

References

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Flaws, Fallacies, and Flimflam: What Is \$100 in Three Years Worth Right Now?

The value of money depends on when you have it. A \$100 bill in my sweaty palm now is worth more to me than the promise of same three years in the future. What is money in the future worth now? Chapter 21 of the popular mathematics-literacy textbook *For All Practical Purposes* (FAPP, eighth edition) uses the concept of *present value* to answer this question, illustrating it with two different scenarios.

First assume the interest rate on *savings* is constant: 4%, for example. This means that \$100.00 yields $100(1.04)^3 = \$112.49$ after three years. In this case, FAPP says, the *present value* of the promise of \$100.00 three years from now is $\$100/1.04^3 = \88.90 , because \$88.90 now will generate \$100 three years later.

Second suppose the rate of *inflation* is constant: say at 3%. Then every year prices go up 3%, so that an item that costs \$100 now will cost $100(1.03)^3 = \$109.27$ after three years. In this case, FAPP says, the *present value* of \$100 three years from now is $100/1.03^3 = \$91.51$, because in three years \$100 will buy what today costs only \$91.51.

An end-of-chapter problem (number 53) asks for the present value when there is both a 4% savings rate *and* 3% inflation rate. It might seem, since both interest and inflation rates reduce the present value, that the present value of \$100 in three years should be $100/1.03^3/1.04^3 = \$81.36$. Three years apparently decreases the value by almost 20 percent!

On the other hand, if I have \$88.90 now, I bank it and withdraw \$100 after three years. When I spend it, I buy stuff costing \$91.51 at today's prices. So it seems that \$88.90 now is worth \$91.15 in three years. At this ratio, \$100 in three years has a present value of \$97.14. Strange. It seems inflation has actually *increased* the present value of \$100 (since it was only \$88.90 without inflation)!

What is the actual present value?

Neither! FAPP's two examples, like apples and oranges, are incomparable. If there's a 4% interest rate, then no matter *what* the rate of inflation it's *still* true that \$88.90 invested now will yield \$100 in three years (even though that \$100 won't be worth what \$100 is now!) And, no matter what the interest rate is, three years of 3% inflation will still reduce the value of \$100 to \$91.51 (in today's prices). The first present value is an *opportunity cost*. It is a fair price now for the opportunity to have \$100 in three years. The second present value is *purchasing power*. It tells what \$100 in three years can purchase at today's prices.

The two notions of present value are quite separate. There's really no meaningful way to combine them taking both inflation and interest rate into account.

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