Undergraduate Preparation for PhD Programs in Mathematics[1]

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For majors intending to pursue doctoral work in the mathematical sciences, faculty must communicate the fact that facility with the tools of "epsilon-delta" calculus and abstract linear algebra is fundamental, as is some knowledge of programming. Students should also be both comfortable with and experienced at writing and reading mathematical proofs.

Coursework Background

Though doctoral programs in pure mathematics vary widely, nearly all require their students to take graduate courses in analysis and algebra and to pass demanding qualifying exams based on these courses. These two areas are at the core of the continuous and discrete aspects of mathematics; cf. *Contrasting but complementary points of view* in the Overview to this *Guide*. Departments that prepare students for doctoral work in mathematics should ensure that their students arrive in graduate school prepared to take such courses. For example, the departmental website at the University of Illinois at Urbana–Champaign[2] lists basic topics with which they assume familiarity. Expectations for real analysis and abstract algebra are as follows:

Real analysis: Completeness properties of the real number system; basic topological properties of n–dimensional space; convergence of numerical sequences and series of functions;

[1] Recommendations (and some language) in this document build on goals laid out in the 1990 and the 2004 CUPM Curriculum Guides.

[2] http://www.math.illinois.edu/GraduateProgram/undergrad-prep.html. This reference should not be interpreted as an endorsement of these topics as the proper undergraduate preparation for a doctoral program in mathematics. It is simply an example of what one respected graduate program expects.

properties of continuous functions; and basic theorems concerning differentiation and Riemann integration.

Abstract algebra: Modular arithmetic, permutations, group theory through the isomorphism theorems, ring theory through the notions of prime and maximal ideals; additional topics such as unique factorization domains and classification of groups of small order.

Some doctoral programs require additional topics in real analysis and abstract algebra or specify that preparation for their programs requires an undergraduate analysis class at the level of Rudin's *Principles of Mathematical Analysis* and an undergraduate algebra class at the level of Herstein's *Topics in Algebra*; others will allow entering students to start with a preparatory course at the level of Rudin or Herstein. Many programs also ask that students see specific areas beyond abstract algebra and real analysis. Again for example, the departmental website at the University of Illinois at Urbana–Champaign states that incoming PhD students should be familiar with the areas of real analysis, abstract algebra, complex analysis, and abstract linear algebra: and at least two of the following six areas: geometry and topology, number theory, differential equations, logic, combinatorics, statistics and probability. See also the list of courses and books in section 2.1, "How to Prepare for Graduate School" of [1] and the section "The Typical educational Path in the Mathematical Sciences Needs Adjustments" on future needs in mathematical training in [2].

Reading and Writing Proofs

Preparing students for doctoral work in mathematics requires more than exposing them to topics in real analysis and abstract algebra. Students need also to be able to read and critique proofs; cf. *Proofs* in the Overview to this *Guide*. This skill includes the ability to determine how assumptions are used and to find counterexamples when any of the hypotheses are weakened. Students need familiarity with the common techniques employed to prove results in analysis and in algebra, and they should be able to use these techniques to prove simple theorems they have not seen before. Above all, students heading into doctoral programs in mathematics need to be able to read rigorous textbooks written in the languages of analysis and algebra. This is a skill that departments should ensure that their students learn; cf. *Effective thinking skills* in the Overview to this *Guide*.

Additional Preparation

Participation in an REU (Research Experiences for Undergraduates) program can be a useful supplemental preparation that introduces students to the excitement and hard work involved in doing mathematical research. Such experience could be especially important for students from colleges where coursework opportunities are limited. Related beyond-the-classroom opportunities include other summer programs aimed at students considering doctoral study in mathematics, semester-long programs for visiting students (like the "Mathematics Advanced Study Semester" at Pennsylvania State University or the "Junior Program of the Center for Women in Mathematics" at Smith College) and semester abroad programs (like "Budapest Semesters in Mathematics" or "Math in Moscow"). Departments should be aware of these programs and direct promising students toward them.

Other experiences that challenge students beyond the usual coursework—undergraduate seminars, capstone courses, independent study or research within a student's department, or undergraduate thesis work—are also highly recommendable for graduate school preparation; cf. *High impact experiences* in the Overview to this *Guide*. Such experience may also encourage talented students who may not yet aspire to earn the doctorate degree.

Departments that cannot afford to provide the full preparation needed for the doctoral program to which a student aspires should consider providing alternative options, such as reading courses, on-line courses, and cross-registration at a nearby institution. These departments might also recommend a transitional fifth year of study, a post-baccalaureate program, or even transfer to another institution. Similar recommendations should be made for students who decide late in their undergraduate careers to pursue a doctoral program. For some students a master's degree in mathematics is an attractive option; it permits them to "try out" graduate school and to improve their preparation for doctoral study if they choose to continue. However, funding opportunities may be more limited for master's programs than for doctoral programs, and some doctoral programs do not admit students who are aiming only at master's degrees.

Advising Students

Some programs offer a wide variety of qualifying topics and students may have some choice of which qualifying examinations to take. Students should be advised to consider a range of graduate programs since departments have varying admission requirements and policies for allowing students to catch up. Departments should advise students accordingly, and appraise realistically how much preparatory work would be needed before taking a particular graduate course. Students should be urged to get to know some of their mathematics professors. Letters of recommendation play a huge role in graduate program admissions, and the strongest letters come from faculty who can give admissions committees an in-depth portrait of the candidate. Students should also be advised to take the subject GRE[3] in mathematics and to maintain a minimum GPA as these are admission requirements in many departments. See [3] for views on the use of GRE and GPA in graduate school admissions.

References

[1] Steven G. Krantz. *A Mathematician's Survival Guide: Graduate School and Early Career Development*. American Mathematical Society, 2003. ISBN: 9780821834558.

[2] Committee on the Mathematical Sciences in 2025; Board on Mathematical Sciences, Their Applications; Division on Engineering, and Physical Sciences; National Research Council. *The Mathematical Sciences in 2025*. The National Academies Press, 2013. ISBN: 9780309284578. URL: `http://www.nap.edu/openbook.php?record_id=15269`.

[3] Julie R. Posselt. "Toward Inclusive Excellence in Graduate Education: Constructing Merit and Diversity in PhD Admissions". In: *American Journal of Education* 120 (2014), pp. 481–514. DOI: `10.1086/676910`.

[3] This reference should not be interpreted as an endorsement by this report's authors of the practice of requiring standardized exams in graduate school admissions. It reflects the requirements in many selective doctoral programs.