to fragment DNAs, we first compare the frequencies of sequences that appear in simulated reads from a reference genome to those of the samples split by the AS and EF methods. We next find the correlation between bases at different positions in order to analyze the significance of start site bias. Finally, we look at the depth at different positions of the sequence and the overall coverage to analyze the appropriate number of reads needed to ensure accuracy.

### 386. Detecting Change Points in Climate Records

**Michelle Yu** College of the Holy Cross

**Advisor(s):** Eric Ruggieri, College of the Holy Cross

Climate change is the result of complex interactions between a wide array of climatic variables. Over a long period of time, climatic patterns can exhibit abrupt shifts. These abrupt shifts occurring over relatively short periods of time are known as change points. During these intervals, different climatic variables may undergo dramatic shifts posing serious consequences for many biological and physical systems. In this talk, we discuss a Bayesian algorithm for detecting the exact timing of change points in time series data.

### 387. Data Mining Graduate School Admissions

**Jiahao Zhu** University of Illinois at Urban-Champaign

**Advisor(s):** A J Hildebrand, University of Illinois

In our project we collected and analyzed admissions data from user forums at [mathematicsgre.com](http://mathematicsgre.com) and similar websites. Each record includes complete GRE and GPA scores of the applicant, as well as lists of schools at which the applicant has been accepted or rejected. We analyzed this data using statistical and machine learning techniques, such as multilinear regression, decision tree analysis, and ranking methods. In this presentation, we describe this analysis and its results.

### 388. Investigating diagonal knot grid diagrams

**Jack Arndt** Simpson College

**Malia Jansen** Simpson College

**Payton McBurney** Simpson College

**Advisor(s):** Katherine Vance, Simpson College

In 2003, Ozsváth, Szabó, and Rasmussen introduced the  $\tau$  invariant for knots, and in 2011, Sarkar published a computational shortcut for the  $\tau$  invariant of knots that can be represented by diagonal grid diagrams. We show that all such knots are positive knots, and we conjecture that the set of these diagonal knots is equal to the set of positive braid knots.

### 389. Geometric realization of nested torus links

**Amie Bray** University of Northern Colorado

**Advisor(s):** Rolland Trapp, California State University, San Bernardino

In this poster, we define nested torus links and discuss their homeomorphisms. We then go on to introduce a cell decomposition of nested torus links. Using the cell decomposition and its resulting circle packing we prove that nested torus links can be geometrically realized via regular ideal octahedra in hyperbolic space.

### 390. State Polynomials and Symmetries of 2-bridge knots

**Hoang Cao** The College of New Jersey

**Advisor(s):** Cynthia Curtis, The College of New Jersey

A mathematical knot is an embedding of a circle in 3-dimensional Euclidean space,  $\mathbb{R}^3$ . In this research, we focus on 2-bridge knots, in which there is a projection of the knot to some plane with exactly two maxima and two minima. Knot theorists work to tell knots apart by defining invariants, which are characteristics of knots that do not change when computed using different projections. By using the state polynomial, a knot invariant due to Curtis and Longo which is a generalization of the Alexander polynomial, we can detect symmetries of arbitrary 2-bridge knots. The results obtained here may be a step towards finding more complicated knot symmetries.

### 391. A first approach to a general theory of filtration functors

**Nathaniel Clause** Vanderbilt University

**Jose Angel Sanchez Gomez** University of Guanajuato

**Advisor(s):** Facundo Mémoli, Ohio State University

The so called Čech and Vietoris-Rips simplicial filtrations are designed to capture information about the topological structure of datasets. These two filtrations are the workhorses of the emerging field of topological data analysis. They enjoy stability, and this stability property allows us to estimate the Gromov-Hausdorff distance between the underlying datasets, when represented as finite metric spaces. Invoking the concepts of Gromov's curvature sets and that of valuations we establish a rich novel theoretical framework that includes these two well known filtrations as well as many novel filtrations that capture diverse characteristics present in data sets. We further explore the concept of globality of filtrations functors and use it to provide a classification of the filtration functors that we identify.

### 392. Geometry and Topology of the Curvature Sets of $\mathbb{S}^1$

**Anna Ellison** Massachusetts Institute of Technology

**Peter Eastwood** Brown University

**Advisor(s):** Facundo Memoli, Ohio State University

The  $n$ th curvature set  $K_n(X)$  of compact metric space  $(X, d_X)$  consists of all the  $n \times n$  matrices that one can obtain by restricting the metric  $d_X$  to  $n$ -tuples of points in  $X$  — in a sense, projecting down from an element of the  $n$ -point configuration space to only the distances between its coordinates. Curvature sets were introduced by Gromov in his green book in order to capture curvature-like information in metric spaces that may not admit a smooth structure. It is known that the collection  $\{K_n(X)\}_{n \in \mathbb{N}}$  of all curvature sets of  $X$  determines  $X$  up to isometry. Little is known about the structure of curvature sets in general. In the case of  $\mathbb{S}^1$  with angular distance it is known that  $K_3(\mathbb{S}^1)$  is homeomorphic to a sphere. Furthermore, it is possible to establish that this curvature set is isometric to a regular tetrahedron in  $\mathbb{R}^3$ . As a first result, in our project we provide a simplicial description of  $K_n(\mathbb{S}^1)$  and prove that its Euler characteristic is  $2^{n-2}$ . As a further exploration of the topological properties of these curvature sets, and exploiting their simplicial representation, we computationally explored the homology groups of  $K_n(\mathbb{S}^1)$  for different values of  $n$ . Our computational results suggested the conjecture that  $H_m(K_n(\mathbb{S}^1)) = \mathbb{Z}^{\binom{n-1}{m}} \oplus (\mathbb{Z}/2\mathbb{Z})^{\sum_{i=0}^{n-m} \binom{n-1}{i}}$  for  $m$  even, between 2 and  $n-1$ ;  $\mathbb{Z}$  if  $m=0$ ; and trivial otherwise. By subsequent analyses involving an inductive gluing procedure, we were able to establish the validity of the conjecture.

### 393. Taming the Loch Ness Monster: Symmetries of Infinite Surfaces

**Samuel Freedman** University of Michigan

**Santana Afton** College of William and Mary

**Advisor(s):** Dan Margalit, Georgia Institute of Technology

The mapping class group of a surface is an algebraic object that encodes the symmetries of the surface. Dehn showed that for many surfaces the mapping class group can be generated by certain infinite-order elements called Dehn twists. However, this result does not apply to “big” surfaces that have an infinite number of holes. Patel and Vlamis recently produced the first generating set for an important subgroup of the mapping class group of a big surface. Their generating set is comprised of Dehn twists as well as symmetries unique to big surfaces called handle shifts. Our work focuses on finding algebraic relations between Dehn twists and handle shifts. As a consequence, we show that big mapping class groups are generated by only a subset of their handle shifts. Additionally, we construct elements of arbitrary order for all surfaces of infinite genus. This is joint work with Justin Lanier and Liping Yin.

### 394. On a Generalization of the Bestvina-Brady Construction

**Abhay Goel** Kalamazoo College

**Advisor(s):** Michele Intermont, Kalamazoo College

This work builds on work in [1] and [2] to study the homotopy fibre of a map between polyhedral products. The construction itself generalizes the definition of Bestvina-Brady group. Here we compute the cellular chain complex of the homotopy fibre. We also use properties of polyhedral products to decompose the homotopy fibre in terms of the map which underlies the construction.

[1] Leary and Saadetoglu. “The Cohomology of Bestvina-Brady Groups.”

[2] Grbic, Intermont, Laude, and Vidaurre. "A homotopical generalisation of the Bestvina-Brady Construction." (to appear)

### 395. Knotty Outcomes in Billiard Boards of Hexagonal Tiles

**Tamara Gomez** University of California, Santa Barbara

**Patricia Commins** Carleton College

**John Bush** University of Washington, Bothell

**Advisor(s):** Jennifer McLoud-Mann, University of Washington, Bothell

Square mosaic knots have many applications in algebra, such as modeling quantum states. We continue the work of the previous REU cohort in extending mosaic knot theory to a theory of hexagonal mosaic knots, which are knots and links embedded in a plane tiling of regular hexagons. We define a new knot invariant, the corona number, by restricting the placement of tiles. We establish the corona number for knots of nine or fewer crossings, excluding  $9_{16}$ . We also examine tile patches with a high number of link crossings, which we describe as saturated link diagrams. Considering patches of varying size and shape, we identify the number of components and what knots are produced in these saturated link diagrams, with a particular focus on patches circumscribed by regular and irregular hexagons. We also examine how the class of  $(2, q)$  torus knots can be constructed on a hexagonal mosaic board, and produce an upper limit on the corona number of these knots. Finally, we discuss open questions relating to the saturated link diagrams and bounds on the corona number for torus knots.

### 396. Characterizing a Physical Embedding of a Loop as the Unknot

**Charles Hesketh** California State University, Fullerton

**Advisor(s):** Matt Rathbun, California State University, Fullerton

A knotted loop embedded in space (almost always) gives rise to a (regular) knot diagram when projected onto a plane, but the diagram depends considerably on the direction of the projection. We seek to understand how different these projections can be, even as the loop itself is unchanging. To this end, we investigate knots embedded on spheres, round cylinders, and certain generalized cylinders, and in each case characterize certain combinatorial restrictions on any diagram arising from a projection of such an embedding.

### 397. Is the Mandelbrot set locally connected?

**Naomi Imbre** Fullerton College

**Advisor(s):** Dana Clahane, Fullerton College

The MLC Conjecture states that the Mandelbrot set is locally connected. We give the first completely self-contained description of this problem, assuming only intermediate algebra as a prerequisite. Along the way, as we present the definitions of the Mandelbrot set and local connectedness, we'll rigorously verify examples of these definitions, and we'll exhibit a topological space that is not locally connected.

### 398. Vietoris-Rips Complexes of Regular Polygons

**Adam Jaffe** Stanford University

**Advisor(s):** Henry Adams, Colorado State University

Persistent homology has emerged as a novel tool for data analysis in the past two decades. However, there are still very few non-convex shapes or manifolds whose persistent homology barcodes (say of the Vietoris–Rips complex) are fully known. Towards this direction, we provide a near-complete characterization of the homotopy types of Vietoris–Rips complexes of the boundary of any regular polygon in the plane. Indeed, for  $P_n$  the boundary of a regular polygon with  $n$  sides, we describe the homotopy types and persistent homology of the Vietoris–Rips complexes of  $P_n$  up to scale  $r_n$ , where  $r_n$  approaches the diameter of  $P_n$  as  $n \rightarrow \infty$ . Surprisingly, these homotopy types include spheres of all dimensions (as  $n \rightarrow \infty$ ) and wedge-sums thereof. Roughly speaking, the number of  $2\ell$ -dimensional spheres in such a wedge sum is linked to the number of equilateral (but not necessarily equiangular) stars with  $2\ell + 1$  vertices that can be inscribed in  $P_n$ . We furthermore show that the Vietoris–Rips complex of an arbitrarily dense subset of  $P_n$  need not be homotopy equivalent to the Vietoris–Rips complex of  $P_n$  itself. As our main tool, we employ the recently-developed theory of cyclic graphs and winding fractions.

### 399. Gaussian Primes and Great Lakes

**Anchala Krishnan** University of Washington

**Rae Helmreich** Wheaton College

**Nathan Schmitz** University of Wisconsin - Stevens Point

**Advisor(s):** John Meier, Lafayette College

The Gaussian primes form an interesting and complicated point cloud inside of the complex plane. Vardi's probabilistic model for the distribution of Gaussian primes is patterned after Cramer's famous model for the integer primes (*Experimental Mathematics*, 7, 1998). Vardi's model provides evidence for Vardi's conjecture, describing the size of the largest prime free zone inside the ball of radius  $R$  about the origin. Using Mathematica, we located large, prime free zones ranging from a radius of  $\sqrt{17}$  to  $\sqrt{65}$ , within the ball of radius  $R = 10,000$  centered at the origin. Our results show that, out to  $R = 10,000$ , Vardi's conjecture overestimates what actually occurs in the Gaussian primes. We also compare patterns in the persistent homology of the Gaussian primes versus the patterns found in Vardi's probabilistic model for the primes. Again, the results suggest that Vardi's model is not closely matching the geometry of the Gaussian primes.

### 400. Adaptive Mapper

**Alisa Leshchenko** City University of New York

**Advisor(s):** Mikael Vejdemo-Johansson, CUNY College of Staten Island

We propose an improvement to the highly successful Mapper algorithm in Topological Data Analysis. Mapper creates a simplicial model of the original data by clustering the fibers of a chosen filter function; the goal is to retain useful topological invariants encoded in the data, but in a structure with only trivial local topological complexity. Fidelity of this representation is guaranteed by the Nerve Lemma in algebraic topology, which requires each cluster in each fiber to be acyclic — devoid of higher-dimensional topological structure. If this condition fails, the resultant model will be a lossy projection of the “true” topological structure, and may hide interesting structure present in the data. Our approach uses persistent homology to dynamically refine the model until the conditions for the Nerve lemma are met. This is achieved by checking each cluster and each intersection of clusters for acyclicity, and using local transformations to split clusters that have non-trivial topology. In this presentation, we will give a detailed description of our method and demonstrate its performance on some illustrative examples.

### 401. Shape-Matching: The Case of Zeolites

**Daniel Lopez** University of California - Berkeley

**Gabrielle Ferra** Brown University

**Advisor(s):** Facundo Memoli, Ohio State University

Zeolites are microporous chemical compounds that are important in a variety of industrial applications. There are about 200 different known types of naturally occurring zeolites, but it is now theoretically possible to synthesize over 10 million new zeolites with unknown properties. Our research is focused on the development of a systematic process to predict the efficiency of a theoretical new zeolite when adsorbing carbon dioxide (CO<sub>2</sub>). In order to conduct our research we used new techniques in Topological Data Analysis (TDA) to analyze and compare the CO<sub>2</sub> adsorption rates of 153 of the 200 extant zeolites. In order to quantify the “difference in shape” between the 153 zeolites, we used the newly developed tool of Persistent Homology with its natural metric “the bottleneck distance” to detect different Persistent Homology features. In particular, this method defines a metric structure on the collection of all extant zeolites. Further analysis on this metric space includes linear regression, linear discriminant analysis, and statistical summaries to determine which topological features showed more prominent correlations between shape and function. Our results indicate that there is a correlation between Persistent Homology features and efficiencies of zeolites when uptaking CO<sub>2</sub>.

### 402. Withdrawn

### 403. Hyperbolicity of Links in Thickened Surfaces

**Daishiro Nishida** Williams College

**Beatrix Haddock** Williams College

**Braeden Reinoso** Haverford College

**Luya Wang** Princeton University

**Advisor(s):** Colin Adams, Williams College

W. Menasco (1984) showed that a non-split, prime link that is not a 2-braid is hyperbolic in  $S^3$  if it has an alternating projection. C. Adams (1994) showed a similar result for toroidally alternating links in  $S^3$  and C. Hayashi (1995) generalized this to links with projections on closed orientable surfaces of positive genus in  $S^3$ . We prove a similar result for links in thickened surfaces  $S \times I$ . We define a link to be fully alternating if it has an alternating projection from  $S \times I$  to  $S$  where the interior of every complementary region is an open disk. We show that a prime, fully alternating link in  $S \times I$  is hyperbolic. Similar to Menasco, we also give an easy way to determine primeness of fully alternating links in  $S \times I$ . Further, we extend our result to show that a prime link with a fully alternating projection to an essential surface embedded in an orientable, hyperbolic 3-manifold has a hyperbolic complement. By associating these projections to tilings of the Euclidean plane, we show how hyperbolicity of links in  $S \times I$  can be used to classify such tilings.

### 404. Proof to Suzuki's Epimorphism Number

**Joshua Ocana** Loyola Marymount University

**Advisor(s):** Patrick Shanahan, Loyola Marymount University

Take an extremely thin piece of string, tangle it up and then connect the ends to form a "knotted" loop in space. Informally, this is how we define a knot in mathematics, and the study of these objects is called knot theory. While knot theory has applications to chemistry and physics, this research project aims to further understand contemporary theoretical results regarding a specific relationship between a class of knots called 2-bridge knots. In 2017, the mathematician Masaaki Suzuki defined the epimorphism number,  $EK(n)$ , as the maximum number of 2-bridge knots a 2-bridge knot with  $n$  crossings can map onto via a construction of Ohtsuki, Riley, and Sakuma. In his research article, Suzuki established several theoretical results about  $EK(n)$  and used a computer program to compute  $EK(n)$  for  $3 \leq n \leq 30$ . He also posed the question of whether  $EK(n) \geq 3$  for  $31 \leq n \leq 44$ . This project aims to answer said question by proving that  $EK(n) \leq 2$  for  $31 \leq n \leq 44$ . We establish this result using techniques developed by Garrabrant, Hoste, and Shanahan associated to parsings of a continued fraction of the 2-bridge knot. Ultimately, the results of this research introduce a method to theoretically bound  $EK(n)$  that do not rely on an exhaustive computation by computer program. Keywords: knot, 2-bridge knot, crossings, epimorphism number, parsing.

### 405. Ordering problem for simple loop and triangulation of surface, and quantum Teichmüller theory

**Do Eun Oh** EWHA Womans University

**So Young Cho** EWHA Womans University

**Hyuna Kim** EWHA Womans University

**Advisor(s):** Hyun Kyu Kim, EWHA Womans University

Let  $S$  be an oriented surface with punctures, and let  $T$  be a triangulation of  $S$ , with edges up to homotopy and vertices being the punctures. In quantum Teichmüller theory, for each edge  $e$  of  $T$  there is a special quantum coordinate function  $\hat{X}_e$ , and to each integral lamination consisting of non-intersecting closed curves on  $S$  with integer weights is associated a special element expressed as Laurent polynomial in  $\hat{X}_e$  with coefficients being Laurent polynomials in quantum parameter  $q$  with integer coefficients. We notice that, in order to show that these integer coefficients are all positive, it is enough to solve the following topological and combinatorial problem. Consider a loop in  $S$ , which is divided into segments by the edges of ideal triangulation. Can we find an ordering on the set of loop segments in each triangle, such that these orderings are compatible at each edge of triangulation? We solve this problem in complete generality, using only elementary combinatorial and topological ideas. In our proof we also introduce a graph associated to a triangulation and a closed curve, which is interesting in its own right.

**406. Polygonal Wheel Graphs and Links**

**Dawn Paukner** University of Wisconsin - Eau Claire  
**Chase Siegfried** University of Wisconsin - Eau Claire  
**Geoffrey Glover** University of Wisconsin - Eau Claire  
**Advisor(s):** Carolyn Otto, University of Wisconsin - Eau Claire

Our research aims to discover more relationships between graph theory and knot theory. We define and use what we call polygonal wheel graphs to create a unique class of alternating knots and links. We were able to discover a relationship between the polygons in our wheel graphs and different knot and link invariants. More specifically, our poster will discuss the connection between these graphs and the determinant, colorability, component number, and genus of the links created.

**407. Withdrawn****408. Mosaic Number of Torus Knots**

**Susanna Rempel** California Baptist University  
**Advisor(s):** Lisa Hernandez, California Baptist University

Mosaic knots are knots that can be represented by combining eight tiles with vertical lines, horizontal lines, corners, or crossings. Each tile will have only one vertical line and one horizontal line. These tiles are organized in  $n \times n$  squares. The mosaic number of a knot,  $K$ , is the smallest square  $n \times n$  that a particular knot can fit on and is denoted  $m(K) = n$ . Torus knots are knots that can be wrapped around the surface of a torus. Torus knots are identified as  $(p, q)$  torus knots where  $p$  and  $q$  are coprime and are the number of times the knot wraps around the torus along the meridian and longitude. The goal is to discover the mosaic numbers of torus knots  $(2, q)$ . Since 2 and  $q$  are coprime,  $q$  is of the form  $2n + 1$  for a natural number  $n$ . Our conjecture is that the mosaic number of a  $(2, 2n + 1)$  torus knot is  $n + 3$ .

**409. Withdrawn****410. Relating Topological Quantum Field Theories and the Cutting and Pasting of Manifolds**

**Matthew Schoenbauer** University of Notre Dame  
**Advisor(s):** Carmen Rovi, Indiana University, Bloomington

In this presentation we shall be concerned with a relation between topological quantum field theories (TQFTs) and cut and paste invariants. These cut and paste invariants, or  $SK$  invariants, are functions on the set of smooth manifolds that are invariant under the cutting and pasting operation. There are also weaker invariants, called  $SKK$  invariants, whose values on manifolds depend only on the gluing diffeomorphisms. Here we investigate a surprisingly natural group homomorphism between the group of invertible TQFTs and the group of  $SKK$  invariants and describe how these groups fit into an exact sequence. We conclude in particular that all positive real-valued  $SKK$  invariants can be realized as restrictions of invertible TQFTs.

**411. Mapping Class Groups, Covering Spaces, and Symplectic Matrices**

**Laura Stordy** Agnes Scott College  
**Ziyi Zhou** Georgia Institute of Technology  
**Advisor(s):** Becca Winarski, University of Wisconsin - Milwaukee

Given a 2-genus 3-fold covering space of a sphere with four branched points, we prove that the matrix representation of the symmetric mapping class group is equal to the normalizer of the matrix representation of the Deck group in the symplectic group. This answers an analogue of a question posed by McMullen.

**412. Topological Modeling of Force Networks in Granular Material**

**Allison Sullivan** College of the Holy Cross  
**Advisor(s):** David Damiano, College of the Holy Cross

The aggregate properties of granular materials are determined by the network of forces between particles. These networks are prominent in a number of fields, including pharmaceutical and agricultural industries. Force networks

are, however, not well understood. The random organization of these networks makes it difficult to derive properties of a granular material from the local structure of its force network. Here, the method of swatch and cloth is instrumental. Swatch and cloth is a new and promising technique developed by Schweinhart et. al. It can be used to accurately describe a force network by analyzing the statistics of the local structure of the network's adjacency graph. We used swatch and cloth to analyze and compare the force networks of planar simulations of granular material. In addition, we analyzed the related community structure of these networks.

#### 413. higher order invariants via quandles

**Michael Vaughan** University of Wisconsin - Eau Claire

**Grace Odegaard** University of Wisconsin - Eau Claire

**Advisor(s):** Christopher Davis, University of Wisconsin - Eau Claire

In 2006 Harrell and Nelson gave a formulation for the linking number of a link in term of the number of colorings by a particular quandle. Thus ideas coming from classical colorability may generalize to link invariants. There is a higher order notion of linking called Milnor's triple linking number. We present in this poster a new quandle which recovers the classical linking number. This quandle has the advantage that it naturally generalizes to a new quandle recovering the triple linking number. Preliminary results about higher order notions of colorability will also be presented.

#### 414. Cohomology and boundaries of relatively hyperbolic groups

**Oliver Wang** Cornell University

**Advisor(s):** Jason Manning, Cornell University

Hyperbolic groups are often studied by their Cayley graphs which are not well-defined up to homotopy. These Cayley graphs can be compactified by attaching a boundary which turns out to be well-defined up to homeomorphism. Thus, one might hope for a relation between a hyperbolic group and homotopy invariant properties of its boundary. Indeed, Bestvina and Mess proved an isomorphism between group cohomology and Cech cohomology of the boundary. I will discuss the analogue of this theorem for relatively hyperbolic groups and some of its consequences.

#### 415. Dynamics of Surface Homeomorphisms

**Yandi Wu** University of California - Berkeley

**Advisor(s):** Dan Margalit, Georgia Institute of Technology

In the 1970s, the Fields medalist William Thurston gave a classification of surface homeomorphisms into three types: periodic, reducible, or pseudo-Anosov. D. Margalit, B. Strenner, and O. Yurttas recently discovered a quadratic time solution for the problem of classifying homeomorphisms. In this poster, I reduce the algorithm, which iterates homeomorphisms on curves, to a linear algebra problem using homotopy and graphs. I also present the challenges and basic building blocks for implementing this algorithm. This is joint work with Balazs Strenner, Ian Katz and Yihan Zhou.

#### 416. Delay embeddings and topological time series analysis

**Boyan Xu** University of Illinois at Urbana-Champaign

**Advisor(s):** Jose Perea, Michigan State University

In a recent paper, J. Perea and C. Tralie introduce a method of detecting quasiperiodic behavior in video data using persistent homology. Their technique is motivated by Takens delay embedding theorem: generically, one can reconstruct a dynamical system by a uniform finite sampling of an observation function along trajectories. In our work we characterize precisely quasiperiodic functions with delay embedding yielding Tori and Klein bottles. Our methods involve studying infinitesimal behavior of observation functions along integral curves and generalize to arbitrary compact manifolds.

#### 417. On Computing Slice Genus of Non-alternating Prime Knots

**Lyujiangnan Ye** Colorado College

**Hanbo Shao** Colorado College

**Advisor(s):** Kathryn Bryant, NYC Data Science Academy

Knot genus, in both the 3- and 4-dimensional settings, is a well-studied knot invariant. In this project, we calculate the smooth slice genus of the last remaining non-alternating prime knots of twelve or fewer crossings for which this



invariant is previously unknown — 11 in all. We do this by performing band moves — additions and deletions — on a knot  $K$  to produce a torus cobordism in 4-ball with another knot  $J$ , and the slice genera of two knots are merely differed by one. Regarding different types of knots, we applied various methods to deduce the slice genus of  $K$  via analysis of  $J$ .

#### **418. Belted Sum Decomposition of Fully Augmented Links**

**Cameron Ziegler** SUNY at Geneseo

**Dean Spyropoulos** Vassar College

**Porter Morgan** Kenyon College

**Advisor(s):** Rolland Trapp, California State University, San Bernardino

Purcell and Adams have introduced notions of nerves and belted sums for fully augmented links (FALs). We prove that all nerves corresponding to FALs are made up of 3-cycles taking 1 of 3 forms. We show that nerve decomposition along one of these forms corresponds to cutting along a specific pair of thrice-punctured spheres in the nerve's FAL. Furthermore, this corresponds to a belt-sum decomposition such that the decomposition is made up of FALs. Finally, we show that decomposition can continue until reaching a finite set of prime FALs, and that this decomposition is unique for a given FAL.

#### **419. Wiener Index and Randic Index for Skeletal Graphs**

**Anna Rossini** Creighton University

**Advisor(s):** Margaret Doig, Creighton University

We present results in mathematical chemistry calculating and comparing the Wiener index, which is a topological index, and Randic index, which is a connectivity index for several families of skeletal graphs of organic molecules, including saturated hydrocarbons and related molecules.

#### **420. Mathematical and Logical Approach to Introduction of Proof in Electrical Engineering**

**Cinthia Calvo Martinez** University of Texas - Rio Grande Valley

**Advisor(s):** Dambaru Bhatta, University of Texas - Rio Grande Valley

Logic is the structure which all proofs are built. Proof is the principal demonstration without conjectures when we want to give evidence to trust or distrust. In order to trust any statement, we must give evidence with arguments for validation. Logical operators are the proper symbolic interpretation and connection that exists in mathematics and a useful tool that will use to prove statements. To make proofs easier to comprehend, I will introduce the physical phenomena of electricity in Electrical Engineering as an example to examine a circuit with pure materials and prove if there is a flow of electrical power or not. Several mathematical techniques as the truth tables, logical equivalences, Venn diagrams and different types of conditional statements will be a very important stage during the analysis. The goal is to illustrate the way of connecting logic and follow the right direction to construct a mathematical representation by using the proper techniques to give the assertion an essential evidence that a specific statement is true.

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